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KUCHSIZ MAGNIT MAYDONIDA IKKI O'LCHOVLI ELEKTRON GAZNING MAGNIT KIRITUVCHANLIGI VA KONSENTRATSIYASINING SONLI VA ANALITIK TAHLILI

Annotatsiya: *Ikki o'lchovli elektron gazning magnit kirituvchanligining temperaturaga bog'lanishini ifodalovchi yangi umumlashgan analitik formula olingan. Shuningdek, parabolik zona modeli uchun kimyoviy potensialning konsentratsiyaga bog'lanishi grafigi keltirilgan. Hisoblashlar cheksiz chuqur kvant o'ra misolida bajarilgan. Magnit kirituvchanlik va konsentratsya uchun keltirilgan ushbu tenglamalarning (* $L \to \infty$ *da, bu yerda kvant o'raning kengligi) 3D gazning o'xshash tenglamalariga o'tishi tekshirildi.*

Kalit so'zlar: *Geterotuzilma, Kvant o'ra, Kimyoviy potensial, Fermi energiyasi, Minizona, Magnitlanganlik, Magnit kirituvchanlik.*

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ЧИСЛЕННЫЙ И АНАЛИТИЧЕСКИЙ АНАЛИЗ МАГНИТНОЙ ВОСПРИИМЧИВОСТИ И КОНЦЕНТРАЦИИ ДВУМЕРНОГО ЭЛЕКТРОННОГО ГАЗА В СЛАБОМ МАГНИТНОМ ПОЛЕ

Аннотация: *Получена новая обобщенная аналитическая формула, представляющая зависимость магнитной восприимчивости двумерного электронного газа от температуры. Также приведен график зависимости химического потенциала от концентрации для модели параболической зоны. Расчеты проводились на примере бесконечно глубокой квантовой ямы. Была проверена конвертация этих уравнений, приведенных для магнитной восприимчивости концентрации (в данном случае ширины квантовая яма), в аналогичные уравнения трехмерного газа.*

Ключевые слова: *Гетероструктура, Квантовая яма, Химический потенциал, энергия Ферми, Минизона, Намагниченность, Магнитная восприимчивости.*

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NUMERICAL AND ANALYTICAL ANALYSIS OF MAGNETIC SUSCEPTIBILITY AND CONCENTRATION OF TWO-DIMENSIONAL ELECTRON GAS IN A WEAK MAGNETIC FIELD

Abstract: *A new generalized analytical formula is obtained, representing the dependence of the magnetic susceptibility of a two-dimensional electron gas on temperature. A graph of the dependence of the chemical potential on concentration for the parabolic zone model is also given. Calculations were performed using an infinitely deep quantum well as an example. The conversion of these equations given for magnetic susceptibility and concentration (in this case, the width of quantum well) into similar equations for a three-dimensional gas was verified.*

Keywords: *Heterostructure, Quantum well, Chemical potential, Fermi energy, Miniband, Magnetization, Magnetic susceptibility.*

KIRISH

Metall plyonkalar yoki yarimo'tkazgichli geterotuzulmalar kvant o'rasida (KO') tashuvchilarning spektri qisman kvantlangan - kvazi-ikki o'lchovlidir (2D). Natijada yangi fizik effektlar kuzatiladi [1-5].

Kuchsiz magnit maydonlarda va tashuvchilarning past konsentratsiyasida ular orasidagi Kulon o'zaro ta'siri sezilarli bo'lishi mumkin va elektron gaz ideal emas. Natijada, spektr qayta normallashtiriladi va ba'zi gaz parametrlari (masalan, effektiv massa, magnit kirituvchanllik) tashuvchining kontsentratsiyasiga bog'liq bo'ladi.

ADABIYOTLAR TAHLILI

Keyingi o‘n yilliklarda bunday qonuniyatlarni o‘rganishga bir qancha eksperimental va nazariy ishlar bag‘ishlangan [6-13]. Ushbu ishlarning aksariyati bitta KO' minizonasidagi ikki o'lchovli gazni ko'rib chiqadi.

Ma'lumki [14] kuchsiz magnit maydonda elektron gazning spin magnit kirituvchanligi termodinamik holatlar zichligi (THZ, $\partial n/\partial\mu$) bilan aniqlanadi, bu yerda n - konsentratsiya va μ - ideal gazning magnit maydonga bog'liq bo'lmagan kimyoviy potentsiali. Bundan tashqari, [1-5] dan ma'lumki, 2D gazda past haroratlarda, n_S kontsentratsiyasining ortishi (yoki μ ortishi bilan) THZ keskin o'zgaradi. Harorat ko'tarilgach, o'tkir sakrashlar asta-sekin silliqlashadi.

