BLACK HOLES SURROUNDED BY PERFECT FLUID DARK MATTER IN STVG

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Abstract

In this paper, we investigate the properties of black holes surrounded by perfect fluid dark matter (PFDM) within the framework of Scalar-Tensor-Vector Gravity (STVG), also known as modified gravity (MOG). The presence of PFDM in the vicinity of black holes introduces deviations from traditional black hole solutions, affecting both the geometry and physical characteristics of the spacetime. We analyze the impact of PFDM on key black hole properties, including the event horizon structure, thermodynamic quantities, and stability conditions. The interplay between the STVG framework and PFDM provides a modified gravitational field, leading to new insights into black hole thermodynamics and energy extraction mechanisms. Furthermore, we explore how PFDM influences particle dynamics and accretion processes near black holes, which could have observable consequences in astrophysical scenarios. Our results suggest that black holes in the presence of PFDM within STVG exhibit unique signatures, providing potential observational tests for both the nature of dark matter and modified gravity theories.

Keywords: Black holes, Perfect Fluid Dark Matter (PFDM), Scalar-Tensor-Vector Gravity (STVG), Modified gravity, Event horizon, Particle dynamics

QORA MATERYA STVGDA SUYIKLIK BILAN O'ROVLANGAN QORA URALAR

ANNOTATSIYA: Ushbu maqolada biz Scalar-Tensor-Vector Gravitatsiya (STVG), ya'ni modifikatsiyalangan gravitatsiya (MOG) doirasida mukammal suyuqlikdan iborat qorong'i materiya (PFDM) bilan o'ralgan qora tuynuklarning xususiyatlarini o'rganamiz. Qora tuynuklar atrofidagi PFDM mavjudligi an'anaviy qora tuynuk yechimlaridan chetlanishga olib keladi va fazoviy vaqtdagi geometrik hamda fizik xususiyatlarga ta'sir ko'rsatadi. Biz PFDMning qora tuynukning asosiy xususiyatlariga, shu jumladan hodisa gorizonti tuzilmasi, termodinamik kattaliklar va barqarorlik sharoitlariga ta'sirini tahlil qilamiz. STVG va PFDM o'zaro ta'siri natijasida modifikatsiyalangan gravitatsion maydon yuzaga keladi, bu esa qora tuynuk termodinamikasi va energiya olish mexanizmlariga yangi tushunchalarni kiritadi. Shuningdek, biz PFDMning qora tuynuklar yaqinidagi zarrachalar dinamikasi va akkresiya jarayonlariga qanday ta'sir qilishini o'rganamiz, bu astrofizik stsenariylarda kuzatilishi mumkin bo'lgan natijalarga ega bo'lishi mumkin. Natijalarimiz shuni ko'rsatadiki, STVG doirasida PFDM bilan o'ralgan qora tuynuklar o'ziga xos imzolarni namoyon etadi va bu qorong'i materiya hamda modifikatsiyalangan gravitatsiya nazariyalarini sinovdan o'tkazish uchun potensial kuzatuv imkoniyatlarini taqdim etadi.

Kalit so'zlar: Qora tuynuklar, Mukammal suyuqlik qorong'i materiya, Scalar-Tensor-Vector Gravitatsiya (STVG), Zarrachalar dinamikasi, Modifikatsiyalangan gravitatsiya, Hodisa gorizonti

ЧЕРНЫЕ ДЫРЫ, ОКРУЖЕННЫЕ ИДЕАЛЬНО ЖИДКОЙ ТЕМНОЙ МАТЕРИЕЙ В СТВГ

АННОТАЦИЯ: В данной работе мы исследуем свойства чёрных дыр, окружённых идеальной жидкостью тёмной материи (PFDM), в рамках скалярно-тензорно-векторной гравитации (STVG), также известной как модифицированная гравитация (MOG). Наличие PFDM вблизи чёрных дыр вносит отклонения от традиционных решений, влияя на геометрию и физические характеристики пространства-времени. Мы анализируем влияние PFDM на ключевые свойства чёрных дыр, включая структуру горизонта событий, термодинамические величины и условия устойчивости. Взаимодействие между STVG и PFDM формирует модифицированное гравитационное поле, что приводит к новым пониманиям термодинамики чёрных дыр и механизмов извлечения энергии. Кроме того, мы изучаем, как PFDM влияет на динамику частиц и процессы аккреции возле чёрных дыр, что может иметь наблюдаемые последствия в астрофизических сценариях. Наши результаты показывают, что чёрные дыры, окружённые PFDM в рамках STVG, проявляют уникальные сигнатуры, что предоставляет потенциальные наблюдательные тесты как для природы тёмной материи, так и для теорий модифицированной гравитации.

