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R^n FAZODA QUVISH DIFFERENSIAL O‘YINI

Annotatsiya: *Ushbu maqolada quvish differensial o‘yini ko‘rib chiqiladi. O‘yinchilarning harakatlari birinchi tartibli chiziqli differensial tenglamalar bilan tavsiflanadi. O‘yinchilarning boshqaruv funksiyalari integral chegaralanishlar qo‘yiladi. Maqolaning asosiy natijasi quvlovchi quvishni chekli vaqt oralig‘ida yakunlashi uchun strategiya va kafolatlangan tutish vaqti uchun formula topishdir.*

Kalit so‘zlar: *quvlovchi, qochuvchi, quvish differensial o’yini, boshqaruv, kafolatlangan tutish vaqti, strategiya, quvishni yakunlash.*

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ДИФФЕРЕНЦИАЛЬНАЯ ИГРА ПРЕСЛЕДОВАНИЯ В R^n

Аннотация: *В настоящей статье рассматривается дифференциальная игра преследования в пространстве R^n. Движения игроков описываются линейными дифференциальными уравнениями первого порядка. На функции управления игроков накладываются интегральные ограничения. Основным результатом данной статьи является нахождение стратегии преследователя для завершения преследования на конечном интервале времени и формула гарантированного времени преследования.*

Ключевые слова: *Преследователь, убегающий, дифференциальная игра преследования, управление, гарантированное время преследования, стратегия, завершение преследования.*

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A PURSUIT DIFFERENTIAL GAME IN R^n

Annotation: *In the present paper, a pursuit differential game in the space* R^n*, is considered. The motions of the players are described by first order linear differential equations. The control functions of the players are subject to integral constraints. The main result of this paper is to find a strategy for the pursuer to complete the pursuit on a finite time interval and a formula for the guaranteed pursuit time.*

Keywords: *Pursuer, evader, pursuit differential game, control, guaranteed pursuit time, strategy, completion of pursuit.*

Kirish

Yigirmanchi asrning o‘rtalarida, Ayzeks tomonidan ziddiyatli muammolarni hal etish bilan shug’ullanuvchi “Differensial o‘yinlar” nomli tadqiqot sohasiga asos solindi. Shundan so‘ng sohaga doir fundamental natijalar [1, 2, 3, 4, 5] kabi ishlarda chop qilina boshlagan. So‘nggi paytlarda differensial o‘yinlar nazariyasi sohasida asosiy e’tibor o‘yinchilarning boshqaruv funksiyalari geometrik, integral yoki aralash chegaralanishlarga ega bo‘lgan holatlarni o‘rganishga qaratilgan (masalan, [6, 7, 8, 9, 10, 10]).

Ibragimov [11] ishda m ta quvlovchili va bitta qochuvchili sodda harakatli qochish differensial o‘yinini o‘rgangan. Bu o‘yinda quvlovchilarning eng katta tezligi qochuvchining eng katta tezligidan kichik bo‘lgan holda qochuvchi cheksiz vaqt davomida qochib yura olishi isbotlangan. Boshqa [12] ishda esa integral chegaralanishga ega bo‘lgan quvlovchilar va bitta qochuvchini optimal tutish differensial o‘yini qaralgan. O‘yinchilarning harakati bir xil turdagi chiziqli sistemalar bilan tavsiflangan. O‘yinning davomiyligi mahkamlab qo’yilgan va o‘yinning to‘lov funksiyasi o‘yin tugagandan so‘ng qochuvchi va quvlovchilar orasidagi minimal masofadir. O‘yinning to‘lov funksiyasini quyidan baholashga erishilgan.

Biz ushbu maqolada R^n fazoda bitta quvlovchi va bitta qochuvchining quvish differensial o‘yinini o‘rganamiz. Bunda o‘yinchilarning harakatlari birinchi tartibli chiziqli differensial tenglamalar bilan tavsiflanadi. O‘yinchilarning boshqaruv funksiyalari integral chegaralanishlarga ega bo‘lsin deb qaraymiz. Mazkur o‘yinda quvlovchi quvishni chekli vaqt oralig‘ida yakunlashi uchun strategiya va kafolatlangan tutish vaqti uchun formula topamiz. Ishning yakunida, maxsus holat uchun misol keltiramiz.

