KASR TARTIBLI HOSILA QATNASHGAN IKKINCHI TARTIBLI O‘ZGARUVCHI KOEFFITSIYENTLI ODDIY DIFFERENSIAL TENGLAMA UCHUN BITSADZE – SAMARSKIY TIPIDAGI MASALA

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Annotatsiya: Ushbu maqolada o‘zgaruvchi koeffitsiyentli, kasr tartibli hosila qatnashgan, bir jinsli bo‘lmagan oddiy differensial tenglama uchun Bitsadze-Samarskiy tipidagi masala o‘rganilgan, ushbu masalaning yechimini mavjud va yagonali alohida, alohida isbotlangan.

Kalit so‘zlar: oddiy differensial tenglama, kasr tartibli hosila, Bitsadze-Samarskiy tipidagi masala, o‘zgaruvchi koyfisentli differensial tenglama.

ЗАДАЧА ТИПА БИЦАДЗЕ-САМАРСКОГО ДЛЯ ОБЫКНОВЕННОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ С ПЕРЕМЕННЫМ КОЭФФИЦИЕНТОМ ВТОРОГО ПОРЯДКА С ДРОБНОЙ ПРОИЗВОДНОЙ

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Аннотация: В данной статье исследуется задача типа бицадзе-самарского для обыкновенного дифференциального уравнения с переменным коэффициентом второго порядка с дробной производной, отдельно доказывается существование и единственность решение этой задачи.

Ключевые слова: обыкновенная дифференциальная уравнения, дробная производная, задача типа Бицадзе-Самарского, дифференциальное уравнение с переменными коэффициентами.

BITSADZE-SAMARSKY TYPE PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION WITH SECOND-ORDER VARIABLE COEFFICIENT WITH FRACTIONAL DERIVATIVE

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Annotation: In this article, the problem of Bitsadze-Samarsky type for a non-homogeneous ordinary differential equation with fractional derivative is considered, and the problem's solution's existence and uniqueness one by one proved.

Key words: ordinary differential equation, fractional derivative, Bitsadze-Samarsky type problem, differential equation with variable coefficient.

KIRISH.

Hozirgi kunda kasr tartibli hosila qatnashgan differensial tenglamalarni o‘rganishga bo‘lgan qiziqish ortib bormoqda, chunki bu tenglamalar biologiya, fizika, mexanika va boshqa sohalarda keng qo‘llanilmoqda, shu nuqtaiy nazardan ushbu masalani o‘rganish muhim hisoblanadi.

ADABIYOTLAR TAHLILI:

Самко С.Г., Килбас А.А., Маричев О.И. lar tomonidan yozilgan [1] kitobda kasr tartibli hosila va uning turlari, xossalari haqida ma‘lumotlar berilgan.

Нахушев A.M. tomonidan yozilgan [2] adabiyotda Riemann–Liouville, Caputo ma’nosidagi kasr tartibli hosila qatnashgan xususuy hosilali va oddiy differensial tenglamalar uchun ba‘zi chgaraviy masalalar o‘rganilgan va Riemann–Liouville ma‘nosidagi kasr tartibli hosila uchun maksimum va minimumlik shartlari keltirilib isbotlangan. O‘rinov A.K.tomonidan yozilgan [3] adabiyotda oddiy differensial va integro-differensial tenglamalar uchun chegaraviy masalalar o‘rganilgan.

TADQIQOT METADOLOGIYASI.

Ushbu maqolada masalaning yagonaligini isbotlashda maksimum prinsipidan foydalanilgan, shuningdek masala yechimining mavjudligini isbotlashda masalaning yechimini Fredgolm ikkinchi tur integral tenglamasiga keltirilgan va Fredgolm alternativasidan foydalanib isbotlangan.

TAHLILLAR VA NATIJALAR.

Ushbu tenglamani o‘rganaylik:

$$y''(x) + p_0(x)y'(x) + \sum_{j=1}^{m} p_j(x)D_{xb}^{\alpha_j}(\omega_j(x)y(x)) + p_{m+1}(x)y(x) = f(x), x\in(a,b). \quad (1)$$

Bu yerda α_j, a, b- oldindan berilgan haqiqiy sonlar, $0<\alpha_j<1$, $\alpha_j\neq\alpha_{j+1}, a<b,$

hamda $p_0(x),\ p_j(x), \omega_j(x),\ p_{m+1}(x),\ f(x)$ oldindan berilgan haqiqiy funksiyalar va $j=1,2,3,...,m,\ m\in\mathrm{N}$, shuningdek

$$D_{xb}^{\alpha}\varphi(x) = -\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_x^b \frac{\varphi(t)\,dt}{(t-x)^{\alpha}},\ 0<\alpha<1$$

Riemann–Liouville ma‘nosidagi kasr tartibli hosila [1]

Masala: (1) tenglamaning $[a,b]$ oraliqda uzluksiz va

$$y(a) = \sum_{i=1}^{n} q_i y(\xi_i) + k_0, \qquad y(b)=k_1 \quad (2)$$

chegaraviy shartlarni qanoatlantiruvchi yechimini toping.