Ключевые слова: Чёрные дыры, Идеальная эидкость тёмной материи, Скалярно-тензорно-векторная гравитачия (СТВГ), Модифицированная гравитачия, Горизонт собътий, Динамика частии

INTRODUCTION

Black holes, as regions of intense gravitational fields, continue to be a focal point of research in modern astrophysics and theoretical physics. The study of black holes in the context of various modified gravity theories offers new insights into their fundamental properties and interactions. One such modified theory is Scalar-Tensor-Vector Gravity (STVG), which extends General Relativity by incorporating additional fields and interactions to address phenomena unexplained by conventional theories. In this paper, we explore the intriguing scenario of black holes surrounded by Perfect Fluid Dark Matter (PFDM) within the STVG framework. PFDM represents a type of dark matter modeled as a perfect fluid, which can significantly influence the spacetime geometry and dynamics around black holes. By analyzing this configuration, we aim to understand how PFDM affects the structural and thermodynamic properties of black holes and to investigate the modifications introduced by the STVG theory. Our study addresses several key aspects: the impact of PFDM on the event horizon and the spatial structure of the black hole, the changes in thermodynamic quantities such as temperature and entropy, and the implications for energy extraction and particle dynamics in the presence of PFDM. These investigations not only contribute to a deeper understanding of black holes in modified gravity theories but also offer potential observational signatures that could be tested with future astronomical observations. This introduction sets the stage for a detailed examination of black holes within STVG and PFDM, highlighting the relevance and potential impact of these theoretical advancements on our understanding of black hole physics and cosmology.

BASIC EQUATIONS

Here, we explicitly study a black hole immersed in the background of PFDM in STVG. The action and the corresponding equation of motion have the form

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}+L_{DM}\right) \quad (1)$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=T^{\phi}_{\mu\nu}-T^{DM}_{\mu\nu} \quad (2)$$

Here, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, $g \equiv det\left(g_{\mu\nu}\right)$ is the determinant of the metric tensor and $R_{\mu\nu}$ is the Ricci tensor, respectively. $B_{\mu\nu}$ is the massless (spin-one graviton) vector field strength tensor related to vector potential ϕ_μ, $B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$. In addition, L_{DM} gives the Lagrangian density of the PFDM. $T^{\phi}_{\mu\nu}$ and $T^{DM}_{\mu\nu}$ give the energy-momentum tensor for the vector field and DM, respectively. One may assume that the vector field is an analog of the electromagnetic field, and its field tensor is defined as

$$T^{\phi}_{\mu\nu} = \frac{-1}{4\pi}\left(B^{\alpha}_{\mu} B_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} B^{\alpha\beta} B_{\alpha\beta}\right)$$

$$(3)$$

with

$$\Delta_\mu B^{\mu\nu} = 0$$

$$(4)$$

$$\Delta_\alpha B^{\mu\nu} + \Delta_\nu B^{\mu\alpha} + \Delta_\mu B^{\alpha\nu} = 0$$

$$(5)$$

The gravitational source charge for the spin-one gravitational vector field ϕ_μ is $Q_g = \sqrt{\alpha} M$, where α is the field parameter that is responsible for running the gravitational constant $G = (1+\alpha) G_N [?]$. The energy-momentum tensors take the form

$$\left(T^{\mu}_{\nu}\right)^{\phi} = \frac{Q_g^2}{r^4} diag(-1, -1, 1, 1)$$

$$(6)$$

$$\left(T^{\mu}_{\nu}\right)^{DM} = diag\left(-\rho, P_r, P_\theta, P_\phi\right)$$

$$(7)$$

with $P_r = -\rho, P_\theta = P_\phi = P[?]$. The equation of state for PFDM takes the form $P = \frac{1}{2}\rho$. We need to solve the gravitational field equations to obtain the desired metric. The first step is to assume a spherical symmetric ansatz for the metric in the form

$$d s^2 = -e^{\chi} d t^2 + e^{-\chi} d r^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$(8)$$