Asosiy qism

Biz quyidagi differensial o‘yinni qaraymiz:

$$\begin{aligned} \dot{\xi}(t) &= -\alpha\xi(t)+u(t), \quad \xi(0)=\xi_0, \\ \dot{\eta}(t) &= -\beta\eta(t)+v(t), \quad \eta(0)=\eta_0, \end{aligned} \tag{1}$$

bu yerda $t\geq 0$, $\xi(t), u(t), \eta(t), v(t), \xi_0, \eta_0 \in R^n$, $\xi_0 \neq \eta_0$, $2\beta \geq \alpha > 0$ va $u(t)$ funksiya quvlovchining boshqaruv parametri, $v(t)$ esa qochuvchining boshqaruv parametri.

Ta’rif 1. *Quyidagi*

$$\int_0^{\infty} \vee u(s) \text{¿}^2 ds \le \rho^2, \tag{2}$$

$$\int_0^{\infty} \vee v(s) \text{¿}^2 ds \le \sigma^2 \tag{3}$$

integral chegarlanishlarni qanoatlaniruvchi $u(t)=(u_1(t), u_2(t), \dots, u_n(t))$ *va* $v(t)=(v_1(t), v_2(t), \dots, v_n(t))$*o‘lchovli funksiyalar mos ravishda, quvlovchi va qochuvchining joiz boshqaruvi deb ataladi, bu yerda* ρ *va* σ *berilgan musbat sonlar.*

Berilgan (1) tenglamaning joiz boshqaruvlarga mos yechimi ushbu

$$\xi(t)=\xi_0 e^{-\alpha t}+\int_0^t e^{-\alpha(t-s)} u(s) ds,$$

$$\eta(t)=\eta_0 e^{-\beta t}+\int_0^t e^{-\beta(t-s)} v(s) ds$$

ko‘rinishda bo‘ladi. Bundan esa quvlovchining boshlang’ich ξ_0 holatdan $\theta \ge 0$ vaqtdagi erishish to‘plami, markazi $\xi_0 e^{\alpha\theta}$ nuqtada bo‘lgan, $\varphi(\alpha,\theta)\rho$ radiusli $B\left(\xi_0 e^{-\alpha\theta}, \varphi(\alpha,\theta)\rho\right)$ shardan iborat bo‘lishi kelib chiqadi. Bu yerda

$$\varphi(\alpha,\theta)=\left(\int_0^{\theta} e^{-2\alpha(\theta-s)} ds\right)^{1/2}=\sqrt{\frac{1}{2\alpha}\left(1-e^{-2\alpha\theta}\right)}. \tag{4}$$

Haqiqatan ham, (2) chegaralanish va Koshi-Shvarts tengsizligiga ko‘ra

$$\text{¿}\xi(\theta)-\xi_0 e^{-\alpha\theta} \vee \le \int_0^t e^{-\alpha(t-s)} |u(s)| ds$$

$$\le \left(\int_0^t e^{-2\alpha(t-s)} ds\right)^{\frac{1}{2}} \cdot \left(\int_0^t \vee u(s) \text{¿}^2 ds\right)^{\frac{1}{2}} \le \varphi(\alpha,\theta)\rho. \tag{5}$$

Huddi shu kabi qochuvchining boshlang’ich y_0 holatdan $\theta \ge 0$ vaqtdagi erishish to‘plami $B\left(\eta_0 e^{-\beta\theta}, \varphi(\beta,\theta)\sigma\right)$ shar bo‘ladi.

