PROPORSIONAL INTENSIVLIKLAR MODELIDAGI APPROKSIMATSIYA

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Annotatsiya. Ushbu maqolada tasodifiy senzurlanishli umumlashgan modelning xususiy holi sifatida proporsional intensivliklar modellari uchun Kats empirik protsesslarini oʻrganishga doir masalalarga bagʻishlangan.
Proporsional intensivliklar modelida yigʻindilar sonini bogʻliqsiz Puasson taqsimotiga ega tasodifiy miqdorlar bilan almashtirish orqali hosil qilingan empirik Kats statistikasi taqsimot funksiyasini baholangan.

Kalit soʻzlar: tasodifiy senzurlanish, empirik baholar, Kats bahosi, kuchli approksimatsiya, Gauss jarayonlari.

АППРОКСИМАЦИЯ В МОДЕЛИ ПРОПОРЦИОНАЛЬНЫХ ИНТЕНСИВНОСТЕЙ

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Аннотация. Данная статья посвящена исследованию эмпирических процессов Каца для моделей пропорциональной интенсивности как частного случая обобщенной модели со случайной цензурой. В статье рассматривается оценка функции распределения с помощью эмпирической статистики Каца, которая получается путем замены количества слагаемых независимыми пуассоновскими случайными величинами.

Ключевые слова: Цензурированные данные, эмпирические оценки, оценка Каца, сильная аппроксимация, гауссовский процесс.

APPROXIMATION IN THE PROPORTIONAL HAZARD MODEL

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Abstract. This article is devoted to the study of empirical Katz processes for proportional intensity models as a special case of a generalized model with random censoring. The article discusses the estimation of the distribution function using empirical Kac statistics, which is obtained by replacing the number of terms with independent Poisson random variables.

Key words: Censored data, empirical estimates, Kac estimate, strong approximation, Gaussian process.

Kirish

Statistik tadqiqotlarning dastlabki bosqichida kuzatish natijalarini qayta ishlash va echimlar qabul qilish qulay ko‘rinishda ifodalanishi asosiy masalalardan biridir. Statistik xulosalarning sifat ko‘rsatkichi nafaqat ularning hajmiga balki ularni olish usullariga ham bog‘liq bo‘lib qoladi. Ammo yon atrofdagi mavjud statistik ma’lumotlar to‘liq bo‘lmay, ular asosida bir yoki bir necha tasodifiy miqdorlarning noma’lum taqsimotlari yoki ularning funksionallari haqida statistik xulosalar chiqarishga to‘g‘ri keladi. To‘liq bo‘lmagan tanlanmalar umr davomiyligi tipidagi ma’lumotlarni tahlil qilishda, sug‘urta ishida, demografiyada, astronomik tadqiqotlarda ko‘plab uchrab turadi. Masalan, chidamlilikka tekshirilayotgan ob’ektlar (texnik uskunalar, individumlar) ni kuzatish jarayonida bunday to‘liq bo‘lmagan sonli ma’lumotlarni uchratish mumkin. Bunday eksperimental holatlarda kuzatilaayotgan tanlanmalar nafaqat bizni qiziqtiradigan tasodifiy miqdorlar, balki bizga xalaqit beruvchi, ya’ni senzuralovchi miqdorlardan ham iborat bo‘lishi mumkin ekan. Bunday to‘liq bo‘lmagan ma’lumotlar orqali baholanishi zarur bo‘lgan asosiy xarakteristika – bu bir yoki bir necha tasodifiy miqdorlarning taqsimot qonunlaridan iboratdir.

Adabiyotlar sharhi

M.Kats empirik protsesslarning maxsus sinfini tadqiq qildi. Bu protsesslar parametri n ga teng bo‘lgan bog‘liq bo‘lmagan Puasson tasodifiy miqdorlar ketma-ketligi yordamida qo‘shiluvchilarni maxsus tanlash yordamida empirik statistika ko‘rinishida tanlanadi. Bunday protsesslarni M.Csörgő, S.Csörgő, P.Gaenssler, W.Stute va boshqalar o‘tgan asr 70-80 yillarida tadqiq qilganlar. Hozirga qadar olib borilgan tadqiqotlar M.Katsning klassik statistikasiga aloqador edi.

