# KO‘P QUVLOVCHI VA BITTA QOCHUVCHI UCHUN “QUTULISH CHIZIG‘I”GA EGA *IG*-CHIZIQLI O‘YIN

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**Annotatsiya.** Ushbu maqolada quvlovchilarning boshqaruvlari integral chegaralanishga, qochuvchining boshqaruvi esa geometrik chegaralanishga ega chiziqli dinamika bilan berilgan differensial o‘yinda quvish masalasi o‘rganilgan. Bunda quvlovchilar uchun parallel quvish strategiyasi (qisqacha, П-strategiya) taklif qilingan va uning yordamida quvish masalasining yangi yetarli shartlari topilgan.

**Kalit so‘zlar.** Differensial o‘yin, quvlovchi, qochuvchi, П-strategiya, geometrik chegaralanish, integral chegaralanish, qutulish chizig‘i.

# *IG*-LINEAR GAME WITH “LIFE-LINE” FOR MULTI-PURSUER AND ONE EVADER

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**Annotation.** This paper studies a pursuit problem in a differential game given by linear dynamics with integral constraints on pursuers’ controls and with geometric constraint on evader’s control. Here, the strategy of parallel pursuit (P-strategy for short) is proposed for pursuers, and with its help, noval sufficient conditions of the pursuit problem are determined.

**Keywords**. Differential game, pursuer, evader, П-strategy, geometric constraint, integral constraint, life-line.

# *IG*-ЛИНЕЙНАЯ ИГРА С «ЛИНИЕЙ ЖИЗНИ» ДЛЯ НЕСКОЛЬКИХ ПРЕСЛЕДОВАТЕЛЕЙ И ОДНОГО УБЕГАЮЩЕГО

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**Аннотация**. В данной статье исследуется задача преследования в дифференциальной игре, заданной линейной динамикой с интегральными ограничениями на управления преследователей и с геометрическими ограничениями на управление убегающих. Здесь для преследователей предлагается стратегия параллельного преследования (сокращенно, П-стратегия) и с ее помощью определяются новые достаточные условия задачи преследования.

**Ключевые слова**. Дифференциальная игра, преследователь, убегающий, П-стратегия, геометрическое ограничения, интегральное ограничения, линией жизни.

## 1. Introduction

Differential games were initiated by Isaacs [23]. Fundamental results were obtained by Bercovitz [6, 7], Basar [4, 5], Chikrii [9], Elliot and Kalton [10], Fleming [11, 12], Friedman [13], Hajek [15, 16], Ho, Bryson and Baron [17], Pontryagin [27], Krasovskii [24], Petrosjan [26], Pshenichnyi [28, 29], Subbotin [36, 37] and others. The book of Isaacs [23] contains many specific game problems that were discussed in details and proposed for further study. One of them is the “Life-line” problem, which rather was comprehensively studied by Petrosjan [26] by approximating measurable controls with most efficient piecewise constant controls that presents the strategy of parallel approach. Later this strategy was called П-strategy. The strategies proposed in [1, 26, 28] for a simple motion pursuit game with geometrical constraints became the starting point for the development of the pursuit methods in games with multiple pursuers (see e.g. [2, 14, 30–34]). A relay pursuit-evasion problem with two pursuers and one evader was studied [36]. The problem is reduced to one pursuer and one evader problem subject to a state constraint. To prolong the capture time, a suboptimal control strategy for the evader was proposed. A relay-pursuit strategy was applied in [3], according to which only one pursuer is assigned to go after the evader at every instant of time. The paper of [35] is devoted to capturing the evader in the shortest time. The Apollonius circles formed by the evader and each pursuer are used to analyze how the evader can find a better strategy to escape or prolong the capture time. The constructing of optimal strategies of players and finding the value of the game are difficult and important problems of differential games. Pashkov and Terekhov [25] studied a simple motion differential game of optimal approach of two pursuers and one evader. In the case where maximal speeds of pursuers are equal, optimal strategies of players were constructed. The works [19–23] are devoted to a simple motion differential game of $k$ pursuers and one evader.