Ta’rif 2. *Ushbu*

$$U(t,\xi,\eta,v), U:[0,\infty) \times R^{3n} \to R^n,$$

uzluksiz funksiya quvlovchining strategiyasi deyiladi, agar qochuvchining har qanday $v(t)$ *joiz boshqaruvi va* $u=U(t,\xi,\eta,v)$ *da (1) Koshi masalasi yagona* $(\xi(t), y(t))$ *yechimga ega bo’lsa, hamda bu yechim bo‘ylab*

$$\int_0^{\infty} \vee U(s,\xi(s),\eta(s),v(s)) \text{¿}^2 ds \le \rho^2$$

tengsizlik bajarilsa.

Demak, quvlovchi har qanday t vaqtda qochuvchining holatini va tanlagan boshqaruv parametrini bilib turadi.

Ta’rif 3. *(1) differensial o‘yinda quvish* $\theta(U)>0$ *vaqt davomida yakunlanadi deyiladi, agar qochuvchi har qanday* $v=v(t)$ *joiz boshqaruvni tanlaganda ham quvlovchining shunday* $U(t,\xi,\eta,v)$ *joiz strategiyasi mavjud bo‘lib, biror* $t \in [0,\theta]$ *uchun* $\xi(t)=\eta(t)$ *tenglik bajarilsa. Biz* $\theta(U)$ *ni kafolatlangan tutish vaqti deb ataymiz.*

Endi ushbu ishning asosiy natijasini keltiramiz.

Teorema. *Agar* $(2\beta-\alpha)\rho^2>\beta\sigma^2$ *bo‘lsa, u holda (1) o‘yinda quvishni chekli* $\theta>0$

vaqtda yakunlash mumkin.

Isbot. Dastlab, kafolatlangan tutish vaqti uchun formula keltiramiz. Undan so‘ng, quvlovchi uchun quvishni yakunlash mumkin bo‘lgan strategiya quramiz.

Kafolatlangan tutish vaqti uchun formula. Quyidagi tenglamani $t>0$ uchun qaraylik

$$\frac{\varphi^2(\beta,t)}{\varphi(2\beta-\alpha,t)}\rho=\text{¿}\,\xi(0)e^{-\alpha t}-\eta(0)e^{-\beta t}\vee+\varphi(\beta,t)\sigma, \tag{6}$$

bu yerda

$$\varphi(2\beta-\alpha,t)=\left(\int_0^t e^{-2(2\beta-\alpha)(t-s)}ds\right)^{1/2}=\sqrt{\frac{1}{2(2\beta-\alpha)}\left(1-e^{-2(2\beta-\alpha)t}\right)}.$$

Yuqoridagi (4) tenglamadan $t\to 0^{+\text{¿}\text{¿}}$ da limitga o‘tib, ushbu $0<\text{¿}\,\xi(0)-\eta(0)\vee+0$ tengsizlikka (bu yerda $t\to 0^{+\text{¿}\text{¿}}$ o‘ng limitni bildiradi), shuningdek, $t\to+\infty$ da esa teorema farazidagi $\frac{\rho}{\sqrt{\beta}}>0+\frac{\sigma}{\sqrt{2\beta-\alpha}}$ tengsizlikka ega bo‘lamiz. Demak, (6) tenglama $(0,+\infty)$ intervalda kamida bitta ildizga ega bo‘ladi. Aytaylik, (6) tenglamaning $(0,+\infty)$ intervaldagi eng kichik ildizi $t=\theta>0$ bo‘lsin. Ushbu $\theta>0$ ni kafolatlangan tutish vaqti ekanligini ko‘rsatamiz.