Tahlil va natijalar

Faraz qilaylik, taqsimot funksiyasi $H(x)=P(Z\le x),\ x\in \mathbb{R}^1$ dan iborat Z tasodifiy miqdor berilgan bo‘lsin. $A^{(1)},\dots,A^{(k)}$ fiksrlangan butun son $k\ge 1$ uchun, juft-jufti bilan birgalikda bo‘lmagan hodisalar bo‘lib, ular orqali tuzilgan subtaqsimotlar esa $H(x;i)=P\left(Z\le x, A^{(i)}\right),\quad i=\{1,\dots,k\}=J$ bo‘lsin. Biz $H(x)$ va $H(x;i),\ i\in J$ funksiyalar uzluksiz deb hisoblaymiz. Agar $p^{(i)}=P(A_i),\ i\in J$ desak, $0<p^{(i)}<1,\ i\in J$ deb hisoblaymiz. Proporsional intensivliklar modellari (PIM) raqobatlashuvchi risklar modelida $\left(Z,A^{(i)}\right)$ juftlikka mos kelgan $1-F(x;i)=\exp(-\Lambda(x;i))$ chidamlilik funksiyasi, $H(x)$ taqsimot funksiyasi orqali quyidagicha ifodalanadi:

$$1-F(x;i)=(1-H(x))^{p^{(i)}},\ i\in J,\ x\in R \tag{1}$$

tenglik orqali ifodalanadi. Ushbu (1) model dastlab [1] da kiritilgan edi. U holda $H(x;i),\ i=1,\dots,k$ subtaqsimotlar va $H(x)$ taqsimot funksiyasi uchun modifikatsiya qilingan quyidagi Kats empirik baholarini qaraymiz:

$$\begin{aligned}\tilde{H}_n(x;i)&=1-\left(1-H_n^*(x;i)\right)I\left(H_n^*(x;i)<1\right),\quad (x;i)\in\mathbb{R}\times J,\\ \tilde{H}_n(x)&=1-\left(1-H_n^*(x)\right)I\left(H_n^*(x)<1\right),\quad x\in\mathbb{R}.\end{aligned} \tag{2}$$

Bu yerda $H^*(x)$ va $H^*(x;i),\quad i=\overline{1,k}$ lar empirik Kats baholari:

$$H_n^*(x)=\begin{cases}\dfrac{1}{n}\sum\limits_{m=1}^{\nu(n)} I(Z_m \le x), & agar\ 1\ ehtimollik\ bilan\ \nu_n \ge 1\ bo`lsa,\\ 0, & aks\ holda,\end{cases}$$

$$H_n^*(x;i)=\begin{cases}\dfrac{1}{n}\sum\limits_{m=1}^{\nu(n)} I\left(Z_m \le x, \delta_m^{(i)}=1\right), & agar\ 1\ ehtimollik\ bilan\ \nu_n \ge 1\ bo`lsa,\\ 0, & aks\ holda.\end{cases}$$

(1) tenglik bilan aniqlangan statistik modelni ko‘ramiz.

Bizning ushbu ishda tadqiq qiladigan bahomiz $f(x,p)=(1-x)^p, \quad (0\le p\le 1,\ 0\le x<1)$ ko‘rinishida bo‘lgani uchun, bunday baholarni asimptotik xossalarini tadqiq qilish uchun uni Teylor qatoriga yoyilmasidan foydalanamiz. Buning uchun dastlab taqsimot funksiyalarning bahosini (1) ifodaga asoslangan holda

$$1-\tilde{F}_n(x;i)=\left(1-\tilde{H}_n(x)\right)^{\tilde{p}_n^{(i)}}, i\in J,\ x\in R, \tag{3}$$

ko‘rinishida qurib olamiz. Bu yerda $\tilde{H}_n(x)$ va $\tilde{p}_n^{(i)}=\tilde{H}_n(\infty;i),\ i\in J$ baholar. Ushbu ikki o‘lchovli $f(x,p)$ funksiyaning birinchi va ikkinchi tartibli hosilalari quyidagicha:

$$f_x'(x,p)=-p(1-x)^{p-1},\quad f_p'(x,p)=(1-x)^p\log(1-x),$$

$$f_{xp}''(x,p)=-(1-x)^{p-1}\left(1+p\log(1-x)\right),\quad f_{xx}''(x,p)=p(p-1)(1-x)^{p-2},$$

$$f_{pp}''(x,p)=(1-x)^p\log^2(1-x),$$

U holda $f\left(\tilde{H}_n(x),\tilde{p}_n^{(i)}\right)=\left(1-\tilde{H}_n(x)\right)^{\tilde{p}_n^{(i)}}, i\in J,\ x\in R$ baholarni $\left(H(x),p^{(i)}\right), i\in J$ nuqtalar atrofida mos yoyilmalarini yozib olamiz:

$$f\left(\tilde{H}_n(x),\tilde{p}_n^{(i)}\right)=\left(1-\tilde{H}_n(x)\right)^{\tilde{p}_n^{(i)}}=\left(1-H(x)\right)^{p^{(i)}}+$$
$$+\left\{-p^{(i)}\left(1-H(x)\right)^{p^{(i)}-1}\left(\tilde{H}_n(x)-H(x)\right)+\left(1-H(x)\right)^{p^{(i)}}\left(\log\left(1-H(x)\right)\right)\left(p_n^{(i)}-p^{(i)}\right)\right\}+$$
$$+\frac{1}{2}\left\{\breve{p}_n^{(i)}\left(\breve{p}_n^{(i)}-1\right)\left(1-\breve{H}_n(x)\right)^{\breve{p}_n^{(i)}-2}\left(\tilde{H}_n(x)-H(x)\right)^2-\right.$$
$$-2\left(1-\breve{H}_n(x)\right)^{\breve{p}_n^{(i)}-1}\left(1+\breve{p}_n^{(i)}\log\left(1-\breve{H}_n(x)\right)\right)\left(\tilde{H}_n(x)-H(x)\right)\left(\tilde{p}_n^{(i)}-p^{(i)}\right)+$$
$$\left.+\left(1-\breve{H}_n(x)\right)^{\breve{p}_n^{(i)}}\left[\log^2\left(1-\breve{H}_n(x)\right)\right]\left(\tilde{p}_n^{(i)}-p^{(i)}\right)\right\},i\in J,\ x\in R \tag{4}$$

Bu yerda

$$\breve{H}_n(x)\in\left[\min\left(\tilde{H}_n(x),H(x)\right),\max\left(\tilde{H}_n(x),H(x)\right)\right],$$
$$\breve{p}_n^{(i)}\in\left[\min\left(\tilde{p}_n^{(i)},p^{(i)}\right),\max\left(\tilde{p}_n^{(i)},p^{(i)}\right)\right].$$

Baholardan iborat vektor protsesslar

$$\tilde{\mathbb{Z}}_n(t)=\left(\tilde{\mathbb{Z}}_n^{(1)}(t_1),\tilde{\mathbb{Z}}_n^{(2)}(t_2),\ldots,\tilde{\mathbb{Z}}_n^{(k)}(t_k)\right),$$

hamda Gauss protsesslaridan iborat

$$\tilde{V}_n(t)=\left(\tilde{V}_n^{(1)}(t_1),\tilde{V}_n^{(2)}(t_2),\ldots,\tilde{V}_n^{(k)}(t_k)\right),$$

vektorni aniqlaymiz. Bu yerda $\tilde{\mathbb{Z}}_n^{(i)}(x)=\sqrt{n}\left(\tilde{F}_n(x;i)-F(x;i)\right),$ $\tilde{F}_n(x;i)=1-\left(1-\tilde{H}_n(x)\right)^{\tilde{p}_n^{(i)}},i\in J,\ x\in R,\ \tilde{p}_n^{(i)}=\tilde{H}_n(\infty;i),\ i\in J$ hamda

$$\tilde{V}_n^{(i)}(x)=\left(1-H(x)\right)^{p^{(i)}}\left\{\frac{p^{(i)}W_n^{(0)}(x)}{1-H(x)}-W_n^{(i)}(\infty)\log\left(1-H(x)\right)\right\}.$$