Ushbu maqola bunday muammolarning faqat bir qismini ko'rib chiqadi. Birinchidan, 2D elektron gaz uchun Pauli magnit kirituvchanligini aniqlaydigan asosiy formulalar va tenglamalar berilgan. Ikkinchidan ushbu tenglamalarning ($L \to \infty$ da, Bu yerda L KO' ning kengligi) 3D gazning o'xshash tenglamalariga o'tishi tekshiriladi. Parabolik spektrga ega bo'lgan ideal gaz ko'rib chiqiladi va KO' cheksiz chuqurlikda deb hisoblanadi. Kimyoviy potensial va kontsentratsiya orasidagi bog'lanish grafikda taqdim etiladi.

TADQIQOT METODOLOGIYASI

KO' tekisligiga perpendikulyar qo'llaniladigan magnit maydon mavjud bo'lganda, elektron spektri quyidagicha ifodalanishi mumkin

$$E_{k,n} = \frac{\hbar^2 k^2}{2m^*} + E_n \pm \beta B, \qquad \beta = \frac{g^*}{g_0}\frac{e\hbar}{2m_0}, \qquad E_n = \frac{\hbar^2 \pi^2 n^2}{2m^* L^2}, \quad n = 1,2,3... \tag{1}$$

Bu yerda $k^2 = k_x^2 + k_y^2$ - kvant qudug'i tekisligidagi elektron to'lqin vektorining kvadrati, m^*-o'tkazuvchanlik zonasining pastki qismidagi elektronning effektiv massasi, E_n-L kenglikdagi KO'dagi energiya sathlari, g^*-spin ajralish faktori va $g_0 = 2$. Ideal Fermi gazining termodinamik potentsiali ko'rinishga ega [14]

$$\Omega = -k_B T \sum_{\mathbf{k}\sigma} \ln(1 + e^{\frac{\mu - E_{\mathbf{k}\sigma}}{k_B T}}) \tag{2}$$

Bu yerda T - gaz harorati, k_B - Boltsman doimiysi. (1) ni (2) ga almashtirib, uni shaklga keltirish mumkin

$$\Omega = -k_B T \frac{L_x L_y}{2} \frac{m^*}{\pi\hbar^2} \sum_{n=1}^{\infty} \int_0^{\infty} d\varepsilon [\ln(1 + e^{\frac{\mu - (\varepsilon + E_n + \beta B)}{k_B T}}) + \ln(1 + e^{\frac{\mu - (\varepsilon + E_n - \beta B)}{k_B T}})] \tag{3}$$

Birlik yuzaga to'g'ri keluvchi termodinamik potentsial quyidagi shaklga ega

$$\Omega_S = -\frac{k_B T}{2} D \sum_{n=1}^{\infty} \int_0^{\infty} d\varepsilon [\ln(1 + e^{\frac{\mu - (\varepsilon + E_n + \beta B)}{k_B T}}) + \ln(1 + e^{\frac{\mu - (\varepsilon + E_n - \beta B)}{k_B T}})], \qquad D = \frac{m^*}{\pi \hbar^2} \tag{4}$$

E'tibor bering (4) formulada μ kimyoviy potentsiali B magnit maydoniga bog'liq. Kuchsiz maydonlar uchun $\beta B << k_B \mathrm{T} \sim \mu$ shart bajarilganida (4) ifodani kichik βB uchun qatorga yoyish mumkin. Natijada quyidagi ko’rinishga keladi

$$\Omega_S = \Omega_S(\mu - \beta B) + \Omega_S(\mu + \beta B) \approx \Omega_0 + \frac{\beta^2 B^2}{2}\frac{\partial^2 \Omega_0}{\partial \mu^2} = \Omega_0 - \frac{\beta^2 B^2}{2}\frac{\partial n_S}{\partial \mu} \quad (5)$$

Bu yerda $n_S = -\left(\frac{\partial \Omega_0}{\partial \mu}\right)_{\mathrm{T,S,B}}$ va Ω_0 magnit maydonsiz ya’ni $B=0$ dagi termodinamik potensial ekanligi hisobga olinadi.

$$\Omega_0 = -k_B T D \sum_{n=1}^{\infty} \int_0^{\infty} d\varepsilon \ln(1 + \mathrm{e}^{\frac{\mu - (\varepsilon + E_n)}{k_B \mathrm{T}}}) \quad (6)$$

(5) formulada n_S-2D elektronlar konsentratsiyasi. U ko’rinishga ega

$$n_S = -\left(\frac{\partial \Omega_0}{\partial \mu}\right)_{\mathrm{T,S,B}} = D \sum_{n=1}^{\infty} \int_0^{\infty} \frac{d\varepsilon}{\mathrm{e}^{\frac{\varepsilon + E_n - \mu}{k_B \mathrm{T}}} + 1} = D k_B T \sum_{n=1}^{\infty} \ln(1 + e^{\frac{\mu - E_n}{k_B T}}) \quad (7)$$