Here, μ and ξ are assumed to be functions of r only. Using the Ansatz metric, the Einstein equations take the form $[??]$

$$e^{\chi}\left(\frac{1}{r^2} + \frac{\chi'}{r}\right) - \frac{1}{r^2} \quad ¿ -\rho - \frac{\alpha M^2}{r^4}$$

Here, prime $(')$ and double prime $('')$ denote the first and second derivatives for r and, we use $M \to (1+\alpha) M$. Assuming $\chi = \ln f(r)$, where $f \equiv f(r)$, the above equation simplifies to

$$r^2 f'' + 3 r f' + f + \frac{\alpha M^2}{r^2} - 1 = 0$$

$$(10)$$

The solution of the above equation is

$$f(r) = 1 - \frac{2(1+\alpha) M}{r} + \frac{\alpha (1+\alpha) M^2}{r^2} + \frac{\lambda}{r} \ln \frac{r}{¿\lambda \vee ¿¿}(11)$$

In the above expression (11), M represents mass of the black hole, α is the MOG parameter, and the parameter λ stands for PFDM contributions. Illustrates the radial profile of the function $f(r)$ under the influence of MOG and PFDM parameter combinations. The graphical behavior shows some interesting $f(r)$ behavior in response to the MOG and PFDM interactions. The solid black curves represent the Schwarzschild black hole case and cross zero at the event horizon at $r=2M$. The existence of α significantly contributes to the event horizon. At $\alpha=1$, the function crosses zero and falls into the black hole beyond $r \approx 3.6\,M$, which is always greater than the Schwarzschild radius corresponding to the positive values of the α parameter. Notably, black holes under the influence of PFDM further reduce the event horizon, making it even smaller than that of the Schwarzschild black hole case. As a result, the black holes become more dense and massive. Depicts the graphical behavior of MOG black holes in PFDM along α and λ parameters in the top and bottom panels, respectively. We observed fascinating behavior of the black hole horizons in response to the MOG interaction and the existence of PFDM near the black hole. Upon its graphical representation along the MOG parameter α, we noticed that α and λ enhance black hole horizons. On the other hand, we can also observe that the horizon behavior along λ changes initially for small values of α; it increases and then starts to decrease along λ. For higher values of α, that is, for almost beyond $\alpha > 0.35$, the horizons behave very interestingly, and it exists only for lower values of λ.

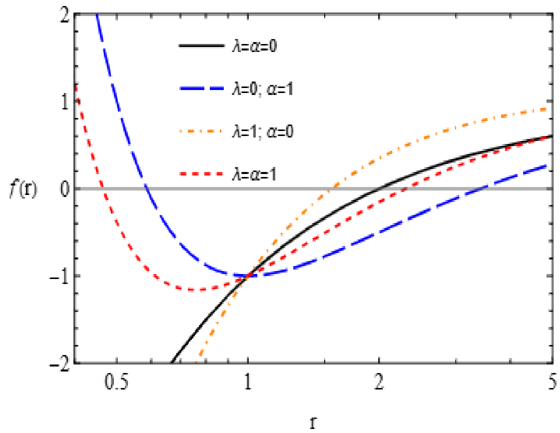

Figure: 1 Graph describing the behavior of the function $f(r)$, at different discrete values of the parameters α and λ.