## 2. The problem statement

Consider differential game when pursuer $P_i$, $i=1,2,\dots,m$, and evader $E$, having radius vectors $x_i$ and $y$ correspondingly, move in space $R^n$. If their velocity vectors are $u_i$ and $v$ then the game will be described by the equations:

$$P_i: \ \dot{x}_i + \delta x_i = u_i, \quad x_i(0) = x_{i0}, \tag{2.1}$$

$$E: \ \dot{y} + \delta y = v, \quad y(0) = y_0, \tag{2.2}$$

where $x_i, y, u_i, v \in R^n$, $n \ge 1$, $\delta \in R$; $x_{i0}$, $y_0$ are initial positions of players $P_i$ and $E$. Here the temporal variation of $u_i$ must be a measurable function $u_i(\cdot):[0,\infty) \to R^n$, which adheres to integral constraint of the form

$$\int_0^\infty |u_i(s)|^2 ds \le \rho_{i0}, \quad i = 1, 2, \dots, m, \tag{2.3}$$

where $\rho_{i0}$ are positive numbers. From the physical viewpoint, $\rho_{i0}$ in (2.3) state the initial resources of players $P_i$. Denote by $U_I^i$ the class of the pursuers' admissible controls satisfying constraint (2.3).

Similarly, the temporal variation of $v$ must be a measurable function $v(\cdot):[0,\infty) \to R^n$, and on this vector-function, we impose geometric constraint (briefly, $G$- constraint)

$$|v(t)| \le \sigma \text{ for almost every } t \ge 0, \tag{2.4}$$

where $\sigma$ is a nonnegative number which states the maximal velocity of player $E$. We denote by $V_G$ the class of the evader's admissible controls satisfying constraint (2.4).

In [32], B.T.Samatov studied the *IG*-Game with one pursuer and one evader for the case $\delta = 0$ in *IG*-Linear Game (2.1)–(2.4).

In *IG*-Linear Game (2.1)–(2.4), the aim of players $P_i$ is to capture player $E$, i.e. to reach the equality $x_i(t) = y(t)$, where $x_i(t)$ and $y(t)$ are motion trajectories of the players. Player $E$ strives to avoid meeting with any player $P_i$, and if this is not possible, to postpone the moment of the meeting as long as possible. Naturally, this is a preliminary problem setting.

At the present, we are going to study mainly the game with phase constraints for the evader being given by a subset $A$ of $R^n$, which is called the "Life-line" [23] (for the evader naturally). Notice that in the case $A = \varnothing$, we have a simple *IG*-Linear Game (2.1)–(2.4).

**Definition 1.** By the equations and initial conditions in (2.1)–(2.2), any pairs $(x_{i0}, u_i(\cdot))$, $u_i(\cdot) \in U_I^i$, and $(y_0, v(\cdot))$, $v(\cdot) \in V_G$, generate the solutions

$$x_i(t) = e^{-\delta t}\left(x_{i0} + \int_0^t e^{\delta s} u_i(s)ds\right), \quad y(t) = e^{-\delta t}\left(y_0 + \int_0^t e^{\delta s} v(s)ds\right). \tag{2.5}$$

In this case, $x_i(t)$ is called the *pursuer's motion trajectory*, and $y(t)$ is called the *evader's motion trajectory*.

**Definition 2.** For each triple $(\rho_{i0}, u_i(\cdot))$, $u_i(\cdot) \in U_I^i$ the scalar quantity

$$\rho_i(t) = \rho_{i0} - \int_0^t |u_i(s)|^2 ds, \quad \rho_i(0) = \rho_{i0}, \quad t \ge 0 \tag{2.6}$$

is called the *pursuer's residual resource* at a time moment $t$.

**Definition 3.** The map $u_i(\cdot): V_G \to U_I^i$ is called the *strategy for pursuer* $P_i$ if the following properties hold:

$1^o$. (Admissibility) For every $v(\cdot) \in V_G$, the inclusion $u_i(\cdot) = u_i[v(\cdot)] \in U_I^i$ is valid.