Bu yerda q_i, k_0, k_1, ξ_i - oldindan berilgan haqiqiy sonlar, $a < \xi_i < \xi_{i+1} < b$, $i = 1,2,3,...,n,\ n \in \mathrm{N}$.

1-Teorema: Agar $p_0(x)$, $p_j(x)$, $p_{m+1}(x)$, $\omega_j(x) \in C[a,b]$ bo‘lib $p_j(x) \le 0$, $p_{m+1}(x) \le 0$, $\forall x \in (a,b)$ tengsizliklar o‘rinli, hamda $\omega_j(x)\, y(x)$ funksiylar jufti $\alpha_j < \beta_j$, $j = 1,2,3,...,m$ ko‘rsatkich bilan Gyolder shartini qanoatlantiruvchi $[a,b]$ oraliqda o‘smaydigan musbat funksiyalar bo‘lsa, u holda

$$y''(x) + p_0(x)\, y'(x) + \sum_{j=1}^{m} p_j(x)\, D_{xb}^{\alpha_j}\left(\omega_j(x)\, y(x)\right) + p_{m+1}(x)\, y(x) = 0 \tag{3}$$

differensial tenglamaning yechimi (a,b) oraliqda musbat maksimum va manfiy minimumga erishmaydi.

Isbot: Teskarisidan faraz qilaylik, ya‘ni (3) tenglamaning yechimi $y(x)$ Bo‘lib, u $x_0 \in (a,b)$ nuqtada musbat maksimum (manfiy minimum) ga ega bo‘lsin deylik. U holda teorema shartidan

$$y''(x_0) < 0\ (>0),\ y'(x_0) = 0,\ y(x_0) > 0\ (<0)$$

$$D_{xb}^{\alpha_j}\left(\omega_j(x)\, y(x)\right)\Big|_{x=x_0} ¿\, 0\ (<0) \qquad j = 1,2,3,\ldots,m$$

ekanligi kelib chiqadi [3].

(3) tenglamani quyidagi hollarda ko‘raylik:

a) $p_j(x) = 0$ va $p_{m+1} = 0$ bo‘lsa (3) tenglikdan quyidagi natija hosil bo‘ladi;

$$y''(x)\big|_{x=x_0} < 0\ (>0),$$

b) $p_j(x) < 0$ va $p_{m+1} = 0$ bo‘lsa (3) tenglikdan quyidagi natija hosil bo‘ladi;

$$\left[y''(x)+\sum_{j=1}^{m} p_j(x) D_{xb}^{\alpha_j}(\omega_j(x)y(x))\right]\Big|_{x=x_0} ¿0(>0)$$
,

c) $p_j(x)=0$ va $p_{m+1}<0$ bo‘lsa (3) tenglikdan quyidagi natija hosil bo‘ladi;

$$y''(x)+p_{m+1}(x)y(x)\big|_{x=x_0}<0(>0)$$
,

d) $p_j(x)<0$ va $p_{m+1}<0$ bo‘lsa (3) tenglikdan quyidagi natija hosil bo‘ladi;

$$\left[y''(x)+p_0(x)y'(x)+\sum_{j=1}^{m} p_j(x) D_{xb}^{\alpha_j}(\omega_j(x)y(x))+p_{m+1}(x)y(x)\right]_{x=x_0} ¿0(>0)$$
,

bu esa (3) tenglikka zid. Demak, farazimiz noto‘g‘ri ekan. Teorema isbotlandi.

2 - Teorema. Agar 1-teorema shartlari bajarilib, $\sum_{i=1}^{n}|q_i|\leq 1$ tengsizlik o‘rinli bo‘lsa {(1),(2)} masala bittadan ortiq yechimga ega bo‘lmaydi.

