MULOHAZALAR HISOBI FORMULALARINING QOLLANISHI

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Annotatsiya. *Mantiqiy fikrlash argumentlar qurishda muhim rol o‘ynaydi, chunki bu tasdiqlangan qoidalar asosida bayonotlarning rostligini tekshirish va tasdiqlash imkonini beradi. Matematika, falsafa va boshqa sohalarda mantiqiy fikrlash teoremalarning isboti, gipotezalarni asoslash uchun qo‘llaniladi. Bunday fikrlash qat’iylik va izchillikni ta’minlaydi, bu esa asosli xulosa va argumentlarni shakllantirishda ayniqsa muhimdir. Mantiqiy amallar yordamida tadqiqotchilar va olimlar bayonotlar o‘rtasidagi munosabatlarni tahlil qilish, ziddiyatlarni aniqlash va yangi g‘oyalarni asoslash imkoniyatiga ega bo‘lishadi. Har qanday sohada, mantiqiy fikrlash va aniq xulosalar chiqarish har qanday faning rivojlanishini davom ettirish uchun asosiy vositadir.*

Kalit so'zlar: *Deduktiv, induktiv, mantiqiy natija, Modus Ponens, Modus Tollens, Modus Ponendo-Tollens, Transitivlik qoidası.*

ПРИМЕНЕНИЕ ФОРМУЛ ЛОГИЧЕСКОГО ВЫВОДА

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Аннотация. *Логическое мышление играет ключевую роль в построении аргументов, поскольку оно позволяет проверять и подтверждать истинность утверждений на основе четко установленных правил. В математике, философии, науке и других дисциплинах логическое мышление используется для доказательства теорем, обоснования гипотез и построения моделей, которые описывают мир. Такое мышление обеспечивает строгость и последовательность в рассуждениях, что особенно важно при формировании обоснованных выводов и аргументов. С помощью логических операций исследователи и ученые могут анализировать отношения между утверждениями, выявлять противоречия и находить оправдания новым идеям. Независимо от области, способность логически мыслить и делать точные выводы является фундаментальным инструментом для проверки знаний и дальнейшего развития любой науки.*

Ключевые слова: *Дедуктивный, индуктивный, предпосылки, логическое следствие, Модус Поненс, Модус Толленс, Модус Понендо-Толленс, правило транзитивности.*

APPLICATION OF LOGICAL INFERENCE FORMULAS

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Abstract. *Logical reasoning plays a key role in building arguments, as it allows for the verification and confirmation of the truthfulness of statements based on clearly established rules. In mathematics, philosophy, science, and other disciplines, logical reasoning is applied to prove theorems, justify hypotheses, and construct models that describe the world. Such reasoning ensures rigor and consistency in thinking, which is especially important when forming well-founded conclusions and arguments. Through logical operations, researchers and scientists can analyze relationships between statements, identify contradictions, and find justifications for new ideas. Regardless of the field, the ability to reason logically and make accurate inferences is a fundamental tool for validating knowledge and furthering the development of any science.*

Keywords: *Deductive, inductive, Premises, Logical consequence, Modus Ponens, Modus Tollens, Modus Ponendo-Tollens, Rule of Transitivity.*

Introduction.

Logical reasoning is the process through which one or more statements (premises) lead to a new statement (conclusion). This process is based on the application of logical rules and operations that determine how premises are connected to the conclusion. Logical reasoning is an essential tool for acquiring knowledge and drawing accurate conclusions[1-3].

The primary purpose of logical reasoning is to derive new true statements based on already known facts. In everyday life and scientific activity, people constantly rely on the process of reasoning to make decisions, draw conclusions, and explain phenomena. Logic, as the foundation of reasoning, ensures that these processes are rigorous, structured, and consistent.

Logical thinking helps to avoid errors in reasoning, make well-founded conclusions, and build sound arguments. Different types of logical reasoning, such as “deduction” and “induction”, are used to address various tasks. Deductive reasoning aids in arriving at absolutely true conclusions when the premises are true, while inductive reasoning allows for probabilistic conclusions based on observations.

Preliminaries

When studying propositional algebra, we could answer many questions using truth tables. However, since some complex problems in logic cannot be solved by this method alone, we now apply formulas from propositional calculus and identify the set of tautologies through a deductive system. In other words, we establish axioms of propositional calculus as the "initial" tautologies and express inference rules that allow deriving new tautologies from these axioms. These rules serve logic by transforming the inference process into purely mechanical calculations, which is why the term propositional calculus came into being.

The process of deriving a new proposition from an existing one using logical operations is called reasoning or inference. In this process, the original proposition is referred to as the premise, and the newly derived proposition is called the conclusion or the inferred statement. In logic, reasoning is divided into inductive and deductive types.

