Fur’e qatorlarining ba’zi tatbiqlari

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Annatatsiya. Ushbu maqolada Fur’e qatorlarining ahamiyati va ba’zi tatbiqlari keltirilgan. Maqolada ba’zi funksiyalarni Fur’e qatoriga yoyish orqali rekkurent formulalar hosil qilingan va bu formulalarni matematik induksiya metodi orqali isbot qilingan. Bundan tashqari isbotlash uchun misollar ham keltirilgan.

Kalit so’z. Fur’e qatori, Fur’e koeffitsienti, yig’indi, Matematik induksiya metodi.

Некоторые приложения рядов Фурье

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Аннотация: В этой статье представлены значение и некоторые применения рядов Фурье. В статье созданы рекуррентные формулы путем разложения некоторых функций в ряд Фурье и доказаны методом математической индукции. В качестве доказательства также приведены примеры.

Ключевые слова: Ряд Фурье, коэффициент Фурье, сумма, метод математической индукции.

Some applications of Fourier series

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Annotation: This article presents the meaning and some applications of Fur’e series. In the article, recurrent formulas are created by expanding some functions

into Fur’e series and proven by the method of mathematical induction. Examples are also provided as evidence.

Key words: Fur’e series, Fur’e coefficient, sum, method of mathematical induction.

KIRISH.

Ma’lumki, Fur’e qatorlari Matematik analiz fanining asosiy tushunchalaridan biri bo’lib, u integrallash qiyin bo’lgan ba’zi funksiyalarni qatorlarga yoyib hisoblash imkonini beradi. Bu qatorlarni yig’indisini hisoblash har doim ham oson bo’lavermaydi [1-5]. Shu sababli ba’zi qatorlarni yig’indilarini hisoblashda Fur’e qatorlaridan keng foydalaniladi [5-12].

Bu maqolada Fur’e qatorlarining ta’rifi, ahamiyati va ba’zi bir tadbiqlari keltirilgan. Maqolada ba’zi funksiyalarni Fur’e qatoriga yoyib, rekkurent formulalar hosil qilingan va ularni matematik induksiya metodi orqali isbotangan. Bundan tashqari isbotlash uchun misollar ham keltirilgan.

Bizga ma’lum bo’lgan quyidagi qatorlarni Fur’e qatori bo’yicha son qiymatini topish masalasini qaraylik:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots; \qquad 1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots;$$

$$1-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots; \qquad 1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots.$$

Buning uchun dastlab Fur’e qatorini ta’rifini beramiz:

Faraz qilaylik, $f(x)$ funksiya $R=(-\infty,\infty)$ da berilgan bo’lsin. Ma’lumki, shunday $T\in R\setminus\{0\}$ son topilsaki, $\forall x\in R$ da

$$f(x+T)=f(x)$$

tenglik bajarilsa, $f(x)$ davriy funksiya, $T\neq 0$ son esa uning davri deyiladi.

Agar $T\neq 0$ son $f(x)$ funksiyaning davr bo’lsa, u holda

$$kT\,(k=\pm 1,\pm 2,\ldots)$$

sonlar ham shu funksiyaning davri bo’ladi.

Aytaylik, $f(x)$ funksiya $\left[-\pi.\pi\right]$ da uzluksiz bo'lsin. Unda

$$f(x)\cos nx,\ f(x)\sin nx\ \left(n=1,2,3,\ldots\right)$$

funksiyalar ham $\left[-\pi.\pi\right]$ da uzluksiz bo'lib, ular $\left[-\pi.\pi\right]$ da integrallanuvchi bo'ladi. Quyidagicha belgilashlar kiritaylik:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx\quad\left(n=1,2,3,\ldots\right) \tag{1}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx\quad\left(n=1,2,3,\ldots\right)$$

Bu belgilashlardan foydalanib, ushbu

$$T\left(f;x\right)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nx+b_n\sin nx\right) \tag{2}$$

qatorni (uni rigonometric qator deyiladi) hosil qilamiz.

(2) qator Funksional qator bo'lib, uning har bir hadi garmonikadan iborat.

Ta'rif: (2) funksinal qator $f(x)$ funksiyaning Fur'e qatori deyiladi. (1) munosabatlar bilan aniqlangan

$$a_0, a_1, b_1, a_2, b_2, \ldots$$

sonlar Fur'e koeffitsiyentlari deyiladi.

