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GILBERT FAZOSIDA PUASSON TAQSIMOTIGA EGA BO‘LGAN TASODIFIY MIQDORLARNING YAQINLASHUVI.

Annotatsiya: Maqolada Gilbert fazosida normal yig‘indining taqsimoti taqdim etilgan bo‘lib, u Puasson taqsimotiga bo‘ysunuvchi tasodifiy miqdorlarni o‘rganish va ularni normal taqsimotga yaqinlashtirish maqsadida ko‘rib chiqiladi.

Kalit so‘zlar: Tasodifiy miqdor, Gilbert fazosi, empirik jarayon, xarakteristika, elementar hodisalar to‘plami, bosqinchilik printsipi.

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СХОДИМОСТЬ СЛУЧАЙНЫХ ВЕЛИЧИН С РАСПРЕДЕЛЕНИЕМ ПУАССОНА В ПРОСТРАНСТВЕ ГИЛБЕРТА.

Аннотация: В статье представлено распределение нормальной суммы в гильбертовом пространстве с целью изучения случайных величин, подчиняющихся распределению Пуассона, и их аппроксимации нормальным распределением.

Ключевые слова: Случайная величина, гильбертово пространство, эмпирический процесс, характеристика, множество элементарных явлений, принцип вторжения.

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THE CONVERGENCE OF RANDOM QUANTITIES WITH THE DISTRIBUTION OF PUASSON IN THE GILBERT SPACE.

Annotation: The article presents the distribution of the normal sum in Hilbert space in order to study these random variables obeying the Poisson distribution and their approximation to the normal distribution.

Keywords: Random quantity, Gilbert space, the emirtic process, a characteristics, set of elemental phenomena, the principle of invasion.

Introduction

The study of random quantities and their behavior within mathematical frameworks is a central topic in probability theory and functional analysis. One particularly intriguing area is the convergence of random variables with a Poisson distribution in Hilbert spaces. The Poisson distribution, often arising in scenarios involving discrete events over time or space, exhibits unique properties that make it a valuable subject of study in stochastic processes.

Hilbert spaces, as complete inner product spaces, provide a versatile framework for analyzing random processes and their convergence. This setting allows for the investigation of both finite-dimensional and infinite-dimensional cases, broadening the applicability of results. The interplay between the Poisson distribution and Hilbert space opens pathways to approximate such random quantities with normal distributions [4]-][6], leveraging tools like the central limit theorem.

This paper delves into the convergence properties of random variables with a Poisson distribution within Hilbert spaces, focusing on their approximation by normal distributions. The analysis includes an exploration of the statistical and functional characteristics that govern such behavior, with implications for fields like mathematical statistics, stochastic modeling, and theoretical physics.

The main part

The purpose of the article is to investigate what infinite dimensional consequences follow from the results of the published work of the authors [1]-[3]. It will be demonstrated that the accuracy of the approximation significantly depends on the rate of decrease of the sequence of eigenvalues of the covariance operator of the terms.

Let's say in the Gilbert space $X_1, X_2, \ldots, X_n$ - let the random quantities that have a Puasson distribution are given, that is, $\Omega = \{0, 1, 2, \ldots\}$, $P = \{P_1, P_2, \ldots\}$,

$$p(x) = \frac{e^{-a} a^x}{x!}; \quad x = 0, 1, \ldots \qquad a>0 \quad \text{or} \quad Mx_n = a.$$

It is known that $M\|x_n\|^2 < \infty$, in that case $\eta_n = n^{-\frac{1}{2}}(X_1 + X_2 + ... + X_n)$

the distribution of the normal sum will approach a normal distribution in Gilbert space. This article shows to study the speed of approaching.

$$Z_n = n^{-1}\int_0^1\left[\sum_{i=1}^{n} p(x+x_i) - h\right]^2 dx \qquad (1)$$

let's enter a definition. Z_n random quantities in the form of (1) are studied in [1].

If

$$\xi_i(x) = p(x+x_i) - 1$$

If we enter, then

$$Z_n = \|\eta\|^2 = \left\|\frac{\xi_1(x) + \xi_2(x) + ... + \xi_n(x)}{\sqrt{n}}\right\|^2$$

we'll have. Here

$$\|p(x)\| = \left(\int_0^1 p^2(x)dx\right)^{\frac{1}{2}}.$$

It is also an empirical $x_n(x) = \sqrt{n}(F_n(x) - x)$ process and the $F_n(x)$ - the empirical distribution function, based on the sale of $x_1, x_2, \ldots, x_n$ the Puasson process. Covera function

$$\cos z(1-x)\ ,\quad 0 \le z \le x \le 1$$

let there be a sort of. [2] да $n\to\infty$ да

$$\Delta_n(\lambda) = P(Z_n < \lambda) - P(\lambda) \qquad (2)$$

The amount is shown to 0. It is a distribution function of $P(\lambda)$, its characteristics function

$$\varphi(x) = \prod_{k=1}^{\infty} (1 - 2 \cdot \lambda_k it)^{-\frac{1}{2}} \qquad (3)$$

equal to function. A sequence $\lambda_1 \geq \lambda_2 \geq \ldots$ of numbers to decrease here.