Isbot: Teoremani isbotlash uchun (3) tenglamaning

$$y(a)=\sum_{i=1}^{n} q_i y(\xi_i), \qquad y(b)=0 \qquad (4)$$

shartlarini qanoatlantiruvchi yechimi faqat $y\equiv 0$ ekanligini isbotlash yetarli.

Teskarisidan faraz qilamiz, {(1),(2)} masala qandaydir trivial bolmagan uzluksiz $y_0(x)$ $x\in[a,b]$ yechimga ega bo‘lsin.

$y_0(x)\in C[a,b]$ bo‘lgani uchun Veyershtrassning 2- teoremasiga asosan, $[a,b]$oraliqda shunday x_0 son mavjud bo‘ladiki,

$$\sup_{[a,b]}|y_0(x)|=|y_0(x_0)|>0$$

munosabt o‘rinli bo‘ladi. 1-teoremaga asosan, $y_0(x)$ funksiya (a,b) oralida musbat maksimum va manfiy minimumga erishmaydi. Buni e‘tiborga olsak,

$y(b)=0$ bo‘lgani uchun $y_0(x)$ funksiyaning $\sup\limits_{[a,b]}|y_0(x)|$ qiymatiga $x=a$ nuqtada erishadi. Demak $\forall\xi\in(a,b)$ bo‘lgani uchun $|y(\xi)|<|y(a)|$ tengsizlik o‘rinli bo‘ladi. Buni va $\sum\limits_{i=1}^{n}|q_i|\le 1$ tengsizlikni e‘tiborga olsak,

$$|y(a)|=\left|\sum_{i=1}^{n}q_i y(\xi_i)\right|<\sum_{i=1}^{n}|q_i y(\xi_i)|<|y(a)|\sum_{i=1}^{n}|q_i|\le|y(a)|$$

tengsizlikka ega bo‘lamiz. Bu qarama–qarshilik farazimiz noto‘g‘ri ekanini ko‘rsatadi. Demak $y_0\equiv 0,\, x\in[a,b]$. Bundan 2 - teorema o‘z tasdig‘ini topdi.

3- Teorema. Agar 1-teorema shartlari bajarilgan bo‘lsa va

$f(x)\in C[a,b],\, p_0(x)\in C^1[a,b],\, p_j(x),\, p_{m+1}(x)\in C^2[a,b]$, $\sum\limits_{i=1}^{n}q_i(b-\xi_i)\ne(b-a)$

bo‘lsa, {(1),(2)} masalaning $C^2[a,b]$ da yagona yechimi mavjud.

Isbot: $\{(1),(2)\}$ masalaning $y(x)$ yechimi mavjud deb faraz qilaylik (1) tenglamani $[x,b]$ oraliq bo‘yicha ikki marta integrallaymiz. So‘ngra $y(b)=k_1$ ekanligini e‘tiborga olib va xosil bo‘lgan takroriy integralda integrallash tartibini pasaytirib, quydagi tenglikka ega bo‘lamiz:

$$y(x)=\int_x^b\Big\{p_0(t)+[p_0'(t)-p_{m+1}(t)](t-x)+$$

$$+\sum_{j=1}^{m}\left(\frac{\omega_j(t)}{\Gamma(1-\alpha_j)}\left[\int_x^t p_j(z)(t-z)^{-\alpha_j}dz+\int_x^t p_j'(z)(t-z)^{-\alpha_j}(z-x)\,dz\right]\right)\Big\}y(t)\,dt=$$

$$=\int_x^b f(t)(t-x)\,dt-y'(b)(b-x)-k_1p_0(b)(b-x)+k_1 \tag{5}$$

bu yerda $y'(b)$- noma‘lum son. (5) tenglikda quydagicha belgilash kiritsak,

$$K_1(x,t)=p_0(t)+\left(p_{m+1}(t)-p_0'(t)\right)(x-t)-$$

$$- \sum_{j=1}^{m} \frac{w_j(t)}{\Gamma(1-\alpha_j)} \left(\int_x^t \frac{p_j(\eta)}{(t-\eta)^{\alpha}} d\eta + \int_x^t \frac{p_j'(\zeta)(\zeta - x)}{(t-\zeta)^{\alpha}} d\zeta \right)$$

$$f_1(x) = \int_x^b f(t)(t-x)dt - p_0(b)k_1(b-x) + k_1$$

ushbu ifodaga ega bo‘lamiz

$$y(x) = \int_x^b K_1(x,t)y(t)dt + f_1(x) - y'(b)(b-x) \qquad (6)$$

$$y(a) = \int_a^b K_1(a,t)y(t)dt + f_1(a) - y'(b)(b-a) \qquad (7)$$

$$y(\xi_i) = \int_{\xi_i}^b K_1(\xi_i,t)y(t)dt + f_1(\xi_i) - y'(b)(b-\xi_i) \qquad (8)$$