The difference between “deductive” and “inductive reasoning” lies in how they approach the process of drawing conclusions and in the level of certainty with which these conclusions can be considered true.

Deductive reasoning assumes that if all premises are true, then the conclusion must also be true. In this case, the conclusion logically follows from the premises with complete certainty.

In deduction, the logical structure guarantees the truth of the conclusion if the premises are true. It moves from the general to the specific, providing strict certainty in the conclusions.

Inductive reasoning, on the other hand, is based on observations and experience, and its conclusions are probabilistic in nature. Even if all the premises are true, the conclusion is likely to be true, but it is not guaranteed.

Induction moves from the specific to the general and leads to probabilistic conclusions. Inductive inferences allow for making assumptions that can be useful, but they do not provide full certainty.

Deduction is more commonly used in mathematics and logic, where precision is required, while induction is widely applied in sciences that rely on observations, such as biology and sociology. Thus, deductive and inductive reasoning represent two approaches to reasoning, each with its strengths and weaknesses depending on the context of application.

Here are definitions of the key terms that play an essential role in the process of logical reasoning:

"Premises" are the initial statements or facts that serve as the foundation for drawing a conclusion. Premises contain the information necessary for deriving the conclusion. If the premises are true and correct logical rules are applied, then the conclusion should also be true.

A "conclusion (inference)" is a new statement that logically follows from the given premises. The truth or falsity of the conclusion depends on the truth of the premises and the logical connection between them.

"Truth" is the conformity of a statement to reality or its accuracy. If a statement corresponds to the actual state of affairs, it is considered true.

"Falsity" is the non-conformity of a statement to reality or its inaccuracy. If a statement does not correspond to the actual state of affairs, it is considered false.

"Logical consequence" is the process of deriving a conclusion from one or more premises based on logical rules. If the conclusion logically follows from true premises according to these rules, it is considered a logical consequence.

These fundamental concepts are essential for constructing a consistent and accurate process of logical reasoning. They help in forming well-founded conclusions and avoiding errors in arguments.

Main part

1."Modus Ponens (Affirming the Antecedent)":

"*If statement A implies statement B, and statement A is true, then statement B is also true.*" This rule is represented as follows:

$$\frac{A \to B, A}{B}$$

Modus Ponens is a fundamental form of deductive reasoning. It ensures that if both the conditional statement and its antecedent are true, the consequent must also be true[1-3].

2. "Modus Tollens (Denying the Consequent)":

"*If statement A implies statement B, but statement B is false, then statement A must also be false.*" This rule is represented as follows:

$$\frac{A \to B, \overline{B}}{\overline{A}}$$

Modus Tollens is another essential form of deductive reasoning, allowing one to infer the falsity of a premise based on the falsity of the consequent in a conditional statement.

3. Modus Ponendo-Tollens (Affirming by Denying):

"*If either statement A or statement B is true (in the exclusive sense), and one of them is true, then the other must be false.*" The structure of this reasoning is as follows:

$$\frac{A \oplus B, A}{\overline{B}}; \frac{A \oplus B, B}{\overline{A}}$$

In this type of reasoning, known as an exclusive disjunction, the truth of one statement implies the falsity of the other. It is often used when two statements cannot both be true simultaneously.

4. Modus Tollendo-Ponens (Denying by Affirming):

a) "*If either statement A or statement B is true (in the exclusive sense), and one of them is false, then the other must be true.*" This rule is represented as follows:

$$\frac{A \oplus B, \overline{A}}{B}; \frac{A \oplus B, \overline{B}}{A}$$

Modus Tollendo-Ponens applies to situations involving exclusive disjunction, where the truth of one statement is guaranteed if the other is known to be false. This reasoning ensures that if one part of the exclusive disjunction is false, the other part must necessarily be true.

b) *"If A or B is true (in the inclusive sense), and one of them is false, then the other must be true."* This logical rule can be represented as follows:

$$\frac{A \vee B, \overline{A}}{B}; \frac{A \vee B, \overline{B}}{A}$$

In this case, since the disjunction is inclusive, at least one of the statements A or B must be true. Therefore, if one statement is known to be false, the other must necessarily be true. This form of reasoning is often used in logic to ensure that the truth of one part of an inclusive disjunction implies the truth of the other part if one is false.

5. Rule of Transitivity:

"*If A implies B, and B implies C, then A implies C.*" The logical structure of this rule is as follows:

$$\frac{A \to B, B \to C}{A \to C}$$

The rule of transitivity allows us to link two conditional statements together. If the first statement implies the second, and the second implies the third, then the first statement must imply the third. This rule is widely used to establish logical sequences and derive conclusions based on a chain of implications.

