# BLACK HOLE THERMODYNAMICS SATISFYING NONLINEAR ELECTRODYNAMICS (NED) CONDITIONS IN CHARGED, NON-ROTATING ANTI-DE SITTER (ADS) SPACE

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***Abstract:*** This study investigates the thermodynamic properties of black holes in charged, non-rotating Anti-de Sitter (AdS) space under the influence of Non-Linear Electrodynamics (NED). We explore the black hole metric solution and calculate key thermodynamic quantities, including Hawking temperature and specific heat. The behavior of the specific heat provides insights into the system's stability, identifying the critical points that signal phase transitions. By examining the thermodynamic characteristics of the horizon, we can find formulas for temperature and particular heat, and we identify considerable changes in the critical points of charge and pressure. The results show that there is a complicated phase arrangement controlled by nonlinear electrodynamics (NED), which is crucial for the stability and basic characteristics of black holes in Anti-de Sitter (AdS) space.

***Key words:*** *Black holes, thermodynamic, event horizon, NED, charge, AdS.*

# ТЕРМОДИНАМИКА ЧЁРНОЙ ДЫРЫ, УДОВЛЕТВОРЯЮЩАЯ УСЛОВИЯМ НЕЛИНЕЙНОЙ ЭЛЕКТРОДИНАМИКИ (НЭД) В ЗАРЯЖЕННОМ, НЕВРАЩАЮЩЕМСЯ ПРОСТРАНСТВЕ АНТИ-ДЕ СИТТЕРА (АдС)

***Аннотация:*** В этом исследовании изучаются термодинамические свойства чёрных дыр в заряженном, не вращающемся пространстве Анти-де Ситтера (АдС) под влиянием нелинейной электродинамики (НЭД). Мы исследуем метрику решения чёрной дыры и вычисляем ключевые термодинамические величины, включая температуру Хокинга и удельную теплоёмкость. Поведение удельной теплоёмкости даёт представление о стабильности системы, определяя критические точки, указывающие на фазовые переходы. Путём анализа термодинамических свойств горизонта мы выводим выражения для температуры и удельной теплоёмкости, наблюдая заметные изменения при критических значениях заряда и давления. Эти результаты указывают на богатую фазовую структуру, управляемую НЭД, которая существенно влияет на термодинамическую стабильность чёрной дыры и её ключевые характеристики в пространстве AdS.

***Ключевые слова:*** *Черные дыры, термодинамика, горизонт событий, NED, заряд,* АдС.

# ZARYADLANGAN, AYLANMAYDIGAN ANTI-DE SITTER (*AdS*) FAZOSIDA NOCHIZIQLI ELEKTRODINAMIKA (NED) SHARTLARINI QONATLANTIRADIGAN QORA TUYNUK TERMODINAMIKASI

***Annotatsiya:*** Ushbu tadqiqotda zaryadlangan, aylanmaydigan Anti-de Sitter (AdS) fazosidagi qora tuynuklarning termodinamik xususiyatlari, nochiziqli elektrodinamika (NED) ta'siri ostida o'rganiladi. tadqiqotimizda qora tuynuklarning metrik funksiyasining yechimi o‘rganild va Xoking harorati va issiqlik sig'imi kabi asosiy termodinamik omillarni hisoblab chiqdik. Solishtirma issiqlik sig‘imining xatti-harakati tizimning barqarorligini ochib beradi va fazoviy o'tishlarni ko'rsatadigan kritik nuqtalarni aniqlaydi. Gorizontning termodinamik xususiyatlarini tahlil qilish orqali harorat va solishtirma issiqlik sig‘imi uchun ifodalarni keltirib chiqardik va zaryad va bosimning kritik qiymatlarida sezilarli o'zgarishlarni kuzatdik. Ushbu natijalar NED bilan boshqariladigan boy fazoviy struktura mavjudligini

ko'rsatadi, bu esa qora tuynukning termodinamik barqarorligi va AdS fazosidagi asosiy xatti-harakatlariga sezilarli ta'sir qiladi.

***Kalit so'zlar:*** *Qora tuynuklar, termodinamika, hodisa gorizonti, NED, zaryad, AdS.*

**Introduction.**The study of black hole thermodynamics is a significant area of research that examines the strong link between gravity, quantum physics, and thermodynamics. When studying black holes in anti-de Sitter (AdS) space, the use of nonlinear electrodynamics (NED) makes the system more complicated, altering the metric function and the thermodynamic properties of the black hole[1]. This research examines electrically charged, non-spinning black holes in the AdS field and explores how this impacts the thermodynamic characteristics of NEDs[2].