Faraz qilaylik, $\left\{T_n,n\geq 1\right\}$ ketma-ketlik shunday tanlansinki,

$$T_n\uparrow T_H,\qquad b_n^{-1}=\left(1-H(T_n)\right)\geq\left(\frac{2\varepsilon\log n}{wn}\right)^{1/2} \tag{5}$$

hamda $\frac{n}{\log n} \geq \max\left\{\frac{r}{2w\left(1-p^{(i)}\right)^2}, \frac{2r}{w\left(p^{(i)}\right)^2}\right\}$ shartlar bajarilsin.

1-teorema. *Faraz qilamiz,* $\{T_n, n \geq 1\}$ *ketma-ketligi (5) shartlarni qanoatlantirsin. U holda [2]dagi 1-teoremaning shartlarini qanoatlantiradigan yetarlicha boy ehtimollar fazosida quyidagi approksimatsiya o‘rinli bo‘ladi:*

$$P\left(\sup_{t\in(-\infty;T_n]^k}\left\|\tilde{\mathbb{Z}}_n(t)-\tilde{V}_n(t)\right\|^{(k)} > C_n\frac{\log n}{\sqrt{n}}\right) \leq kQn^{-r}, \tag{6}$$

Bu yerda $\varepsilon = r,$ $C_n = \frac{r}{4w}b_n^2 + \left(A_4 + \frac{r\left(1+e^{-1}\right)}{w}\right) + \frac{1}{ep^{(i)}}\left(A_4 + \frac{4r}{wep^{(i)}}\right),$

$Q = 2A_5 + 16, \quad r \geq 2$ *va* A_4, A_5 *–o‘zgarmas sonlar*.

*1-teoremaning isboti***.** $i = 1,\dots,k$ uchun

$$\left|\tilde{\mathbb{Z}}_n^{(i)}(x)-\tilde{V}_n^{(i)}(x)\right| \leq R_{1n}^{(i)}(x)+R_{2n}^{(i)}(x)+R_{3n}^{(i)}(x)+R_{4n}^{(i)}(x)+R_{5n}^{(i)}(x) \tag{7}$$

tengsizlik o‘rinli. U holda (4) yoyilmaga hamda (7) tengsizlikka asosan,

$$P\left(\sup_{x\leq T_n}\left|\tilde{\mathbb{Z}}_n^{(i)}(x)-\tilde{V}_n^{(i)}(x)\right| > C_n\frac{\log n}{\sqrt{n}}\right) \leq \sum_{m=1}^{5} P\left(\sup_{x\leq T_n}\left|R_{mn}^{(i)}(x)\right| > \mu_m(n)\right), \tag{8}$$

bu yerda $\mu_1(n)+\mu_2(n)+\mu_3(n)+\mu_4(n)+\mu_5(n) = C_n\frac{\log n}{\sqrt{n}}.$ Endi (8) dagi har bir ehtimollikni alohida baholaymiz.

U holda

$$P\left(\sup_{x\leq T_n}\left|R_{1n}^{(i)}(x)\right| > A_4 b_n\frac{\log n}{\sqrt{n}}\right) =$$

$$P\left(\sup_{x\leq T_n}\left|p^{(i)}\left(1-H(x)\right)^{p^{(i)}} b_n\sqrt{n}\left(\tilde{H}_n(x)-H(x)\right)-W_n\left(H(x)\right)\right| > A_4 b_n\frac{\log n}{\sqrt{n}}\right) \leq$$

$$\leq P\left(\sup_{x\leq T_n}\left|\sqrt{n}\left(\tilde{H}_n(x)-H(x)\right)-W_n\left(H(x)\right)\right| > A_4\frac{\log n}{\sqrt{n}}\right) \leq A_5^1 n^{-r} \tag{9}$$

tengsizlik $\frac{n}{\log n} \geq \frac{r}{8(1+e/3)^2}$, bo‘lganida o‘rinli bo‘ladi. Endi $u \in [0,1]$ hamda $\alpha, \gamma > 0$ bo‘lganida