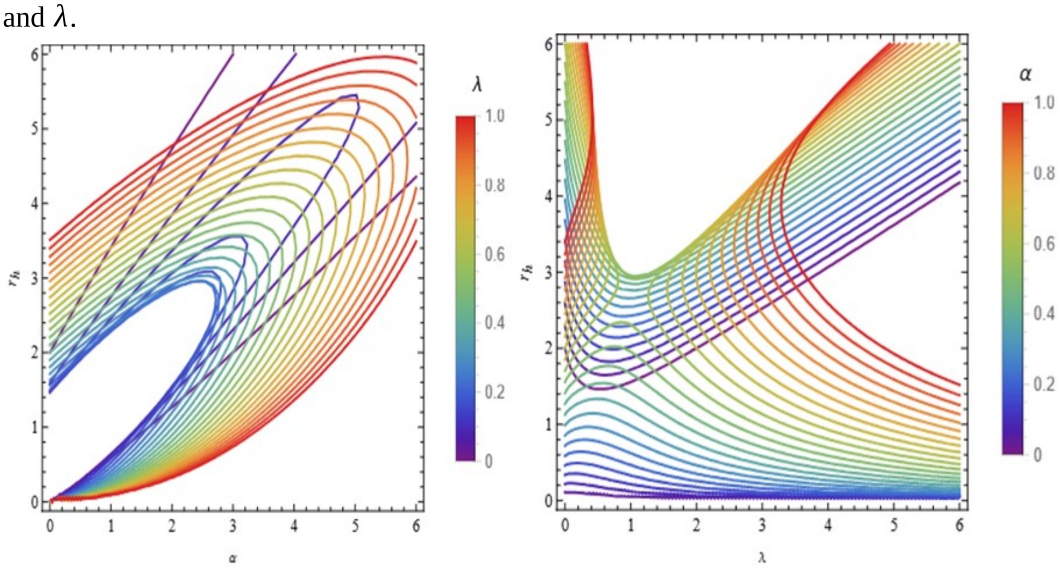

Figure: 2 Graphs showing the behavior of black hole's event horizon in response to the MOG (top) and FPDM (bottom) parameters.

NEUTRAL PARTICLE MOTION

Neutral particle motion refers to the movement of particles that possess no net electric charge. Electric fields do not influence these particles due to their lack of charge. However, they may still

be subject to other forces, such as gravitational, magnetic, or particle interactions. Examples of neutral particles include neutrons, neutrinos, and certain atoms or molecules with equal protons and electrons. Understanding the motion of neutral particles is crucial in various scientific fields, including nuclear physics, astrophysics, and materials science. Studying how neutral particles move and interact with their surroundings provides insights into the behavior of matter under different conditions and helps understand fundamental processes in the universe.
The Hamilton-Jacobi equation, which takes into account both interactions between the magnetized particles with scalar and magnetic fields, has the form

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}}-q\Phi_{\mu}\right)\left(\frac{\partial S}{\partial x^{\nu}}-q\Phi_{\nu}\right)=-m^2 \tag{12}$$

where $q=\sqrt{\alpha}m$ is gravitational test particle charge, and $q\Phi_{\mu}$ is the term that defines MOG interaction between the particles and the scalar field. There is an additional interaction between the particles and the scalar field,

$$\Phi_{\mu}=\frac{\sqrt{\alpha}M}{r}(-1,0,0,0) \tag{13}$$

The Lagrangian of the magnetized particles near the black-bounced black holes in MOG has a form that includes the MOG interaction,

$$L=\frac{1}{2}m g_{\mu\nu}u^{\mu}u^{\nu}+q\Phi_{\mu}u^{\mu} \tag{14}$$

Using the above Lagrangian, one can easily find the integrals of motion of magnetized particles $(p_{\phi}=L=mu^{\phi}$ and $p_t=-E=mu^t$ denoting the total angular momentum and total energy of the particle, respectively) in the following form,

$$-E=g_{tt}\dot{t}+\frac{q}{m}\Phi_t \tag{15}$$

$$l=g_{\phi\phi}\dot{\phi} \tag{16}$$

where $E=E/m$ and $l=L/m$ are the specific energy and angular momentum. In solving the above Eqs. (15) and (16), we obtain

$$\dot{t}=\frac{E+\frac{q}{m}\Phi_t}{-g_{tt}} \tag{17}$$

$$\dot{\phi}=\frac{l}{g_{\phi\phi}} \tag{18}$$

The Hamilton-Jacobi action for the motion of magnetized particles in the equatorial plane $(\theta=\pi/2)$ can be separated as follows

$$S=-Et+L\phi+S_r(r) \tag{19}$$

By making use of the Hamilton-Jacobi equation (12), we can easily obtain the equation of radial motion of the magnetized particles:

$$g_{rr}\dot{r}^2 = -\left[g^{tt}\left(E + \frac{q\Phi_t}{m}\right)^2 + g^{\phi\phi}l^2 + 1\right]$$

(20)