Bu yerda:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx,\quad a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx,\quad b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx$$

TAHLILLAR VA NATIJALAR.

Faraz qilaylik $f(x)=x^p$ bo'lsin.

a) $p=2k$ juft bo'lsa Fur'e koeffisientlari quyidagi ifodaga teng bo'ladi.

$$a_0=\frac{2\pi^{2k}}{2k+1}, b_n=0$$

$$a_n=4(-1)^n\left[\frac{k\pi^{2k-2}}{n^2}-\frac{k(2k-1)(2k-2)\pi^{2k-4}}{n^4}+\frac{k(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^6}+\cdots\right.$$

$$\left.\cdots+(-1)^{k+1}\frac{k(2k-1)(2k-2)\cdots 4}{n^{2(k-1)}}+(-1)^{k+1}\frac{k(2k-1)(2k-2)\cdots 1}{n^{2k}}\right],$$

Natijada $f(x)=x^p$ quyidagi ko'rinishga keladi:

$$f(x)=x^{2k}=\frac{2\pi^{2k}}{2k+1}+\sum_{n=1}^{\infty}4(-1)^n\left[\frac{k\pi^{2k-2}}{n^2}-\frac{k(2k-1)(2k-2)\pi^{2k-4}}{n^4}+\right.$$

$$\left.+\frac{k(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^6}+\cdots+(-1)^{k+1}\frac{k(2k-1)(2k-2)\cdots 1}{n^{2k}}\right]\cos nx$$

Bu yerda $x\in[-\pi;\pi]$ va $\cos n\pi=(-1)^n$ bo'lganidan quyidagilarga ega bo'lamiz:

$$\pi^{2k}=\frac{\pi^{2k}}{2k+1}+4\sum_{n=1}^{\infty}\left[\frac{k\pi^{2k-2}}{n^2}-\frac{k(2k-1)(2k-2)\pi^{2k-4}}{n^4}+\right.$$

$$\left.+\frac{k(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^6}+\cdots+(-1)^{k+1}\frac{k(2k-1)(2k-2)\cdots 1}{n^{2k}}\right]$$

Yuqoridagi tenglikdagi $k=\overline{1...s}$ qiymat berib quyidagi yig'indiga ega bo'lamiz:

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}, \qquad \sum_{n=1}^{\infty}\frac{1}{n^8}=\frac{\pi^8}{9450},$$

$$\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}, \qquad \sum_{n=1}^{\infty}\frac{1}{n^{10}}=\frac{\pi^{10}}{93555},$$

$$\sum_{n=1}^{\infty}\frac{1}{n^6}=\frac{\pi^6}{945}, \qquad \sum_{n=1}^{\infty}\frac{1}{n^{12}}=\frac{691\pi^{12}}{467775}.$$

Yuqoridagi tengliklardan yana quyidagilar kelib chiqadi:

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}, \qquad \sum_{n=1}^{\infty}\frac{1}{(2n-1)^6}=\frac{\pi^6}{960},$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}=\frac{\pi^4}{96}, \qquad \sum_{n=1}^{\infty}\frac{1}{(2n-1)^8}=\frac{51\pi^8}{483840}.$$

b) $p=2k-1$ bo’lsin Fur’e koeffisentlari quyidagi ifodaga teng bo’ladi:

$$a_0=0, \qquad a_n=0,$$

$$b_n=2(-1)^{n+1}\left[\frac{\pi^{2k-2}}{n}-\frac{(2k-1)(2k-2)\pi^{2k-4}}{n^3}+\frac{(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^5}+\cdots+\right.$$

$$\left.+\frac{(2k-1)(2k-2)\cdots\pi^2}{n^{2k-3}}+\frac{(2k-1)\cdots(2k-4)}{n^{2k-1}}\right].$$