The black hole metric function is an important part of comprehending the thermodynamics of a system. The effects of nonlinear electrodynamic fields are captured by obtaining a modified metric through solving the Einstein field equations using the NED Lagrangian[3-4]. This changed measure is important for computing thermodynamic values like the Hawking temperature and specific heat. The Hawking temperature, connected to the gravitational pull of the black hole's surface, is crucial in deciding the thermal radiation given off by the black hole[5].

The specific heat is also measured to find out how stable the black hole is. Studying the heat capacity helps identify important points where changes in the stability of the black hole occur during phase transitions [6-7]. These important points offer understanding into how black holes behave in the NED-modified AdS field, indicating a complicated phase arrangement when NED and AdS boundary conditions are at play.

In this work, we adopt the signature ¿ for the space-time and utilize the unit system where $G=c=1$. However, in our expressions related to astrophysical applications, we explicitly include the speed of light. The Latin indices span from 1 to 3 , while the Greek indices range from 0 to 3[8] .

**I. Action Integral.**

For a charged, non-rotating (static) Anti-de Sitter (AdS) black hole, the action integral is given by:

$$S=\int d^4x\sqrt{-g}\left(\frac{R-2\Lambda}{16\pi}+\mathcal{L}_{NED}\right) \tag{1}$$

where $g$ is the determinant of the metric, $R$ is the Ricci scalar, $\Lambda$ is the cosmological constant, and $\mathcal{L}_{\mathrm{NED}}$ represents the Lagrangian of Non-Linear Electrodynamics (NED). For a simple electromagnetic field, the Lagrangian is:

$$\mathcal{L}_{EM}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. In the case of NED, the Lagrangian is more general and depends on the field invariant[9]:

$$\mathcal{L}_{NED}=\mathcal{L}(F), F=F_{\mu\nu}F^{\mu\nu} \tag{3}$$

The total Lagrangian for the charged black hole, incorporating the EinsteinHilbert term and the NED term, is expressed as:

$$\mathcal{L}_{\mathrm{total}}=\frac{R-2\Lambda}{16\pi}+\mathcal{L}_{NED} \tag{4}$$

Here, the Einstein-Hilbert term, $\frac{R-2\Lambda}{16\pi}$, describes the gravitational dynamics, while $\mathcal{L}_{\mathrm{NED}}$ captures the influence of the nonlinear electromagnetic field[10].

The Einstein field equations in the presence of a cosmological constant and a nonlinear electromagnetic field are given by:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=8\pi T_{\mu\nu}$$
(5)

where $R_{\mu\nu}$ is the Ricci tensor, $R$ is the Ricci scalar, and $T_{\mu\nu}$ is the energy-momentum tensor associated with the electromagnetic field, which for NED is given by:

$$T_{\mu\nu}=2\left(\mathcal{L}_F F_{\mu\lambda}F_{\nu}{}^{\lambda}-g_{\mu\nu}\mathcal{L}\right)$$
(6)

where $\mathcal{L}_F=\frac{d\mathcal{L}}{dF}$ and $F_{\mu\nu}$ is the field strength tensor.

For a spherically symmetric black hole, the Ricci scalar is expressed as:

$$R=-\frac{2}{r^2}\left(1-f(r)-rf'(r)\right)$$
(7)

where $f(r)$ is the metric function to be determined by solving the Einstein equations.

The metric for a charged, non-rotating AdS black hole under the conditions of Non-Linear Electrodynamics is given by[11-13]:

$$ds^2=-f(r)dt^2+\frac{dr^2}{f(r)}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$
(8)

where the metric function $f(r)$ is

$$f(r)=1-\frac{2M}{r}+\frac{q^2}{r^2}-\frac{\Lambda r^2}{3}$$
(9)

with $M$ being the mass of the black hole, $q$ its charge, and $\Lambda$ the cosmological constant, which is related to the AdS pressure $P$ by