$2^o$. (Volterranianity) For every $v_1(\cdot), v_2(\cdot) \in V_G$ and $t \ge 0$, the equality $v_1(s) = v_2(s)$ a.e. on $[0, t]$ implies $u_{i_1}(s) = u_{i_2}(s)$ a.e. on $[0, t]$ with $u_i(\cdot) = u_i[v(\cdot)] \in U_I^i$, $i = 1, 2, \dots, m$.

**Definition 4.** A strategy $u_i(v)$ is called *winning* for pursuer $P_i$ on the interval $[0,T]$ in *IG*-Linear Game (2.1)–(2.4) if for every $v(\cdot) \in V_G$, there exists a moment $t^* \in [0,T]$ such that $x_i(t^*) = y(t^*)$.

**Definition 5.** Assume $y_0 \notin \bar{A} \subset R^n$ and a strategy $u_i(v)$ is winning for pursuer $P_i$ on the interval $[0,T]$ while evader $E$ stays in zone $R^n \setminus \bar{A}$, i.e., $y_t(\cdot) = \{y(s) : 0 \le s \le t\} \notin \bar{A}$ and $t \in [0,T]$. Then $u_i(v)$ is called *winning* for pursuers $P_i$, $i = 1, 2, \dots, m$ on the interval $[0,T]$ in *IG*-Linear Game with the "Life-line" (2.1)–(2.4).

Notice that $\bar{A}$ doesn't restrict any motion of $P_i$. Here $\bar{A}$ is the closure of the set $A \subset R^n$.

Let $x_{i0} \ne y_0$ and let a current value of control $v(t)$, $t \ge 0$, be given, where $v(\cdot) \in V_G$. Suppose that the triple $(\nu_{i0}, \sigma)$ is a parametric state of *IG*-Linear Game (2.1)–(2.4) and denoted it by $s_i$. We find the following nonempty simply connected set of such states $s_i$:

$$\mathbf{S}_{IG}^i = \mathbf{S}_1^i \cup \mathbf{S}_2^i \cup \mathbf{S}_3^i,$$

where $\mathbf{S}_1^i = \{s_i : \nu_{i0} \ge 0, \sigma \ge 0\}$, $\mathbf{S}_2^i = \{s_i : \nu_{i0} > 2\sigma, \sigma \ge 0\}$, $\mathbf{S}_3^i = \{s_i : \nu_{i0} \ge 4\sigma, \sigma \ge 0\}$, and $\mathbf{S}_1^i, \mathbf{S}_2^i, \mathbf{S}_3^i$ are mutually disjoint sets.

**Definition 6.** The function

$$u_{IG}^i(v) = v - \lambda_{IG}^i(v)\xi_{i0} \tag{2.7}$$

is called *the parallel pursuit strategy*, or the $\Pi_{IG}^i$*-strategy* for player $P_i$ in *IG*-Linear Game (2.1)–(2.4), where

$$\lambda_{IG}^i(v) = \frac{\nu_{i0}}{2} + \langle v, \xi_{i0}\rangle + \sqrt{\left(\frac{\nu_{i0}}{2} + \langle v, \xi_{i0}\rangle\right)^2 - |v|^2}, \quad \xi_{i0} = \frac{z_{i0}}{|z_{i0}|}, \quad z_{i0} = x_{i0} - y_0, \quad \nu_{i0} = \frac{\rho_{i0}}{|z_{i0}|}.$$

**Property 1.** The function $\lambda_{IG}^i(v)$ is defined, positive, continuous with $v, |v| \le \sigma$, for every $s_i \in \mathbf{S}_{IG}^i$.