(6) va (7) formulalardagi μ kimyoviy potentsial B maydoniga bog'liq emas. Magnitlanish va magnit kirituvchanlik quyidagi munosabatlar bilan aniqlanadi

$$M_S = -\left(\frac{\partial \Omega_S}{\partial B}\right)_{\mathrm{T,S,\mu}} = \beta^2 B \frac{\partial n_S}{\partial \mu}, \qquad \chi_S = \left.\frac{\partial M_S(B)}{\partial B}\right|_{S, n_S} = \beta^2 \frac{\partial n_S}{\partial \mu} \quad (8)$$

(7) ni hisobga olgan holda (8) dagi ikkinchi formula quyidagi shaklga keladi

$$\chi_S = \beta^2 D \sum_{n=1}^{\infty} \frac{1}{\mathrm{e}^{\frac{E_n - \mu}{k_B \mathrm{T}}} + 1} \quad (9)$$

TAHLILLAR VA NATIJALAR

(7) tenglama μ kimyoviy potentsialini $n_S,\ T,\ L$ funktsiyasi sifatida belgilanadi. (7) va (9) tenglamalar sistemasidan μ ni chiqarib tashlab, χ_S magnit kirituvchanligini $n_S,\ T,\ L$ funksiyasi sifatida topish mumkin. $T=0$ holat uchun $\mu = E_F$ (Bu yerda E_F- Fermi energiyasi) deb faraz qilsak, (7) va (9) tenglamalar sistemasini quyidagi shaklga keltirish mumkin.

$$\chi_S = \beta^2 D \sum_{n=1}^{\infty} \theta(E_F - E_n), \qquad n_S = D \sum_{n=1}^{\infty} (E_F - E_n)\theta(E_F - E_n) \quad (10)$$

Bu yerda $\theta(E_F - E_n)$ Heaviside funktsiyasidir. Ushbu tenglamalarda yig'indi belgisi ostidagi hadlar musbat bo'lishi kerak. Manfiy shartlar (ya'ni, $E_F < E_n$ holatida) Heaviside funktsiyasi($\theta(E_F - E_n)$) ta'sirida nolga aylanadi. Ikkinchi tenglama Fermi energiyasi E_F va umumiy 2D elektron kontsentratsiyasi n_s o'rtasidagi munosabatni o'rnatadi.

KO' kengligi makroskopik $L \to \infty$ bo'lganda, bu holda, $E_1 \to 0$ va to'lqin vektori ***k*** va ***n*** kvant soni deyarli uzluksiz miqdorlar ((1) ga qarang) deb olingan cheklovda (7) va (9) munosabatlarda yig'indidan integralga o'tishimiz mumkin. Bunday holda, gaz uch o'lchovli bo'lib, biz $n_V = n_S / L$ va $\chi_V = \chi_S / L$ belgilaridan foydalanishimiz mumkin. So'ngra, (7) tenglamani bo'laklab integrallashdan so'ng, quyidagini olamiz

$$n_V = \frac{2^{1/2} m^{*3/2}}{\pi^2 \hbar^3} \int_0^\infty \frac{\sqrt{E}\,dE}{e^{\frac{E-\mu}{k_B T}} + 1} \qquad (11)$$

Xuddi shunday, (9) tenglamadan biz magnit kirituvchanlik ifodasini yozamiz

$$\chi_V = \beta^2 \frac{m^{*3/2}}{2^{1/2} \pi^2 \hbar^3} \int_0^\infty \frac{1}{e^{\frac{E-\mu}{k_B T}} + 1} \frac{dE}{\sqrt{E}} \qquad (12)$$

$T = 0$ holatida (11) va (12) dan quyidagi natijalar olinadi

$$n_V = \frac{2\sqrt{2}}{3} \frac{(m^*)^{3/2}}{\pi^2 \hbar^3} (E_F)^{3/2}, \qquad \chi_V = \beta^2 \frac{2^{1/2} m^{*3/2} E_F^{\ 1/2}}{\pi^2 \hbar^3} \qquad (13)$$

(13) dan E_F ni istisno qilish Pauli spin magnit kirituvchanligiga olib keladi [15]