Bundan esa $f(x)=x^p$ quyidagi ko’rinishga keladi:

$$f(x)=x^{2k-1}=\sum_{n=1}^{\infty}2(-1)^{n+1}\left[\frac{\pi^{2k-2}}{n}-\frac{(2k-1)(2k-2)\pi^{2k-4}}{n^3}+\right.$$

$$\left.+\frac{(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^5}+\cdots+(-1)^{k+1}\frac{(2k-1)(2k-2)\cdots2}{n^{2k-1}}\right]\sin nx$$

$$\left(\frac{\pi}{2}\right)^{2k-1}=\sum_{n=1}^{\infty}2(-1)^{n+1}\left[\frac{\pi^{2k-2}}{n}-\frac{(2k-1)(2k-2)\pi^{2k-4}}{n^3}+\right.$$

$$\left.+\frac{(2k-1)\cdots(2k-4)\pi^{2k-6}}{n^5}+\cdots+(-1)^{k+1}\frac{(2k-1)(2k-2)\cdots2}{n^{2k-1}}\right]\sin n\frac{\pi}{2}$$

Yuqoridagi tenglikdagi k ga $k=\overline{1...s}$ qiymat berib quyidagilarga ega bo’lamiz:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n-1}=\frac{\pi}{4},\qquad \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^7}=\frac{61\pi^7}{2880},$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^3}=\frac{\pi^3}{32},\qquad \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^9}=\frac{144857\pi^9}{41287680},$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^5}=\frac{5\pi^5}{1536},\qquad \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^{11}}=\frac{\pi^{11}}{32}.$$

Yuqoridagilardan xulosa qilib shuni aytish mumkinki, $\sum_{n=1}^{\infty}\frac{1}{n^{2s}}$ va $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^{2s-1}}$ qiymatlarini aniqlash uchun bizga bu qatorlarni $s=1$ dan $s=s-1$ gacha bo’lgan qiymatlari ma’lum bo’lishi kerak.

$$\sum_{i=1}^{n} i = 1+2+3+\ldots+n$$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2$$

$$\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3$$

… … … ….

$$\sum_{i=1}^{n} i^{\alpha-1} = 1^{\alpha-1}+2^{\alpha-1}+3^{\alpha-1}+\ldots+n^{\alpha-1}$$

Demak, biz ixtiyoriy α darajali yig‘indini topishimiz uchun $(\alpha-1)$ darajali yig‘indini topish formulasini bilishimiz kerak.

Biz quydagi rekkurent formulani olamiz:

$$\sum_{i=1}^{n} i^{\alpha+1}+(n+1)^{\alpha+1}=\sum_{i=1}^{n}(i+1)^{\alpha+1}+1^{\alpha+1} \tag{3}$$

Bu formulani matematik induksiya yordamida isbotlash mumkin.

Isbot:

a) $n=2$ da tekshiramiz

$$\sum_{i=1}^{2} i^{\alpha+1}+(2+1)^{\alpha+1}=\sum_{i=1}^{2}(i+1)^{\alpha+1}+1$$

Demak, tenglik o‘rinli ekan.

b) $n=k$ da tog‘ri deb faraz qilib quyidagini xosil qilamiz

$$\sum_{i=1}^{k} i^{\alpha+1}+(k+1)^{\alpha+1}=\sum_{i=1}^{k}(i+1)^{\alpha+1}+1^{\alpha+1}$$

c) $n=k+1$ uchun isbotlaymiz. Ya’ni

$$\sum_{i=1}^{k+1} i^{\alpha+1}+(k+2)^{\alpha+1}=\sum_{i=1}^{k+1}(i+1)^{\alpha+1}+1^{\alpha+1} \Rightarrow$$

$$\Rightarrow \sum_{i=1}^{k} i^{\alpha+1}+(k+1)^{\alpha+1}+(k+2)^{\alpha+1}=\sum_{i=1}^{k}(i+1)^{\alpha+1}+(k+2)^{\alpha+1}+1^{\alpha+1} \Rightarrow$$

$$\Rightarrow \sum_{i=1}^{k} i^{\alpha+1}+(k+1)^{\alpha+1}=\sum_{i=1}^{k}(i+1)^{\alpha+1}+1^{\alpha+1}$$

Biz bu tenglikni $n=k$ da to‘griligini bilamiz. Demak, (3) rekkurent formula o’rinli ekanini isbotlandi.

Biz quyida (3) tenglikdagi α ning ma’lum qiymatlarida yig‘indi formulalarini ko‘rib chiqamiz.

1) $\sum_{i=1}^{n} i=1+2+3+\ldots+n$ yig’indini hisoblang.