Say

$$|P'(x)| \leq c \qquad (4)$$

Let there be a sort of. Here c positive number

The theorem 1. For adequate large n and all $\lambda > 0$

$$|\Delta_n(\lambda)| \leq C(H) \frac{\ln_n}{\sqrt{n}} e^{-\sqrt{\lambda}}$$

Inequality will be appropriate. Here it depends on the *C(H).*

Proven Z_n

$$Z_n = \sqrt{n} \left(\frac{1}{n} \sum_{i=1} p(x + x_i) - 1 \right] = \sqrt{n} \int_0^1 p(x+u) - 1)\, dF_n(u) =$$

$$= \int_0^1 (P(x+u) - 1) dx_n(u)$$

We write like. (3) and (1) from

$$Z_n = \int_0^1 \left[\int_0^1 (p(x+u) - 1) dx_n(u) \right]^2 dx$$

we will have. According to the principle of invasion, Z_n

$$Z = \int_0^1 \left[\int_0^1 (p(x+u) - 1) dx(u) \right]^2 dx$$

strives for.

Comlosh- Mayor Tushnadi [2] descends according to the traine (Ω,F,P)

there is a distinctive space that is empirical for the optional *n* in this space $x_n^{(1)}(u)$ the process can build the process and $B_n(u)$ Brown bridge. Voluntary $x>0$ for them

$$P\left(\sqrt{n}\sup_{0\le u\le 1}|\,y_n^{(1)}(u)-B_n(u)|>x\right)\le kn^{a}e^{-bx} \tag{5}$$

Inequality will be appropriate. Here there are positive constant prime numbers of *a,b,k.*

we will enter the below

$$Z_n^{(1)}=\int_0^1\left[\int_0^1(P(x+u)-1)dy_n^{(1)}(u)\right]^2dx,\ Z^{(1)}=\int_0^1\left[\int_0^1(P(x+u)-1)dB_n(u)\right]^2dx$$

(6)

In that case

$$P(Z_n<\lambda)=P(Z_n^{(1)}<\lambda),\ P(Z<\lambda)=P(Z^{(1)}<\lambda) \tag{7}$$

from (6) using

$$Z_n^{(1)}=Z^{(1)}+\int_0^1\left[\int_0^1(f(x+u)-1)d\left(y_n^{(1)}(x)-B_nu\right)\right]^2du+$$

$$+2\int_0^1\left[\int_0^1(f(x_1+u)-1)d\left(y_n^{(1)}(x_1)-B_n(x_1)\right)\right]\times\left[\int_0^1 f(x_2+u)-1)dB_n(x_1)\right]du$$

we will have. The elementary accidents set

$$\Omega_1=\left|\omega:\sqrt{n}\sup_{0\le u\le 1}|\,y_n^{(1)}(u)-B_n(u)|\le x\right| \tag{8}$$

If we can be (here $x>0$ optional number) $\omega\in\Omega_1$ then for all

$$\left|\int_0^1\left[\int_0^1 (f(x_1+u)-1)d\left(y_n^{(1)}(x_1)-B_n(x_1)\right)\right]^2 du\right| \le C_1\frac{|x^2|}{n}, \tag{9}$$

$$\left|2\int_0^1\left[\int_0^1 (f(x_1+u)-1)d\left(y_n^{(1)}(x_1)-B_n(x_1)\right)\right]\times\left[\int_0^1 (f(x_2+u)-1)dB_n(x_2)]du\right| \le C_2\frac{x}{\sqrt{n}}\sqrt{Z^1}\right| \tag{10}$$

Inequality will be appropriate.

(9) when we first applied (4), (8), after we first applied to the x_1 variable. In addition to the formation of (10), we have applied Koshi-Bunyakovsky's inequality other than the above.