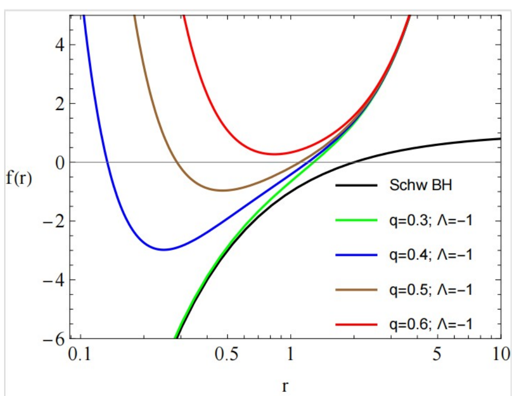


**Fig. 1:** Radial dependence of lapse function $f(r)$ for varying values of $q$ and $\Lambda$.

In Fig.1 shows the behavior of the metric function $f(r)$ as a function of radial distance $r$, comparing different black hole charge values $q$ with a fixed cosmological constant $\Lambda=-1$. The Schwarzschild black hole $(q=0)$ is included for reference. The vertical axis represents $f(r)$, the metric function of the black hole, while the horizontal axis represents the radial distance $r$. As the charge $q$ increases from 0.3 (green) to 0.6 (red), the curves shift upward, particularly at smaller radii. For charged black holes, $f(r)$ dips below zero and then rises back, indicating the formation of an inner horizon (Cauchy horizon) and an outer event horizon, which is characteristic of charged black hole solutions. The higher the charge, the more pronounced this behavior becomes. For example, the red curve $(q=0.6)$ shows both the outer and inner horizons clearly, as the curve crosses $f(r)=0$ twice. As the charge increases, the black hole develops both inner and outer horizons, unlike the Schwarzschild black hole, which has only one event horizon. The outer horizon is pushed to larger values of $r$ as the charge increases. The Schwarzschild black hole has a single event horizon where $f(r)=0$, and for larger values of $r, f(r)$ increases more slowly compared to charged black holes.

$$P=-\frac{\Lambda}{8\pi}$$
$$(10)$$

**II. Thermodynamics of AdS BH.**

The Hawking temperature $T$ of the black hole is related to the surface gravity at the event horizon $r_h$ and can be expressed as[14]:

$$T=\frac{f'(r_h)}{4\pi}=-\frac{3q^2-3Mr+r^4\Lambda}{6\pi r^3}$$
$$(11)$$

The entropy of the black hole is proportional to the area of the event horizon, as per the Bekenstein-Hawking entropy formula:

$$S=\frac{A}{4}=\pi r_h^2$$
$$(12)$$

The mass of the black hole, which is interpreted as the internal energy $M$, is given by:

$$M=\frac{r_h}{2}\left(1+\frac{q^2}{r_h^2}-\frac{\Lambda r_h^2}{3}\right)$$
$$(13)$$

The electrostatic potential $\Phi$ for a charged black hole is:

$$\Phi=\frac{q}{r_h}$$
$$(14)$$

In the extended thermodynamics framework, the pressure $P$ is associated with the cosmological constant $\Lambda$, and the thermodynamic volume $V$ is related to the horizon radius $r_h$ by:

$$V=\frac{4}{3}\pi r_h^3$$
$$(15)$$

The first law of black hole thermodynamics is modified to:

The specific heat at constant charge $C$ is defined as:

$$C=T\left(\frac{\partial S}{\partial T}\right)$$
$$(16)$$

Since $S=\pi r_h^2$, we have:

$$\frac{\partial S}{\partial r_h}=2\pi r_h$$
$$(17)$$

Next, we need to differentiate the temperature $T$ from $r_h$. From the temperature expression[15-16]:

$$C=\frac{2\pi r^2\left(3q^2-3Mr+r^4\Lambda\right)}{6Mr-9q^2+r^4\Lambda} \tag{18}$$

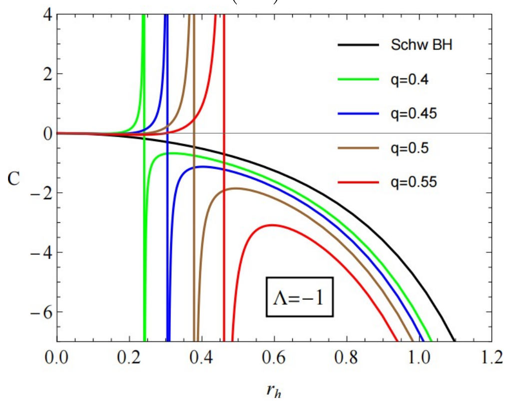


**Fig. 2:** The vertical axis represents the heat capacity $C$, while the horizontal axis represents the event horizon radius $r_h$