Ma’lumki, ushbu

$$\int_0^\theta e^{-\beta(\theta-s)}\vee v(s)\vee ds\le\left(\int_0^\theta e^{-2\beta(\theta-s)}ds\right)^{1/2}\cdot\left(\int_0^\theta \vee v(s)\text{¿}^2 ds\right)^{1/2} \tag{7}$$

tengsizlikda tenglik

$$\text{¿}\,v(s)\vee\text{¿}\,k e^{-\beta(\theta-s)},0\le s\le\theta,k=const \tag{8}$$

bo‘lganda bajariladi. Agar qochuvchi $\int_0^\theta \vee v(s)\text{¿}^2 ds=\sigma^2$ ga teng energiya sarflasa, u holda (8) ga ko‘ra $k=\frac{\sigma}{\varphi(\beta,0,\theta)}$ bo‘ladi. Bundan quyidagiga ega bo‘lamiz:

$$\text{¿}\,v(s)\vee\text{¿}\frac{\sigma}{\varphi(\beta,0,\theta)}e^{-\beta(\theta-s)},0\le s\le\theta. \tag{9}$$

Biz keyinroq foydalanadigan ushbu

$$u_0(s)=\frac{\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta}}{\varphi^2(\beta,\theta)}e^{(\alpha-2\beta)(\theta-s)} \tag{10}$$

boshqaruv quvlovchining holatini $\xi_0 e^{-\alpha\theta}$ nuqtadan $\eta_0 e^{-\beta\theta}$ nuqtaga o‘tkazadi. Haqiqatan ham ,

$$\xi(\theta)\qquad\text{¿}\,\xi_0 e^{-\alpha\theta}+\int_0^\theta e^{-\alpha(\theta-s)}u_0(s)ds$$

$$\text{¿}\qquad\text{¿}\,\xi_0 e^{-\alpha\theta}+\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta}=\eta_0 e^{-\beta\theta}.$$

Quvlovchi uchun strategiya. Aytaylik, quvlovchi quyidagi

$$u(s)=\begin{cases}u_0(s)+e^{(\beta-\alpha)(\theta-s)}v(s),0\le s\le\theta,\\ 0,s>\theta\end{cases} \tag{11}$$

strategiyani qo‘llasin. Ushbu (11) strategiyaning joiz ekanligini ko‘rsatamiz. Koshi-Shvarts tengsizligiga ko‘ra

$$\left(\int_0^{\infty} \vee u(s)¿^2 ds\right)^{1/2} = \left(\int_0^{\theta} \vee u(s)¿^2 ds\right)^{1/2}$$
$$\leq \left(\int_0^{\theta} \vee u_0(s)¿^2 ds\right)^{1/2} + \left(\int_0^{\theta} e^{2(\alpha-\beta)(\theta-s)} \vee v(s)¿^2 ds\right)^{1/2}.$$

Endi navbatma-navbat (10), (9), (6) lardan foydalanib, quyidagilarni yoza olamiz:

$$\left(\int_0^{\infty} \vee u(s)¿^2 ds\right)^{1/2} \leq \left(\int_0^{\theta} \frac{¿\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta}¿^2}{\varphi^4(\beta,\theta)} e^{-2(2\beta-\alpha)(\theta-s)} ds\right)^{1/2}$$
$$¿ \quad ¿¿\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta} \vee \frac{¿}{\varphi^2(\beta,\theta)} \left(\int_0^{\theta} e^{-2(2\beta-\alpha)(\theta-s)} ds\right)^{1/2} ¿ + \frac{\sigma}{\varphi(\beta,\theta)} \left(\int_0^{\theta} e^{-2(2\beta-\alpha)(\theta-s)} ds\right)^{1/2}$$
$$¿¿\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta} \vee \frac{¿}{\varphi^2(\beta,\theta)} \varphi(2\beta-\alpha,\theta) + \frac{\sigma}{\varphi(\beta,\theta)} \varphi(2\beta-\alpha,\theta) = \rho.¿$$

Bu esa (11) strategiyaning joiz ekanligini bildiradi.