¿¿
¿

After certain calculations, we obtain the effective potential in the following form:

$$V_{\text{eff}}^{\pm} = \frac{\alpha M}{r} \pm \sqrt{-g_{tt}\left(1 + g^{\phi\phi}l^2\right)}$$

(21)

In case we do not consider the MOG interaction, the effective potential takes the value

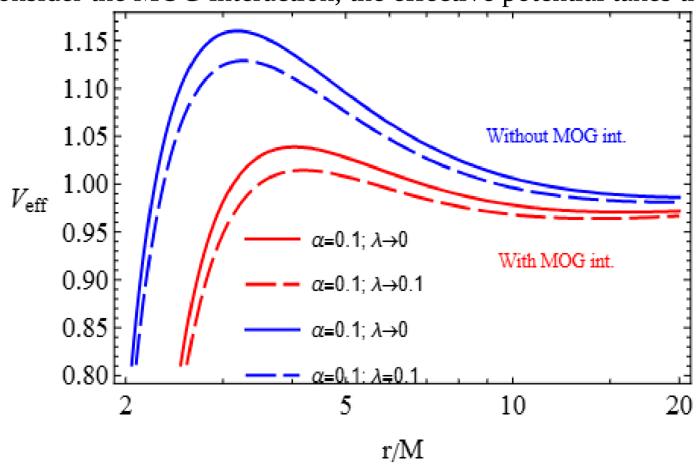

Figure: 3 Radial dependence of the effective potential at various discrete values of the parameters λ and α.

ISCO

The ISCO can mathematically be represented by the equation $\partial_{rr} V_{\text{eff}} = 0$. Typically, this criterion is combined with the circularity condition given in Eq. (??), which defines the Innermost Stable Circular Orbit (ISCO). As a result, the equation for the effective potential provided in Eq. (21) undergoes modification to fulfill these conditions.

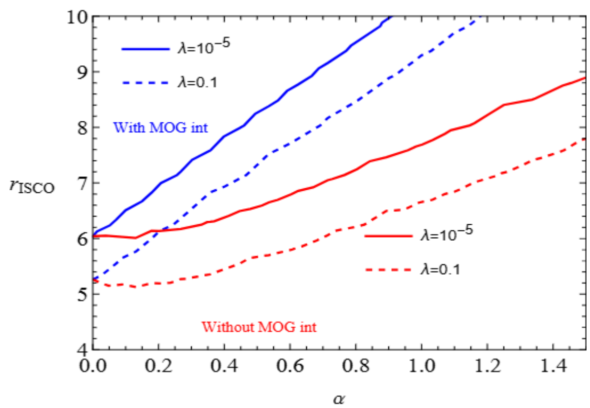

Figure: 4 Graphical representation of the ISCO radius along the dimensionless parameter α for two different values of the blackbounce parameter.

Illustrates the graphical description highlighting the significant impact of both MOG interactions and the black-bounce parameter on the ISCO radius. Particularly, the MOG interactions amplify the increase in r_{ISCO} along α, and this effect becomes more intense in the absence of black-bounce impacts $(a = 0)$. The black-bounce parameter decreases the r_{ISCO} in both cases, i.e., in the presence and absence of the MOG interactions. Thus, one can have higher r_{ISCO} values for the Schwarzschild case at $a=\alpha=0$. Furthermore, it is worth noting that the MOG interactions vanish at $\alpha=0$ and the r_{ISCO} of "With MOG int"meets the r_{ISCO} of "Without MOG int"case. It is also observed that MOG interaction weakens the black-bounce effects.