In Fig. 2, the Schwarzschild black hole shows a relatively smooth behavior with negative heat capacity, decreasing gradually as $r_h$ increases. The vertical lines where the heat capacity diverges represent second-order phase transitions, where the black hole shifts between thermodynamically stable (positive $C$ and unstable (negative $C$ states. Charged Black Holes: Increasing the charge introduces significant thermodynamic phase transitions in black hole behavior. These transitions are characterized by divergences in the heat capacity and indicate shifts between stable and unstable states. Schwarzschild Black Hole: The Schwarzschild black hole does not exhibit phase transitions in this range, maintaining a consistently negative heat capacity. General Behavior: For charged black holes, the heat capacity transitions from positive to negative as $r_h$ increases, with higher charge values shifting the point of phase transition to a larger $r_h$. The appearance of these divergences for charged black holes contrasts with the smoother behavior of the Schwarzschild black hole, which does not exhibit such transitions in this region. Physically, this means that the black hole undergoes a second-order phase transition. Then, this phase transition leads the black hole to move from the stable phase $(C>0)$ to the unstable phase $(C<0)$.

**III. $P-V$ Criticality.**

The $P-V$ criticality of black holes shows an analogy to Van der Waals fluids, where phase transitions occur between small and large black holes[17]. The equation for temperature is:

$$T=\frac{f'(r_h)}{4\pi}=-\frac{3q^2-3Mr+r^4\Lambda}{6\pi r^3} \tag{19}$$

$$P=\frac{3\left(q^2-Mr+2\pi r^3 T\right)}{r^4} \tag{20}$$

In Fig. 3 illustrates the relationship between a black hole's temperature and pressure as functions of radial distance. Each graph represents different values of the black hole's charge $q$ and compares them with the Schwarzschild black hole. Each curve shows how the temperature of a black hole changes with radius for different values of charge $q$.