6. Law of Contradiction:

"*If A implies B, and A also implies not-B, then A must be false.*" The structure of this rule is as follows:

$$\frac{A \to B, A \to \overline{B}}{\overline{A}}$$

The law of contradiction states that a statement A cannot logically lead to both B and not- B . If this occurs, it indicates a contradiction, implying that A cannot be true. This law is a fundamental principle in logic, ensuring that contradictory statements cannot both be true simultaneously.

7. Rule of Contraposition:

"*If A implies B, then if B is false, A must also be false.*" This rule can be represented as follows:

$$\frac{A \rightarrow B}{\overline{B} \rightarrow \overline{A}}$$

The rule of contraposition states that a conditional statement is logically equivalent to its contrapositive. In other words, if the truth of A guarantees the truth of B, then the falsity of B guarantees the falsity of A. This rule is commonly used in proofs and logical reasoning to infer information about the negation of premises.

8. Complex Contraposition:

"*If A and B together imply C, then if A is true and C is false, B must also be false.*" This rule can be represented as follows:

$$\frac{A \cdot B \rightarrow C}{A \cdot \overline{C} \rightarrow \overline{B}}$$

The complex contraposition rule states that if a conjunction of two statements implies a third statement, then if the first statement is true and the third is false, the second statement must be false. This is an extension of the contraposition principle applied to conjunctions, and it is useful in reasoning where multiple premises work together to lead to a conclusion.

9. Rule of Section (Cut Rule):

"*If A implies B, and B and C together imply D, then A and C together imply D.*" The logical structure of this rule is as follows:

$$\frac{A \rightarrow B, B \cdot C \rightarrow D}{A \cdot C \rightarrow D}$$

The rule of section allows for combining two implications by linking them through a shared intermediate statement. It states that if the first condition A leads to B , and B together with another condition C leads to D, then A and C together will imply D. This rule is useful in situations where a chain of dependencies connects multiple premises to a final conclusion.

10. Rule of Importation (Conjunction of Premises):

"*If A implies that B implies C, then A and B together imply C.*" This rule can be represented as follows:

$$\frac{A \to (B \to C)}{A \cdot B \to C}.$$

The rule of importation allows us to combine two conditional statements into a single statement where both premises must be true for the conclusion to follow. Essentially, it transforms nested implications into a conjunction, showing that the truth of A and B together directly leads to C. This rule is helpful for simplifying logical expressions and reasoning with multiple premises.

11. Rule of Exportation (Disjunction of Premises):

"*If A and B together imply C, then A implies that B implies C.*" This rule can be represented as follows:

$$\frac{A \cdot B \to C}{A \to (B \to C)}$$

The rule of exportation allows us to transform a conjunction of premises that leads to a conclusion into a nested implication. Essentially, it expresses that if two conditions together imply a result, then the first condition alone implies that the second condition will lead to the same result. This rule is useful for restructuring logical statements and breaking down complex premises into simpler components.

12. Simple Dilemmas:

A simple dilemma involves two conditional premises that lead to a common conclusion. It can be expressed as follows:

$$\frac{A \to C, B \to C, A \vee B}{C}; \frac{A \to B, A \to C, \overline{B} \vee \overline{C}}{\overline{A}}$$

In a simple dilemma, there are two possible conditions (A and B), either of which leads to the same outcome (C). This form of reasoning is useful when two different premises support a single conclusion, regardless of which premise is true. It is often applied when reasoning about multiple scenarios that yield the same result.

13. Complex Dilemmas:

A complex dilemma involves two conditional premises that lead to two different conclusions, depending on which premise is true. It can be expressed as follows:

$$\frac{A\to B, C\to D, A \vee C}{B \vee D};\frac{A\to B, A\to C, \overline{B} \vee \overline{D}}{\overline{A} \vee \overline{C}}$$

In a complex dilemma, there are two possible conditions (A and B), each leading to a different outcome (C and D). The conclusion reflects that if either condition holds, at least one of the corresponding conclusions will also hold. Complex dilemmas are useful in situations where multiple possible scenarios lead to different but relevant results, allowing one to account for various outcomes within a single reasoning framework[5-6].

Let's consider a few examples to apply these rules and evaluate the validity of the conclusions (correctness of reasoning).

Example 1. Check the correctness of the reasoning: "*If this polygon is regular, then a circle can be inscribed in it. This polygon is regular. Therefore, a circle can be inscribed in this polygon.*"

Solution. To construct a rule of reasoning, we formalize the problem, i.e. introduce the notation of statements:

X - this polygon is regular;

Y - a circle can be inscribed in this polygon.

Then the scheme (rule) of reasoning will be written as:

$$\frac{X\to Y, X}{Y}$$ (modus ponens).