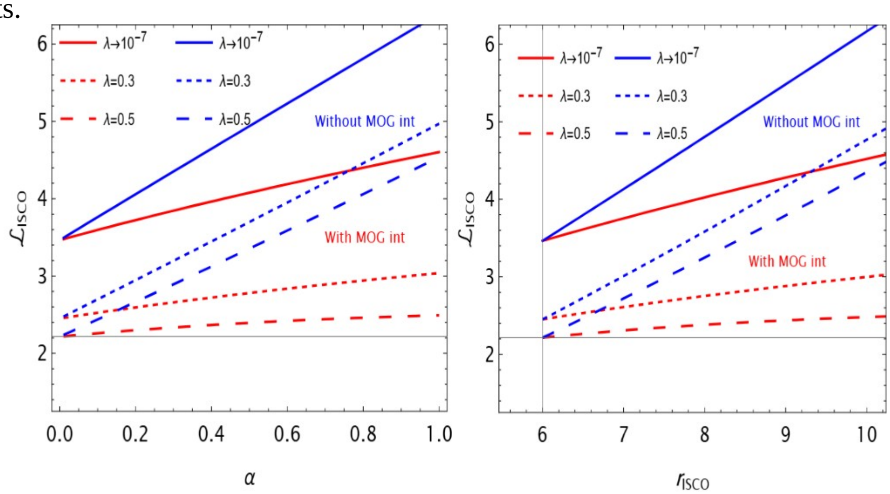

Figure: 5 The graph illustrates the behavior of angular momentum at ISCO along α (left) and r_{ISCO} (right), both with and without considering the MOG interactions.

We have plotted the graphical behavior of angular momentum at ISCO of particles in the SV-MOG spacetime along α and r_{ISCO} in the left and right panels, respectively. We noted that for $a=0$, ISCO increases along α in both cases (with and without MOG interaction). However, in the absence of MOG interactions, the increase becomes more pronounced. Hence, we can conclude that the MOG interactions diminish the angular momentum interaction) show higher ISCO values than the blue curves for the same α. The angular momentum at ISCO behaves similarly along r_{ISCO}; it has smaller values when MOG interactions are taken into account than when they are not (for details, please see Fig. 5). Furthermore, it increases linearly for both $a=0$ and $a\neq 0$ along r_{ISCO} for both cases (with and without MOG interactions). Notably, black holes with non-zero black-bounce parameters have higher ISCO than those without black-bounce effects. The graphical illustration of Fig. ?? shows the radial dependence of energy at the ISCO along iSco (right panel) and r_{ISCO} (left). For both $a=0$ and $a\neq 0$, energy at the ISCO reveals the same behavior along iSCO and r_{ISCO}. As in both cases, the ISCO decreases without considering the MOG interaction while showing a much higher increase by considering the MOG interaction along ISCO. In addition, the black-bounce parameter considerably contributes to the ISCO in both cases. In contrast, it impacts on ISCO decrease effects of the MOG interactions, particularly at higher values of ISCO. Moreover, the left panel shows how the ISCO energy changes with the parameter α under different conditions. For instance, the MOG interaction and a have influenced these relationships.

CONCLUSIONS AND DISCUSSIONS

In this study, we have investigated the properties of black holes surrounded by Perfect Fluid Dark Matter (PFDM) within the framework of Scalar-Tensor-Vector Gravity (STVG). Our analysis reveals several key findings that advance our understanding of black hole physics in modified gravity theories. Impact of PFDM on Black Hole Structure The presence of PFDM significantly alters the structural properties of black holes, including the event horizon and the overall geometry of the spacetime. PFDM influences the distribution of mass and energy around the black hole, leading to modifications in the shape and extent of the event horizon. Energy Extraction and Particle Dynamics: The presence of PFDM influences the mechanisms of energy extraction from black holes, such as the Penrose process. Additionally, PFDM affects the dynamics of particles near the black hole, impacting the efficiency of energy conversion processes. These changes could lead to observable differences in high-energy astrophysical phenomena. Observational Signatures: The theoretical modifications introduced by PFDM in STVG offer potential observational signatures. Future astronomical observations and experiments could provide insights into the nature of dark matter and validate or refute the predictions of modified gravity theories. Comparing theoretical models with observational data will be crucial for testing the validity of STVG and PFDM effects. Future Research Directions: This study opens several avenues for future research. Further investigations could focus on detailed numerical simulations to explore the full impact of PFDM on black hole dynamics. Additionally, exploring the interplay between PFDM and other forms of dark matter or modified gravity theories could provide a more comprehensive understanding of these phenomena.

In summary, our findings highlight the importance of considering PFDM and modified gravity theories in the study of black holes. The interplay between PFDM and STVG introduces new dimensions to our understanding of black hole physics and offers promising directions for future research and observational endeavors.