**Property 2.** If $s_i \in \mathbf{S}_{IG}^i$, then for the $\Pi_{IG}^i$-strategy the following equality holds:

$$|u_{IG}^i(v)|^2 = \nu_{i0}\lambda_{IG}^i(v). \tag{2.8}$$

## 3. MAIN RESULTS

For the players' trajectories

$$x_i(t) = e^{-\delta t}\left(x_{i0} + \int_0^t e^{\delta s} u_{IG}^i(v(s))ds\right), \quad y(t) = e^{-\delta t}\left(y_0 + \int_0^t e^{\delta s} v(s)ds\right)$$

we define the multi-valued mapping

$$W_{IG}^i(x_i(t), y(t), \rho_i(t)) = \left\{ w : |w - x_i(t)|^2 \ge (e^{-\delta t}\Lambda_{IG}^i(t, v(\cdot))\rho_i(t)/\sigma)\,|w - y(t)| \right\}. \tag{3.1}$$

**Theorem 1.** If $s_i \in \mathbf{S}_{IG}^i$, then

$$W_{IG}^i(t) = x_i(t) + e^{-\delta t}\Lambda_{IG}^i(t, v(\cdot))[W_{IG}^i(0) - x_{i0}], \quad t \in [0, t_i^*], \tag{3.2}$$

in *IG*-Linear Game (2.1)–(2.4), where

$W^i_{IG}(t) = W^i_{IG}(x_i(t), y(t), \rho_i(t))$, $t_i^* = \min\{t : z_i(t) = 0\}$, $i = 1, 2, \ldots, m$ , $W^i_{IG}(0) = \left\{ w : |w - x_{i0}|^2 \geq (\rho_{i0}/\sigma)\,|w - y_0| \right\}$.

*Proof.* Consider the situation when pursuer $P_i$ makes use of the $\Pi^i_{IG}$- strategy while evader $E$ on $[0, t_i^*]$ applies any control $v(\cdot) \in V_G$. Assume that $x_i(t),\ y(t)$ are the current positions of the players, and $\rho_i(t)$ is the current resource of pursuer $P_i$. Then from (2.9) and (3.1) we have

$$W^i_{IG}(t) - x_i(t) = \left\{ w : |w|^2 \geq (e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))\rho_i(t)/\sigma)\,|w + z_i(t)| \right\} =$$

$$= \left\{ w : |w|^2 \geq (e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))\rho_i(t)/\sigma)\,|w + e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))z_{i0}| \right\} =$$

$$= \left\{ w : |w / e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))|^2 \geq (\rho_{i0}/\sigma)\,|\left(w / e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))\right) + z_{i0}| \right\} = e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))[W^i_{IG}(0) - x_{i0}],$$

and this concludes the proof.

**Property 3.** If $s_i \in S^i_{IG}$, then the $W^i_{IG}(t)$ is convex on $[0, t_i^*]$.

Consider the set $W^*_{IG}(t) = W^i_{IG}(t) + \delta\left(\int_0^t x_i(s)ds\right) + \delta\left(\int_0^t e^{-\delta s}\Lambda^i_{IG}(s, v(\cdot))ds\right)\left(W^i_{IG}(0) - x_{i0}\right)$, $0 \leq t \leq t_i^*$.

**Theorem 2.** Let $s_i \in S^i_{IG}$. Then $W^*_{IG}(t_2) \subset W^*_{IG}(t_1)$ for arbitrary $t_1,\ t_2 \in [0, t_i^*],\ 0 \leq t_1 \leq t_2$ in *IG*-Linear Game (2.1)–(2.4).

*Proof.* From (2.8) we have

$$\left|v(t)|z_{i0}| - \lambda^i_{IG}(v(t))z_{i0}\right|^2 = \lambda^i_{IG}(v(t))\rho_{i0}|z_{i0}|.$$

Then from $|v(t)| \leq \sigma$ and from Property 1 we obtain

$$\left|(v(t)|z_{i0}| / \lambda^i_{IG}(v(t)) + y_0) - x_{i0}\right|^2 \geq (\rho_{i0}/\sigma)\left|(v(t)|z_{i0}| / \lambda^i_{IG}(v(t)) + y_0) - y_0\right|$$

or

$$\left(v(t)|z_{i0}| / \lambda^i_{IG}(v(t))\right) + y_0 \in W^i_{IG}(0).$$

From this and from properties of the supporting function $F(W, \psi) = \sup_{w \in W} \langle w, \psi \rangle$ for any $\psi \in R^n,\ |\psi| = 1$ in [8] we have