$$0 \leq u^{\gamma} |\log u|^{\alpha} \leq \left(\frac{\alpha}{\gamma}\right)^{\alpha} e^{-\alpha}. \tag{10}$$

elementar tengsizlikdan foydalanamiz. Agar $u = 1 - H(x),\ \alpha = 1,\ \gamma = p^{(i)}$ bo‘lsa, u holda (10) ga asosan $(1 - H(x))^{p^{(i)}} |\log(1 - H(x))| \leq (p^{(i)})^{-1} e^{-1}$ bo‘ladi. Demak, $i \in J = 1,\dots,k$ uchun

$$P\left(\sup_{x \leq T_n} \left|R_{2n}^{(i)}(x)\right| > \frac{A_4}{ep^{(i)}} \frac{\log n}{\sqrt{n}}\right) =$$

$$= P\left(\sup_{x \leq T_n} \left|(1 - H(x))^{p^{(i)}} |\log(1 - H(x))| \left(\sqrt{n}\left(\tilde{p}_n^{(i)} - p^{(i)}\right) - W_n(H(\infty, i))\right)\right| > \frac{A_4}{ep^{(i)}} \frac{\log n}{\sqrt{n}}\right) \leq$$

$$\leq P\left(\sup_{x \leq T_n} \left|\left(\sqrt{n}\left(\tilde{p}_n^{(i)} - p^{(i)}\right) - W_n(H(\infty, i))\right)\right| > \frac{A_4}{ep^{(i)}} \frac{\log n}{\sqrt{n}}\right) \leq A_5^1 n^{-r} \tag{11}$$

tengsizlik $\frac{n}{\log n} \geq \max\left\{\frac{r}{2w\left(1 - p^{(i)}\right)^2}, \frac{r}{8(1+e/3)^2}\right\}$ bo‘lganida o‘rinli bo‘ladi.

$u \in [0,1]$ uchun $u(1 - u) \leq \frac{1}{4}$ ekanligidan,

$$P\left(\sup_{x\le T_n}\left|R_{3n}^{(i)}(x)\right|>\frac{b_n^2 r}{4w}\frac{\log n}{\sqrt{n}}\right)=$$

$$=P\left(\sup_{x\le T_n}\left|\frac{\sqrt{n}}{2}\hat{p}_n^{(i)}\left(\hat{p}_n^{(i)}-1\right)\left(1-\hat{H}_n(x)\right)^{-2}\left(\tilde{H}_n(x)-H(x)\right)^2\right|>\frac{b_n^2 r}{4w}\frac{\log n}{\sqrt{n}}\right)\le$$

$$\le P\left(\sup_{x\le T_n}\left|\left(1-\hat{H}_n(x)\right)^{-2}\left(\tilde{H}_n(x)-H(x)\right)^2\right|>\frac{2b_n^2 r}{w}\frac{\log n}{\sqrt{n}}\right)\le$$

$$\le P\left(\sup_{x\le T_n}\left|\left(1-\hat{H}_n(x)\right)^{-2}\right|>4b_n^2\right)+P\left(\sup_{x\le T_n}\left|\tilde{H}_n(x)-H(x)\right|>\left(\frac{r}{w}\frac{\log n}{\sqrt{n}}\right)^{1/2}\right)=$$

$$=p_{31}+p_{32}.$$

Endi $y=\frac{1}{x}$ funksiyaning $(0,1]$ oraliqda monotonligi va lemmaga asosan,

$$p_{31}\le\max\left\{P\left(\sup_{x\le T_n}\left|\left(1-H_n(x)\right)^{-1}\right|>2b_n\right),P\left(\sup_{x\le T_n}\left|\left(1-\tilde{H}_n(x)\right)^{-1}\right|>2b_n\right)\right\}\le$$

$$\le\max\left\{P\left(\frac{1}{1-H(x)}>\frac{2}{1-H(x)}\right),4n^{-r}\right\}=\max\left\{0,4n^{-r}\right\}=4n^{-r},$$