Buning uchun (3) tenglikda $\alpha=1$ desak

$$\sum_{i=1}^{n} i^2+(n+1)^2=\sum_{i=1}^{n}(i+1)^2+1^2$$

ifodani soddalashtirsak,

$$\sum_{i=1}^{n} i^2+n^2+2n+1=\sum_{i=1}^{n} i^2+2\sum_{i=1}^{n} i+n+1$$

$$2\sum_{i=1}^{n} i = n^2 + 2 \quad \Rightarrow \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Demak, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

2) $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$ yig'indini hisoblang.

Buning uchun (3) tenglikda $\alpha = 2$ desak,

$$\sum_{i=1}^{n} i^3 + (n+1)^3 = \sum_{i=1}^{n} (i+1)^3 + 1^3$$

ifodani soddalashtirsak, unda

$$\sum_{i=1}^{n} i^3 + n^3 + 3n^2 + 3n + 1 = \sum_{i=1}^{n} i^3 + 3\sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + n + 1^3$$

$$3\sum_{i=1}^{n} i^2 = n^3 + 3n^2 + 2n - 3\sum_{i=1}^{n} i = n^3 + 3n^2 + 2n - \frac{3n(n+1)}{2} =$$

$$= \frac{2n^3 + 3n^2 + n}{2} = \frac{n(n+1)(2n+1)}{2}$$

Demak, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

3) $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3$ yig'indini hisoblang.

Buning uchun (3) tenglikda $\alpha = 3$ desak,

$$\sum_{i=1}^{n} i^4 + (n+1)^4 = \sum_{i=1}^{n} (i+1)^4 + 1^4$$

ifodani soddalashtirsak unda,

$$\sum_{i=1}^{n} i^4 + n^4 + 4n^3 + 6n^2 + 4n + 1 = \sum_{i=1}^{n} i^4 + 4\sum_{i=1}^{n} i^3 + 6\sum_{i=1}^{n} i^2 + 4\sum_{i=1}^{n} i + n + 1^n$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2 ,$$

Demak, $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2$.

4. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \ldots + n^4$ yig'indini hisoblang.

Buning uchun (3) tenglikda $\alpha = 4$ desak,

$$\sum_{i=1}^{n} i^5 + (n+1)^5 = \sum_{i=1}^{n} (i+1)^5 + 1^5$$

ifodani soddalashtiramiz

$$\sum_{i=1}^{n} i^5 + n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1 =$$

$$\sum_{i=1}^{n} i^5 + 5\sum_{i=1}^{n} i^4 + 10\sum_{i=1}^{n} i^3 + 10\sum_{i=1}^{n} i^2 + 5\sum_{i=1}^{n} i + n + 1,$$

$$\Rightarrow \sum_{i=1}^{n} i^4 = \frac{1}{6} n(n+1)(2n+1)(3n^2 + 3n - 1)$$

Demak,

$$\sum_{i=1}^{n} i^4 = \frac{1}{6} n(n+1)(2n+1)(3n^2 + 3n - 1).$$

Xulosalar

Xulosa qilib aytganda funksiyalarni Fur'ye qatoriga yoyish va uning yig'indisini xisoblash har doim ham oson bo'lmas ekan. Qatorga yoyilgan funksiyaning yigindi formulasi to'g'ri ekanligini isbotlash muhim hisoblanadi. Bu yig'indilarni isbotlashda Matematik induksiya metodi birmuncha qulaylik tug'diradi. Yuqorida bu yig'indilarni ushbu metod orqali isbot qilingan. Yuqoridagilardan foydalanib quidagi yig'indilarni to'g'ri ekanligini tekshirib ko'rish mumkin.

1. $\sum_{i=1}^{n} i^5 = \frac{1}{12} n^2 (n+1)^2 (2n^2 + 2n - 1)$;

2. $$\sum_{i=1}^{n} i^6 = \frac{1}{42} n(n+1)(2n+1)(3n^4+6n^3-3n+1)$$

3. $$\sum_{i=1}^{n} i^7 = \frac{1}{24} n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)$$;

4. $$\sum_{i=1}^{n} i^9 = \frac{1}{20} n^2(n+1)^2(n^2+n-1)(2n^4+4n^3-n^2-3n+3)$$;

5. $$\sum_{i=1}^{n} i^{10} = \frac{1}{66} n(n+1)(2n+1)(n^2+n-1)\cdot(3n^6+9n^5+2n^4-11n^3+3n^2+10n-5)$$;

Foydalanilgan adabiyotlar

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