REFERENCES

1. B. Toshmatov, O. Rahimov, B. Ahmedov, A. Ahmedov, Galaxies 9(3), 65 (2021). doi:10.3390/galaxies9030065

2. A. Abdujabbarov, J. Rayimbaev, B. Turimov, F. Atamurotov, Physics of the Dark Universe 30, 100715 (2020). doi:https://doi.org/10.1016/j.dark.2020.100715.URL https://www.sciencedirect.com/science/article/ pii/S2212686420304283

3. L. Amendola, M. Kunz, D. Sapone, Journal of Cosmology and Astroparticle Physics2008(4), 013 (2008).doi:10.1088/1475-7516/2008/04/013

4. M. Pitkin, S. Reid, S. Rowan, J. Hough, Living Reviewsin Relativity 14(1), 5 (2011). doi:10.12942/lrr-2011-5

5. T.Liu, M.Biesiada, S.Tian, K.Liao, Phys.Rev.D109(8),084074(2024). doi:10.1103/PhysRevD.109.084074

6. E.H.T. Collaboration, K. Akiyama, A. Alberdi, W. Alef, K. Asada, R. Azuly, et al., Astrophys. J. Lett 875(1), L1(2019)

7. E.H.T. Collaboration, et al., Astrophysical Journal Letters 930(2), L12 (2022)

8. R. Shaikh, Monthly Notices of the Royal Astronomical Society523(1), 375 (2023). doi:10.1093/mnras/stad1383

9. J. Mazza, E. Franzin, S. Liberati, Journal of Cosmology and Astroparticle Physics 2021(4), 082 (2021). doi:10.1088/1475-7516/2021/04/082

10. J. D. Jackson, R. F. Fox, Classical Electrodynamics, 3rd ed., American Journal of Physics 67 (1999) 841-842.

11. P. A. M. Dirac, Classical Theory of Radiating Electrons, Proceedings of the Royal Society of London Series A 167 (1938) 148−169.

12. L. D. Landau, E. M. Lifshitz, The Classical Theory of Fields, Course of Theoretical Physics, Volume 2, Elsevier ButterworthHeinemann, Oxford, 2004.

13. A. Tursunov, M. Kološ, Z. Stuchlík, D. V. Gal'tsov, Radiation Reaction of Charged Particles Orbiting a Magnetized Schwarzschild Black Hole, ApJ 861 (2018) 2.

14. A. A. Tursunov, M. Kološ, Z. Stuchlík, Orbital widening due to radiation reaction around a magnetized black hole, Astronomische Nachrichten 339 (2018) 341-346.

15. A. A. Shoom, Synchrotron radiation from a weakly magnetized Schwarzschild black hole, Phys. Rev. D 92 (2015) 124066.

16. T. Damour, N. Deruelle, Radiation reaction and angular momentum loss in small angle gravitational scattering, Physics Letters A 87 (1981) 81-84.

17. T. Damour, Gravitational Radiation Reaction in the Binary Pulsar and the Quadrupole-Formula Controversy, Phys. Rev. Lett. 51 (1983) 1019-1021.

18. L. Blanchet, T. Damour, Radiative gravitational fields in general relativity. I. General structure of the field outside the source, Proceedings of the Royal Society of London Series A 320 (1986) 379-430.

19. C. Cutler, D. Kennefick, E. Poisson, Gravitational radiation reaction for bound motion around a Schwarzschild black hole, Phys. Rev. D 50 (1994) 3816-3835.

20. B. Turimov, H. Alibekov, P. Tadjimuratov, A. Abdujabbarov, Gravitational synchrotron radiation and Penrose process in STVG theory, Phys. Lett. B 843 (2023) 138040.

21. J. W. Moffat, Scalar tensor vector gravity theory, JCAP 2006 (2006) 004.

22. J. W. Moffat, Black holes in modified gravity (MOG), European Physical Journal C 75 (2015) 175.

23. J. W. Moffat, Time delay predictions in a modified gravity theory, Classical and Quantum Gravity 23 (2006) 6767-6771.

24. J. W. Moffat, V. T. Toth, Testing Modified Gravity with Globular Cluster Velocity Dispersions, ApJ 680 (2008) 1158-1161.

25. J. R. Brownstein, J. W. Moffat, The Bullet Cluster 1E0657-558 evidence shows modified gravity in the absence of dark matter, MNRAS 382 (2007) 29-47.