$\frac{n}{\log n}\ge\frac{r}{8(1+e/3)^2}$ tengsizlik bajarilganida o‘rinli. Endi [2]dagi 6-teoremaga asosan, agar $\frac{n}{\log n}\ge\frac{r}{8(1+e/3)^2}$ bo‘lsa,

$$p_{32}\le P\left(\sup_{-\infty<x<\infty}\left|\tilde{H}_n(x)-H(x)\right|>\left(\frac{r}{w}\frac{\log n}{\sqrt{n}}\right)^{1/2}\right)\le4n^{-r}.$$

Demak, $i\in J=1,...,k$ uchun

$$P\left(\sup_{x\le T_n}\left|R_{3n}^{(i)}(x)\right|>\frac{b_n^2 r}{4w}\frac{\log n}{\sqrt{n}}\right)\le4n^{-r} \tag{12}$$

ekan. Endi (10) tengsizlikdan $u=1-\hat{H}_n(x),\ \gamma=\hat{p}_n^{(i)},\ \alpha=1$ deb tanlasak,

$$\left(1-\hat{H}_n(x)\right)^{\hat{p}_n^{(i)}}\left|\log\left(1-\hat{H}_n(x)\right)\right|\le\frac{1}{e\hat{p}_n^{(i)}}$$

tengsizlikni olamiz. Bu tengsizlikni qo‘llasak,

$$P\left(\sup_{x\le T_n}\left|R_{4n}^{(i)}(x)\right|>r\,\frac{(1+e^{-1})b_n\log n}{w}\right)\le$$

$$\le P\left(\sup_{x\le T_n}\left|\left(1-\hat{H}_n(x)\right)^{\hat{p}_n^{(i)}}\left(1+\hat{p}_n^{(i)}\log\left(1-\hat{H}_n(x)\right)\right)\frac{\left(\tilde{H}_n(x)-H(x)\right)\left(\tilde{p}_n^{(i)}-p^{(i)}\right)}{\left(1-\hat{H}_n(x)\right)}\right|>\right.$$

$$\left.>r\,\frac{(1+e^{-1})b_n\log n}{wn}\right)\le$$

$$\le P\left(\sup_{x\le T_n}\left|(1+e^{-1})\frac{1}{1-\hat{H}_n(x)}\left(\tilde{H}_n(x)-H(x)\right)\left(\tilde{p}_n^{(i)}-p^{(i)}\right)\right|>r\,\frac{(1+e^{-1})b_n\log n}{wn}\right)\le$$

$$\le P\left(\sup_{x\le T_n}\left(1-\hat{H}_n(x)\right)^{-1}>2b_n\right)+P\left\{\sup_{x\le T_n}\left(\tilde{H}_n(x)-H(x)\right)\left(\tilde{p}_n^{(i)}-p^{(i)}\right)>\frac{r\log n}{wn}\right\}=$$

$$=p_{41}+p_{42}.$$

Ko‘rish mumkinki, $p_{41}=p_{31}<4n^{-r}$.

Agar $\frac{n}{\log n}\ge\max\left\{\frac{r}{8(1+e/3)^2},128w^2r\right\}=r\max\left\{\frac{1}{8(1+e/3)^2},\frac{1}{2(1+e/3)^2}\right\}=$

$=\frac{r}{2(1+e/3)^2}=128w^2r$ bo‘lgan holda, qolgan qo‘shiluvchilar uchun tengsizliklarni

qo‘llasak,

$$p_{42}\le\max\left\{P\left(\sup_{x\le T_n}\left(\tilde{H}_n(x)-H(x)\right)>\sqrt{\frac{r\log n}{2wn}}\right),P\left(\left|\tilde{p}_n^{(i)}-p^{(i)}\right|>\sqrt{\frac{r\log n}{2wn}}\right)\right\}\le 4n^{-r}$$

bo‘ladi.