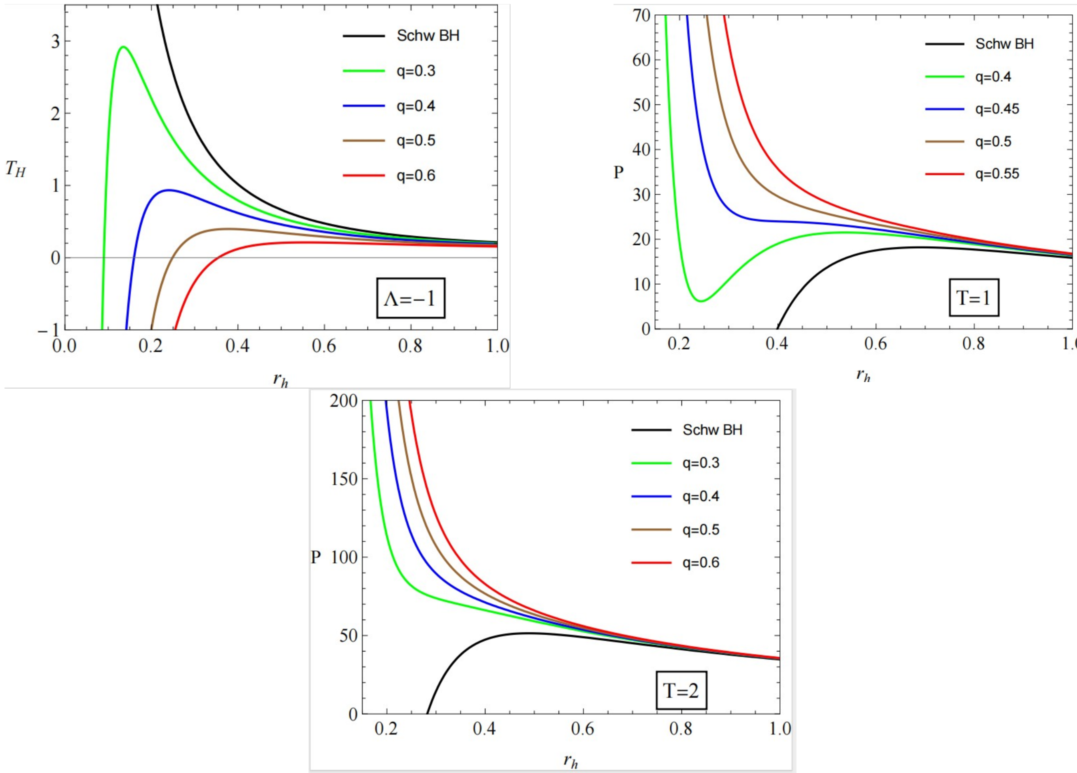


**Fig. 3:** Change of the black hole temperature $T_h$ and pressure P

The black line represents the Schwarzschild black hole, which starts at a higher temperature and decreases as $r_h$ increases. As the charge $q$ increases, the temperature starts lower, peaks at a certain point, and then decreases. This graph shows how the pressure varies with $r_h$ when the temperature $T=1$. For the Schwarzschild black hole (black line), the pressure is initially low but increases rapidly as $r_h$ increases. As the charge $q$ increases, the initial pressure is higher, but it quickly drops as $r_h$ increases. At a higher temperature, the initial pressure for all charged black holes starts very high. The Schwarzschild black hole has relatively lower pressure initially while increasing charge leads to a sharp drop in pressure as $r_h$ increases. Increasing the charge $q$ significantly impacts the temperature and pressure of black holes. In the temperature graph, increasing charge decreases the maximum temperature, which occurs at smaller radii. In the pressure graphs, increasing charge leads to a higher initial pressure, but the pressure quickly decreases as $r_h$ increases. Higher temperatures (as seen in the difference between the second and third graphs) lead to substantially higher initial pressures, but increasing charge causes a steeper drop in pressure.

This formula represents the pressure $P$ inside the black hole system, also depending on the charge $q$, the black hole mass $M$, and the temperature $T$. The term $r$ refers to the radial coordinate, representing the radius of the horizon. The expression $2\pi r^3 T$ shows how temperature and horizon volume influence the pressure in this AdS space[18-19].

$$\left(\frac{\partial P}{\partial V}\right)_T = 0, \left(\frac{\partial^2 P}{\partial V^2}\right)_T = 0 \tag{21}$$

To find the critical points, these two conditions are used. The first condition Eq. 21 ensures that the pressure is stationary concerning the volume $V$ at constant temperature $T$. The second condition Eq.

21 ensures that the system is at a point of inflection, which corresponds to the critical point of a phase transition.

Solving these equations gives the critical radius $r_c$, pressure $P_c$, temperature $T_c$, and volume $V_c$. For a Reissner-Nordström AdS black hole, the critical values are:

$$r_c=\sqrt{6}q, P_c=\frac{1}{96\pi q^2}, T_c=\frac{\sqrt{6}}{18\pi q}, V_c=\frac{8\sqrt{6}\pi q^3}{3} \quad (22)$$

The formulas describe how a charged black hole behaves in AdS space. They show how temperature, pressure, and volume interact at critical points. These important points are linked to changes between small and large black holes, similar to the shifts between liquid and gas in regular thermodynamics. The equations explain how the physical characteristics of a black hole, like its mass, charge, and cosmological constant, affect its thermodynamic properties.

## IV. Conclusion

This article covers the thermodynamic characteristics and criticality of charged, non-spinning black holes in AdS space influenced by nonlinear electrodynamics. The important characteristics of these black holes are almost the same as those of Van der Waals liquids. They show sudden changes between small and large black holes at temperatures lower than the critical point. These findings offer more details about the complex thermodynamic makeup of AdS black holes with nonlinear electrodynamics.

To sum up, this research emphasizes the important impact of nonlinear electrodynamics (NED) on the heat and energy dynamics of charged, non-spinning black holes in Anti-de Sitter (AdS) space. We examined important thermodynamic properties like the Hawking temperature and specific heat by creating a changed measurement function, which helped us find critical points in the system. The temperature of a black hole is directly related to its surface in conditions of gravitational NED. These black holes have provided significant information about the thermal radiation they emit.

Our examination of the heat capacities showed that black holes in this NED-altered AdS field have a diverse phase structure with critical points indicating changes in phase and stability. The way the specific heat behaves near these critical points shows that the NED is very important in determining the thermodynamic stability of the black hole.