$$\left\langle \left(v(t)|z_{i0}| / \lambda^i_{IG}(v(t))\right) + y_0,\ \psi \right\rangle \leq F(W^i_{IG}(0), \psi)$$

or

$$\langle v(t), \psi \rangle - \frac{1}{|z_{i0}|}\lambda^i_{IG}(v(t))F(W^i_{IG}(0) - y_0, \psi) \leq 0 \tag{3.5}$$

for all $\psi \in R^n,\ |\psi| = 1$. From (1.1), (1.7), (3.2), (3.4) we have

$$F(W^*_{IG}(t), \psi) = F\left(W^i_{IG}(t) + \delta\left(\int_0^t x_i(s)ds\right) + e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))\left(W^i_{IG}(0) - x_{i0}\right), \psi\right) =$$

$$= \langle x_i(t), \psi \rangle + e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))F(W^i_{IG}(0) - x_{i0}, \psi) + F\left(\delta\left(\int_0^t x_i(s)ds\right), \psi\right) +$$

$$+ \delta\left(\int_0^t e^{-\delta s}\Lambda^i_{IG}(s, v(\cdot))ds\right)F\left(W^i_{IG}(0) - x_{i0}, \psi\right) \Rightarrow$$

$$\Rightarrow \frac{d}{dt}F(W^*_{IG}(t), \psi) = \langle u^i_{IG}\left(v(t)\right), \psi \rangle - \delta\langle x_i(t), \psi \rangle - \frac{1}{|z_{i0}|}\lambda^i_{IG}(v(t))F(W^i_{IG}(0) - x_{i0}, \psi) -$$

$$- \delta e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))F(W^i_{IG}(0) - x_{i0}, \psi) + \delta\langle x_i(t), \psi \rangle + \delta e^{-\delta t}\Lambda^i_{IG}(t, v(\cdot))F(W^i_{IG}(0) - x_{i0}, \psi) =$$

$$= \langle v(t) - \lambda^i_{IG}(v(t))\xi_{i0}, \psi \rangle - \frac{1}{|z_{i0}|}\lambda^i_{IG}(v(t))F(W^i_{IG}(0) - x_{i0}, \psi) =$$

$$= \langle v(t), \psi \rangle - \frac{1}{|z_{i0}|} \lambda_{IG}^{i}(v(t)) F(W_{IG}^{i}(0) - y_0, \psi) \leq 0 .$$

This finishes the proof.

**Property 4.** If $s_i \in S_{IG}^{i}$, then $y(t) \in W_{IG}^{i}(0) - \delta\left(\int_0^t x_i(s)ds\right) - \delta\left(\int_0^t e^{-\delta s}\Lambda_{IG}^{i}(s, v(\cdot))ds\right)\left(W_{IG}^{i}(0) - x_{i0}\right)$ is valid on the time interval $t \in [0, t_i^*]$.

**Theorem 3.** If $s_i \in S_{IG}^{i}$ for some $i = 1, 2, ..., m$, then $y(t) \in \tilde{W}_{IG}$ is satisfied on the time interval $[0, T_{IG}]$ in *IG*-Linear Game (2.1)–(2.4), where $T_{IG} = d / \sigma$,
$\tilde{W}_{IG} = \bigcup_0^{T_{IG}}\left[\bigcap_{i=1}^{m}\left\{W_{IG}^{i}(0) - \delta\left(\int_0^t x_i(s)ds\right) - \delta\left(\int_0^t e^{-\delta s}\Lambda_{IG}^{i}(s, v(\cdot))ds\right)\left(W_{IG}^{i}(0) - x_{i0}\right)\right\}\right]$, $d = \max\{|w_1 - w_2| : w_1, w_2 \in \tilde{W}_{IG}\}$.

*Proof*. This follows from Theorem 2 and Property 4.

**Theorem 4.** If $\tilde{W}_{IG} \cap A = \varnothing$, then the players $P_i$, $i = 1, 2, ..., m$ win on the interval $[0, T_{IG}]$ in *IG*-Linear Game (2.1)–(2.4) with the "Life-line".

*Proof.* This follows from Theorem 3.

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