SHunday qilib,

$$P\left(\sup_{x\le T_n}\left|R_{4n}^{(i)}(x)\right|>r\,\frac{(1+e^{-1})b_n\log n}{w}\right)\le 8n^{-r}. \tag{13}$$

Endi (10) tengsizlikni $u=1-\hat{H}_n(x),\ \gamma=\hat{p}_n^{(i)},\ \alpha=2$ bo‘lgan holda qo‘llasak,

$$\left(1-\hat{H}_n(x)\right)^{\hat{p}_n^{(i)}-1}\log^2\left(1-\hat{H}_n(x)\right)\le\frac{4}{\left(\hat{p}_n^{(i)}\right)^2e^2}.$$

Bu tengsizlik va [2]-lemmani qo‘llasak, quyidagiga ega bo‘lamiz:

$$P\left(\sup_{x\le T_n}\left|R_{5n}^{(i)}(x)\right|>\frac{4r}{we^2\left(p^{(i)}\right)^2}\frac{\log n}{\sqrt{n}}\right)=$$

$$=P\left(\sup_{x\le T_n}\left|\left(1-\hat{H}_n(x)\right)^{\hat{p}_n^{(i)}-1}\log^2\left(1-\hat{H}_n(x)\right)\left(\tilde{p}_n^{(i)}-p^{(i)}\right)^2\right|>\frac{8r}{we^2\left(p^{(i)}\right)^2}\frac{\log n}{n}\right)\le$$

$$\le P\left(\left|\frac{4}{\left(\hat{p}_n^{(i)}\right)^2}\left(\tilde{p}_n^{(i)}-p^{(i)}\right)^2\right|>\frac{8r}{w\left(p^{(i)}\right)^2}\frac{\log n}{n}\right)\le$$

$$\le P\left(\frac{4}{\left(\hat{p}_n^{(i)}\right)^2}>\frac{16}{\left(p^{(i)}\right)^2}\right)+P\left(\left(\tilde{p}_n^{(i)}-p^{(i)}\right)^2>\frac{r\log n}{2wn}\right)\le$$

$$\le P\left(\frac{1}{\hat{p}_n^{(i)}}>\frac{2}{p^{(i)}}\right)+P\left(\left|\tilde{p}_n^{(i)}-p^{(i)}\right|>\sqrt{\frac{r\log n}{2wn}}\right)\le$$

$$\le\max\left\{P\left(\frac{1}{\hat{p}_n^{(i)}}>\frac{2}{p^{(i)}}\right),P\left(\frac{1}{\tilde{p}_n^{(i)}}>\frac{2}{p^{(i)}}\right)\right\}+P\left(\left|\tilde{p}_n^{(i)}-p^{(i)}\right|>\sqrt{\frac{r\log n}{2wn}}\right)\le$$

$$\le 2n^{-r}+2n^{-r}=4n^{-r},$$

bunda $\frac{n}{\log n}\ge\max\left\{\frac{2r}{w\left(p^{(i)}\right)^2},128w^2r\right\}.$ Hosil bo‘lgan (9), (10)-(13) tengsizliklardan (6) ga ega bo‘lamiz.

Endi 1-teoremaning shartlarini tekshiramiz.

$$\frac{r}{2w\left(1-p^{(i)}\right)^2}\ge\frac{r}{2w}=8(1+e/3)r>\frac{r}{2(1+e/3)^2}=128w^2r$$

ekanligidan

$$\frac{n}{\log n} \geq \max\left\{\frac{r}{8(1+e/3)^2}, \frac{r}{2w\left(1-p^{(i)}\right)^2}, 128w^2 r, \frac{2r}{w\left(p^{(i)}\right)^2}\right\} =$$

$$= \max\left\{\frac{r}{2w\left(1-p^{(i)}\right)^2}, 128w^2 r, \frac{2r}{w\left(p^{(i)}\right)^2}\right\} =$$

$$= \max\left\{\frac{r}{2w\left(1-p^{(i)}\right)^2}, \frac{2r}{w\left(p^{(i)}\right)^2}\right\}$$

shartlar bajarilar ekan. 1-teorema isbotlandi.

Xulosa

Ushbu ishda tasodifiy senzurlanishning raqobatli risklar modelida modifikatsiya qilingan Kats statistikasi va uning funksionallarining asimptotikasiga oid masalalar o‘rganildi.

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