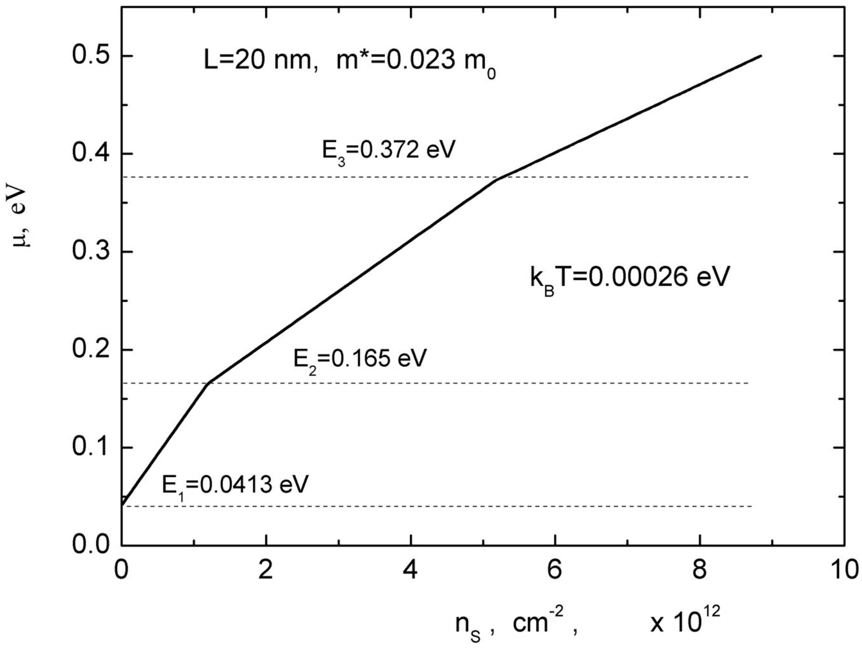
$$\chi_V = \beta^2 D \left(\frac{3 n_V}{\pi} \right)^{1/3} \qquad (14)$$

Umumiy holatda KO' qalinligi L yoki harorat(T)ning istalgan qiymati uchun (7) va (9) tenglamalar tizimini analitik yechish mumkin emas. Quyida $\mu(n_S, T, L)$ bog'lanishning sonli natijalari keltirilgan.

(7) ifodada $D = m^* / (\pi \hbar^2) = m^* / m_0 \cdot 417 \cdot 10^{12} \cdot (\mathrm{eV})^{-1} \cdot \mathrm{c}m^{-2}$ va biz InAs yarimo'tkazgichiga mos keladigan effektiv massa uchun $m^* = 0{,}023 \cdot m_0$ ni oldik. KO' (1) dagi elektron energiya spektrini $E_n = E_1 n^2, \quad n = 1,2,3..$ ko'rinishida ifodalaymiz,

bu yerda $E_1 = 0.38\text{eV} \times (m^*/m_0)\,(L/nm)^2$. KO’ kengligi $L = 20\,nm$ bo'lsa, birinchi uchta energiya sathi $E_n = [\ 0.0413,\ \ 0.165,\ \ 0.372\ ]\ eV$ ga teng bo'ladi.

Berilgan harorat va KO’ kengligining qat'iy qiymati uchun (7) tenglama n_S kontsentratsiyasi va μ kimyoviy potentsiali o'rtasidagi munosabatni aniqlaydi. Ushbu bog'liqlik 1-rasmda keltirilgan.



1-Rasm. Ideal ikki o'lchovli elektron gazning kimyoviy potentsialining $T \approx 3K$ va $L = 20\,nm$ qat'iy qiymatlaridagi konsentratsiyaga bog'liqligi.

1-Rasmdagi grafikdan ko'rinib turibdiki, bunda noaniqliklar mavjud. Ushbu bog’liqlikning tabiati turli xil minizonlarning ustma-ust tushishi bilan tushuntiriladi, masalan, [16] ga qarang.

XULOSA

Ushbu maqoladagi tahlillar va sonli natijalar shuni ko'rsatdiki, ikki o'lchovli ideal elektron gazda barcha termodinamik kattaliklar kabi gazning magnit kirituvchanligi konsentratsiyaga kuchli bog'liq. Tabiiyki, past konsentratsiyalarda ko'rib chiqilayotgan model kirish qismida muhokama qilingan sabablarga ko'ra gaz ideal emas. Bunday holda, elektronlar orasidagi Kulon o'zaro ta'sirini hisobga olish kerak, bu esa spektrni qayta tahlil qilishga va tashuvchining effektiv massasiga olib kelishi mumkin. Kvant o’ra kengligi makroskopik ya’ni, $L \to \infty$ bo’lganda, (7) va

(9) ifodalar uch o’lchovli gazning o’xshash tenglamalarga o’tar ekan. Biror bir minizona yuqoridagi minizonalar bilan ustma-ust tushganligi uchun kimyoviy potensial kvantlangan energetik sathlarni kesib o’tayotganda konsentratsiya keskin ortishi kuzatildi.

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