(9), are optional from (10) $\omega\in\Omega_1$ for $|Z_n^{(1)}-Z_0^{(1)}|\le t_1$ (11)

we create. Here $t_1=c_1\cdot\frac{x^2}{n}+c_2\cdot\frac{x}{\sqrt{n}}\sqrt{T^{(1)}}$

(11) according to

$$P(Z_n<\lambda)=P\left((Z_n^{(1)}<\lambda)\cap\Omega_1\right)+P(T_n^{(1)}<\lambda)\cap\bar{\Omega}_1) \tag{12}$$

we appreciate the second added accounting (5) on the right of (12).

$$P\left((Z_n^{(1)}<\lambda)\cap\Omega_1\right)\le P\left((Z^{(1)}<\lambda+t_1)\cap\Omega_1\right), \tag{13}$$

$$P\left((Z_n^{(1)}<\lambda)\cap\Omega_1\right)\le P\left((Z^{(1)}<\lambda-t_1)\cap\Omega_1\right) \tag{14}$$

(13) and from (14)

$$P\left((Z_n^{(1)}<\lambda)\cap\Omega_1\right)\le P\left(Z^{(1)}<\lambda+\alpha_n\right), \tag{15}$$

$$P\left((Z_n^{(1)}<\lambda)\cap\Omega_1\right)\ge P\left(Z^{(1)}<\lambda-\beta_n\right)-P\left(\bar{\Omega}_1\right) \tag{16}$$

we create inequalities. Here

$$\alpha_n=C_3\frac{x^2}{n}+2x\sqrt{\frac{c_n}{n}}\left[x\sqrt{\frac{c_n}{n}}+\sqrt{\lambda+2\frac{x^2}{n}c_3}\right],$$

$$\beta_n = C_3 \frac{x^2}{n} + 2x\sqrt{\frac{c_n}{n}}\left[\sqrt{\lambda} - x\sqrt{\frac{c_3}{n}}\right]$$

x the fact that the voluntary value of the variable can be selected,

$$x = \frac{1}{b}\sqrt{\lambda} + \frac{a+2}{b}\ln n$$

can select as. In that case, an optional are $\lambda > 0$ in and adequate n large

$$\alpha_n \le C_4 \lambda \frac{\ln n}{\sqrt{n}}, \qquad \beta_n \le C_5 \lambda \frac{\ln n}{\sqrt{n}}$$

we create (6), (15), (16).

$$\Delta_n(\lambda) \le P(T < \lambda + \alpha_n) - P(T < \lambda) + P(\bar{\Omega}_1), \tag{17}$$

$$\Delta_n(\lambda) \ge P(T < \lambda - \beta_n) - P(T < \lambda) - P(\bar{\Omega}_1)$$

we'll have. In view of the selected x and (5)

$$P(\bar{\Omega}_1) \le C_6 \frac{1}{n} e^{-\sqrt{\lambda}} \tag{18}$$

Voluntary it is formed for u>0. (7) mainly

$$P'(u) \le C_7 u^{m_0} e^{-\frac{\alpha}{2}}$$

We create. Here's C_7 and m_0 - permanent numbers.

From the last inequality

$$|P(Z < \lambda + \alpha_n) - P(Z < \lambda)| \le C_8 \frac{\ln}{\sqrt{n}} e^{-\sqrt{\lambda}} \tag{19}$$

$$|P(Z < \lambda - \beta_n) - P(Z < \lambda)| \le C_9 \frac{\ln}{\sqrt{n}} e^{-\sqrt{\lambda}} \tag{20}$$

are formed.

(17) - (18), (19) from uniting (20)

$$- C_6 \frac{1}{n} e^{-\sqrt{\lambda}} - C_9 \frac{\ln}{\sqrt{n}} e^{-\sqrt{\lambda}} \le \Delta_n(\lambda) \le C_8 \frac{\ln}{\sqrt{n}} e^{-\sqrt{\lambda}} + C_6 \frac{1}{n} e^{-\sqrt{\lambda}}$$

lnequality occurs. This proves that the theorem is true.

Based on the Komlish-Mayor-Tushnadi theorem [2], there exists a probability space (Ω,F,P) in which the empirical process for arbitrary *n* was $x_n^{(1)}(u)$ determined.

Conclusion

The convergence of random quantities with a Poisson distribution in Hilbert spaces underscores the rich interplay between stochastic processes and functional analysis. By examining their behavior under the framework of Hilbert spaces, this study highlights the conditions under which these random variables can be approximated by normal distributions.

The findings contribute to a deeper understanding of the theoretical properties of Poisson-distributed random variables in infinite-dimensional spaces. Such insights have significant implications for both theoretical and applied sciences, including the development of advanced probabilistic models and the refinement of numerical simulations in various disciplines.

Future research could expand on these results by exploring convergence under alternative metrics or by applying these principles to specific applications in physics, biology, and engineering, where Poisson processes play a critical role. This work establishes a foundation for further exploration and innovation in the field of stochastic analysis.

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