This scheme corresponds to formula $(X\to Y)\cdot X\to Y$. This formula is a tautology. Indeed

$$(X\to Y)\cdot X\to Y=(\overline{X}\vee Y)\cdot X\to Y=\overline{(\overline{X}\vee Y)\cdot X}\vee Y=\overline{(\overline{X}\vee Y)}\vee\overline{X}\vee Y=$$
$$=(X\cdot\overline{Y})\vee\overline{X}\vee Y=\overline{X}\vee\overline{Y}\vee Y=\overline{X}\vee(\overline{Y}\vee Y)=\overline{X}\vee 1=1$$

(Note that when transforming the formula we applied the formula for deleting $(X\cdot\overline{Y})\vee\overline{X}=\overline{X}\vee\overline{Y}$ and the associativity property for disjunction.) Since as a result we found that the formula is a tautology, then, consequently, the reasoning is correct.

Answer: The reasoning is correct.

Example 2. Check the correctness of the reasoning: "*If this polygon is regular, then a circle can be inscribed in it. A circle cannot be inscribed in this polygon. Therefore, this polygon is not regular*."

Solution: Leaving the previous notations, we obtain the following reasoning scheme:

$$\frac{X \to Y, \overline{Y}}{\overline{X}} \text{ (modus tollens),}$$

which corresponds to formula $(X \to Y) \cdot \overline{Y} \to \overline{X}$, which is also a tautology, which can be verified by constructing a truth table or by performing simple transformations:

$$(X \to Y) \cdot \overline{Y} \to \overline{X} = (\overline{X} \vee Y) \cdot \overline{Y} \to \overline{X} = \overline{(\overline{X} \vee Y) \cdot \overline{Y}} \vee \overline{X} = (X \cdot \overline{Y}) \vee Y \vee \overline{X} =$$
$$= X \vee Y \vee \overline{X} = (X \vee \overline{X}) \vee Y = 1 \vee Y = 1$$

Answer: The reasoning is correct.

Example 3. Check the correctness of the reasoning: "*If this polygon is regular, then a circle can be inscribed in it. A circle can be inscribed in this polygon. Therefore, this polygon is regular*."

Solution: With the same notations of statements as in the previous cases, the reasoning scheme will look like this:

$$\frac{X \to Y, Y}{X},$$

which in this case will correspond to the formula: $(X \to Y) \cdot Y \to X$, and which is not a tautology. Indeed:

$$(X \to Y) \cdot Y \to X = (\overline{X} \vee Y) \cdot Y \to X = \overline{(\overline{X} \vee Y) \cdot Y} \vee X = (X \cdot \overline{Y}) \vee \overline{Y} \vee X = \overline{Y} \vee X \neq 1.$$

Therefore, the reasoning is not correct.

Answer: The reasoning is incorrect.

Example 4. Check the correctness of the reasoning: "*If a number is rational, then it can be represented as a ratio of two integers. Therefore, if a number cannot be represented as a ratio of two integers, then it is not rational.*"

Solution. Let's introduce the notation of statements:

X- is a rational number;

Y- is a number that can be represented as a ratio of two integers.

Let's write the reasoning scheme as follows:

$$\frac{X \to Y}{\overline{Y} \to \overline{X}}$$ (Rule of Contraposition).

This rule corresponds to formula $(X \to Y) \to (\overline{Y} \to \overline{X})$, which is a tautology (the law of contraposition), and, therefore, the reasoning is correct.

Answer: The reasoning is correct.

Example 5. Check the correctness of the reasoning: "*If a triangle is isosceles, then two of its sides are equal. If two sides of a triangle are equal, then two of its angles are equal. Therefore, if a triangle is isosceles, then two of its angles are equal.*"

Solution. Let's introduce the notation of the statements:

X - the triangle is isosceles;

Y - two sides are equal;

Z - two angles are equal.

Let's write the reasoning scheme as follows: $\dfrac{X \to Y, Y \to Z}{X \to Z}$,

which is a rule of syllogism, and therefore the formula 1 corresponding to it is a tautology, which determines the correctness of the reasoning.

Answer: The reasoning is correct.

Thus, logical reasoning serves as the basis for many forms of reasoning and proof. Understanding and using logical rules helps make reasoning more consistent, sound, and accurate, which is important not only for science but also for everyday life.

Conclusions

In the logic of propositions, reasoning is divided into deductive and inductive. In deductive reasoning, the connections between premises and conclusion represent formal logical laws, which means that with true premises, the conclusion is always true. In inductive reasoning, the connections between premises and conclusion are such that they provide only a plausible conclusion when the premises are true. In these types of reasoning, the premises only support the conclusion. During reasoning, sometimes inferences are mistakenly taken as deductive when they are not. The former are called incorrect, while (properly) deductive ones are correct. Above are examples of the most commonly used rules for obtaining logically correct inferences.

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