The findings offer a better grasp of the thermodynamic characteristics of black holes under NED circumstances and create new opportunities for examining the diverse phase arrangements that arise from the interplay of gravity, electrodynamics, and the AdS boundary.

## References


1. Kahya, E. O., Khurshudyan, M., Pourhassan, B., Myrzakulov, R., & Pasqua, A. (2015). Higher order corrections of the extended Chaplygin gas cosmology with varying G and $\Lambda$. The European Physical Journal C, 75(2)

2. Hassaine, M., & Martinez, C. (2007). Higher-dimensional black holes with a conformally invariant Maxwell source. Physical Review D, 75(2), 027502 .

3. Hoffmann, B. (1935). Gravitational and electromagnetic mass in the Born-Infeld electrodynamics. Physical Review, 47(11), 877.

4. Sekhmani, Y., Luciano, G., Rayimbaev, J., Jasim, M., Al-Badawi, A. & Maurya, S. Topological AdS black holes surrounded by Chaplygin dark fluid: From stability to geometrothermodynamic analysis. Physics Of The Dark Universe. 46 pp. 101567 (2024,12)

5. Rahmatov, B., Zahid, M., Rayimbaev, J., Rahim, R. & Murodov, S. QPOs and circular orbits around black holes in Chaplygin-like cold dark matter. Chinese Journal Of Physics. 92 pp. 143-165 (2024,12)

6. Dey, T. K. (2004). Born-Infeld black holes in the presence of a cosmological constant. Physics Letters B, 595(1-4), 484-490.

7. Cai, R. G., Pang, D. W., & Wang, A. (2004). Born-Infeld black holes in (A) dS spaces. Physical Review D, 70(12), 124034.

8. Kamenshchik, A., Moschella, U., & Pasquier, V. (2001). An alternative to quintessence. Physics Letters B, 511(2-4), 265-268.
9. Mustafa, G., Maurya, S., Ditta, A., Ray, S. & Atamurotov, F. Circular orbits and accretion disk around AdS black holes surrounded by dark fluid with Chaplygin-like equation of state. Eur. Phys. J. C. 84, 690 (2024)
10. Arora, D., Yasir, M., Chaudhary, H., Javed, F., Mustafa, G., Tiecheng, X. & Atamurotov, F. Joule-Thomson expansion and tidal force effects of AdS black holes surrounded by Chaplygin dark fluid. ArXiv E-prints. pp. earXiv:2312.16224 (2023,12)
11. Ditta, A., Javed, F., Mustafa, G., Atamurotov, F. & Salimov, S. Particle dynamics and fundamental frequencies of black hole coupled with a nonlinear electrodynamics field. Journal Of High Energy Astrophysics. 43 pp. $51-60(2024,8)$
12. Ashraf, A., Ditta, A., Sofuoğlu, D., Ma, W., Javed, F., Atamurotov, F. & Mahmood, A. Quasi-periodic oscillations and particle motion around charged black hole surrounded by a cloud of strings and quintessence field in Rastall gravity. Physica Scripta. 99, e065011 (2024,6)
13. Bilić, N., Tupper, G. B., & Viollier, R. D. (2002). Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas. Physics Letters B, 535(1-4), 17-21.
14. Bento, M. C., Bertolami, O., & Sen, A. A. (2002). Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification. Physical Review D, 66(4), 043507.
15. Carturan, D., & Finelli, F. (2003). Cosmological effects of a class of fluid dark energy models. Physical Review D, 68(10), 103501.
16. Amendola, L., Finelli, F., Burigana, C., & Carturan, D. (2003). WMAP and the generalized Chaplygin gas. Journal of Cosmology and Astroparticle Physics, 2003(07), 005.
17. Bean, R., & Dore, O. (2003). Are Chaplygin gases serious contenders for the dark energy?. Physical Review D, 68(2), 023515.
18. R Ibadov, B Rahmatov, D Abduraimov, TOPOLOGY OF EQUATORIAL TIME-LIKE CIRCULAR ORBITS AROUND QUIESCENT BLACK HOLES, Issue 1 of 2023 (143/2), 2024
19. Mushtaq, F., Tiecheng, X., Ditta, A., Atamurotov, F., Abduvokhidov, A. & Asalkhon, A. Weak gravitational lensing and fundamental frequencies of Einstein-Euler-Heisenberg black hole. New Astronomy. 108 pp. e102185 (2024,5)