Quvishni yakunlash. Biz $t=\theta$ vaqtda quvishni yakunlash mumkinligini ko‘rsatamiz. Agar quvlovchi (8) strategiya bo‘yicha harakatlansa, u holda $t=\theta$ vaqtda $\xi(\theta)=\eta(\theta)$ tenglikka ega bo‘lamiz. Haqiqatan ham,

$$\xi(\theta) \quad ¿\xi_0 e^{-\alpha\theta} + \int_0^{\theta} e^{-\alpha(\theta-s)} \left(\frac{\eta_0 e^{-\beta\theta}-\xi_0 e^{-\alpha\theta}}{\varphi^2(\alpha,\theta)} e^{-\alpha(\theta-s)} + e^{(\beta-\alpha)(\theta-s)} v(s)\right) ds$$
$$¿ \quad + \int_0^{\theta} e^{-\alpha(\theta-s)} e^{(\alpha-\beta)(\theta-s)} v(s) ds$$
$$¿ \quad ¿\xi_0 e^{-\alpha\theta} + \eta_0 e^{-\beta\theta} - \xi_0 e^{-\alpha\theta} + \int_0^{\theta} e^{-\beta(\theta-s)} v(s) ds = \eta(\theta).$$

Isbot yakunlandi.

Misol. Aytaylik R^2 fazoda quyidagi differensial o‘yin berilgan bo‘lsin

$$\dot{\xi}(t) = \frac{-3}{2}\xi(t) + u(t), \quad \xi_0 = (0,0),$$
$$\dot{\eta}(t) = -\eta(t) + v(t), \quad \eta_0 = (4,4), \tag{12}$$

bu yerda $t \geq 0$, $\xi(t), u(t), \eta(t), v(t) \in R^2$, shuningdek o‘yinchilarning boshqaruv parametri mos ravishda quyidagi chegaralanishlarni qanoatlantirsin

$$\int_0^{\infty} \vee u(s)¿^2 ds \leq 4, \int_0^{\infty} \vee v(s)¿^2 ds \leq 1,$$

ya'ni, $\rho=2$, $\sigma=1$, $\alpha=3/2$ va $\beta=1$ bo‘lsin. Yuqoridagi (6) formulaga ko‘ra, kafolatlangan tutish vaqti $\theta \approx 16.45$ bo‘ladi. Berilgan (12) o‘yinda, quvlovchi quyidagi strategiyani qo‘llash orqali quvishni $\theta \approx 16.45$ vaqtda yakunlaydi

$$u(s) = \begin{cases} u_0(s) + e^{\frac{-1}{2}(\theta-s)} v(s), 0 \leq s \leq \theta, \\ 0, s > \theta, \end{cases}$$

bu yerda $u_0(s) = \dfrac{2e^{\frac{1}{2}(\theta+s)}}{e^{2\theta}-1} y_0.$

Xulosa

Ushbu maqolada R^n fazoda bitta quvlovchi va bitta qochuvchining quvish differensial oʻyini haqida yozilgan. Bunda oʻyinchilarning harakatlari birinchi tartibli chiziqli differensial tenglamalar bilan tavsiflangan va oʻyinchilarning boshqaruv funksiyalari integral chegaralanishlarga ega boʻlsin deb qaralgan. Maqolaning asosiy natijasi sifatida quvlovchi quvishni chekli vaqt oraligʻida yakunlashi uchun strategiya va kafolatlangan tutish vaqtini topish uchun formula koʻrsatilgan. Ishning yakunida, maxsus holat uchun misol keltirilgan.

Bu ishda asosiy e'tibor oʻyinchilarning harakat tenglamalari chiziqli differensial tenglamalar bilan berilgan hol uchun quvish masalasini hal qilishga qaratildi. Oʻyinchilarning harakat tenglamalari chiziqlimas tenglamalar bilan berilgan, shuningdek, oʻyinchilar uchun turli chegaralanishlar qoʻyilgan differensial oʻyinlar uchun quvish-qochish masalalarini oʻrganish kelgusi tadqiqotlarimiz uchun reja qilingan.

Tashakkurnoma. Mualliflar ushbu masalani tavsiya qilgani va uning ustida ishlash jarayonidagi koʻmagi uchun professor Ibragimov G.I.ga chuqur minnatdorlik bildiradi.

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