(7) va (8) dan hamda (2) shartning birinchisidan foydalanib $y'(b)$ ni quyidagicha topamiz

$$y'(b) = \frac{\sum_{i=1}^{n} q_i \int_{x_i}^{b} K_1(x_i,t)y(t)dt - \int_a^b K_1(a,t)y(t)dt - f_1(a) + \sum_{i=1}^{n} q_i f_1(x_i) + k_0}{\sum_{i=1}^{n} q_i(b-x_i) - (b-a)}.$$

Topilgan $y'(b)$ ni (6) tenglikka qo‘yib ba‘zi shakl almashtirishlardan keyin quyidagi Fredgolmning ikkinchi tur integral tenglamasini xosil qilamiz [4]:

$$y(x) = \int_a^b K(x,t)y(t)dt + f_2(x)\,. \qquad (9)$$

Bu yerda $x \in (\xi_n, b)$ bo‘lgan hol qaralgan bo’lib,

$$K_{2,i}(x,t)=\frac{(b-x)K_1(x_i,t)}{\sum\limits_{j=1}^{n} q_j(b-x_j)-(b-a)}\;,\; K_3(x,t)=\frac{(b-x)K_1(a,t)}{\sum\limits_{j=1}^{n} q_j(b-x_j)-(b-a)},$$

$$f_2(x)=f_1(x)-\frac{(b-x)\sum\limits_{i=1}^{n} q_i f_1(x_i)+k_0(b-x)-(b-x)f_1(a)}{\sum\limits_{j=1}^{n} q_j(b-x_j)-(b-a)}.$$

$$K(x,t)=\begin{cases} K_1(x,t)+\sum\limits_{i=1}^{n} q_i K_{2,i}(x,t)-K_3(x,t), & x\le t\le b,\\ \sum\limits_{i=1}^{n} q_i K_{2,i}(x,t)-K_3(x,t), & x_n\le t\le x,\\ \sum\limits_{i=1}^{n-1} q_i K_{2,i}(x,t), & x_{n-1}\le t\le x_n,\\ \sum\limits_{i=1}^{n-2} q_i K_{2,i}(x,t)-K_3(x,t), & x_{n-2}\le t\le x_{n-1},\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ \sum\limits_{i=1}^{n-(n-1)} q_i K_{2,i}(x,t)-K_3(x,t), & x_{n-(n-1)}\le t\le x_{n-(n-2)},\\ -K_3(x,t), & a\le t\le x_1 \end{cases}$$

$K(x,t),\ \{a\le x\le b, a\le t\le b\}$ to‘rtburchakda chegaralangan va bo‘lakli uzluksiz bo‘lgan ma‘lum funksiya. Shuningdek $f_2(x)$, $[a,b]$ oraliqda uzluksiz bo‘lgan ma‘lum funksiya.

Agar {(3),(4)} masalani qarasak, u

$$y(x)=\int_a^b K(x,t)\,y(t)\,dt \qquad (10)$$

ko‘rinishdagi birjinsli integral tenglamaga ekvivalent. Yuqorida ko‘rsatdikki {(1),(2)} masala (9) ga ekvivalent. {(3),(4)} masala faqat trivial yechimga ega bo‘lgani uchun (10) tenglama ham faqat trivial yechimga ega. U holda Fredgolm alternativiga asosan[4], (9) tenglama yagona yechimi mavjud bo‘ladi.

Bundan {(1),(2)} masalaning ham yechimi yagona ekanligi kelib chiqadi. (5) va (9) tengliklardan foydalanib ko‘rish mumkinki $f(x)\in C[a,b], p_0(x)\in C^1[a,b], p_j(x), p_{m+1}(x)\in C^2[a,b]$ shartlar tenglamaning yechimi $y(x)\in C^2[a,b]$ bo‘lishini ta‘minlaydi. Teorema isbotlandi.

XULOSALAR.

Xulosa o‘rnida shuni aytish mumkinki ushbu maqolada masalaning umumiy yechiming aniq ko‘rinishi topilmagan, uning topish metodikasi ko‘rsatilgan, mavjud va yagona ekanligi isbotlangan.

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