Liuvill tenglamasi bilan bog‘liq rekurrent-differensial tenglama haqida

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Annotatsiya. *Ushbu* $y''+xy=0$ *Liuvill tenglamasiga tengkuchli bo‘lgan* $z'(x)=x+z^2(x)$ *Rikkati tenglamasi yechimi uchun Teylor formulasi bilan bog‘liq rekurrent-differensial tenglamalar sistemasi o‘rganilgan.*

Kalit so‘zlar: *rekurrent-differensial tenglama, polinomial uchburchak, cheksiz sistema, Liuvill tenglamasi, taqribiy yechim.*

О рекуррентно-дифференциальном уравнении, связанном с уравнением Лиувилля

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Аннотация. *Исследована система рекуррентно-дифференциальных уравнений, связанная с формулой Тейлора для решения уравнения Риккати* $z'(x)=x+z^2(x)$, *эквивалентная уравнению Лиувилля* $y''+xy=0$.

Ключевые слова: *рекуррентно-дифференциальное уравнение, полиномиальный треугольник, бесконечная система, уравнение Лиувилля, приближенное решение.*

About the recurrent-differential equation related to the Liouville equation

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Abstract. *It is studied a system of recurrent differential equations related to the Taylor formula for the solution of the Liouville equation* $y''+xy=0$ *that is equivalent to Riccati equation* $z'(x)=x+z^2(x)$.

Key words: *recurrent-differential equation, polynomial triangle, infinite system, Liouville's equation, approximate solution.*

Parametrga bog‘liq chiziqli tenglamalarni kichik parametr usuli bilan

$$y'_n(x)=a_n(x)y_n(x)+b_n(x)y_{n-1}(x)+c_n(x), \quad n=0,1,2,... \tag{1}$$

ko‘rinishidagi rekurrent-differensial tenglamalar cheksiz sistemasiga keltirish mumkinligi yaxshi ma’lum, bunda $y_0(x)$ berilgan deb qaraladi. Bu sistemada no’malumlar birin-ketin integrallash yo‘li orqali hisoblanadi. Amaliyotda (1) sistemadan farqli

$$P_{n+1}^k(x)=a_n^k(x)P_n^{k+1}(x)+b_n^k(x)(P_n^k)'+c_n^k(x)P_n^{k-1}(x) \tag{2}$$

ko‘rinishdagi tenglamalar sistemasiga keltiriladigan masalalar uchraydi. Xususan, $a_n^k(x)=(k+1)x, \ \ b_n^k(x)=1, \ \ c_n^k(x)=k-1$ bo‘lgan hol Liuvill tenglamasi uchun kombinatorik masalaga tengkuchli. Bu yerda $P_n^k(x)$ ko‘phadlardan iborat bo‘lib, u binomial koeffitsientlar kabi uchburchak hosil qiladi.

$$
\begin{array}{ccccccccccc}
 & & & & & 0 & & & & & \\
 & & & & 1 & & x & & & & \\
 & & & 2 & & 2x & & 1 & & & \\
 & & 6 & & 8x & & 2 & & 2x^2 & & \\
 & 24 & & 40x & & 10 & & 16x^2 & & 6x & \\
120 & & 240x & & 60 & & 136x^2 & & 52x & & 16x^3+6
\end{array}
$$

(3)

(2) munosabatdan bevosita quyidagi xossalar kelib chiqadi: 1) agar $k-$ juft bo‘lsa, $P_n^k(x)$ bu $\left(\frac{k}{2}-1\right)$-darajali, agar toq bo‘lsa, $\left(\left[\frac{k}{2}\right]+1\right)$-darajali ko‘phad; 2) $d_n^k=\deg P_n^k(x)$ deyilsa, $d_{n+1}^{k+2}=d_n^k+1$, $n=2,3,\ldots,n-1;\ k=1,2,\ldots,n-1.$

(2) formuladan

$$P_{n+1}^n(x)=(n+1)!x+(n-1)P_n^{n-1}(x),\quad P_{n+1}^{n-1}(x)=\left(P_n^{n-1}(x)\right)'+(n-2)P_n^{n-2}(x),$$

$$P_{n+1}^k(x)=(k+1)xP_n^{k+1}(x)+(P_n^k)'+(k-1)P_n^{k-1}(x),\quad P_{n+1}^1=\left(P_n^1(x)\right)'+2xP_n^2(x),$$

$$P_{n+1}^0(x)=\left(P_n^0(x)\right)'+xP_n^1(x)$$

tengliklarga ega bo‘lamiz, agarda $P_n^k(x)=\begin{cases}0, & k<0\\ 0, & k=n\\ n!, & k=n+1\end{cases}$ bo‘lsa (n=0, 1, 2…, k=0, 1, 2, …).

Endi $y''+xy=0$ tenglamani qaraymiz. Uning yechimi elementar funksiyalar orqali, ular ustida algebraik amallar va integrallash amali bilan ifodalanmaydi (J.Liuvill teoremasi) [1, 2]. Bu tenglama

$$z'(x)=x+z^2(x) \tag{4}$$

Rikkati tenglamasiga tengkuchli.

Teorema. (4) tenglamaning yechimi $z(x)$ uchun

$$z^{(n)}=n!z^{n+1}+\sum_{k=0}^{n-1}P_n^k(x)z^k \tag{5}$$

formula o‘rinli.

Isbot. $n=1$ bo‘lsa (4) formula $P_1^0(x)=x$ ekanligidan kelib chiqadi. Xuddi shu singari $n=2$ bo’lsa (4) tenglik (3) jadvalning ikkinchi satridan hosil bo’ladi. (4) tenglik n uchun o‘rinli bo‘lsin [3, 4, 5]. U holda

$$z^{(n+1)}=(n+1)!z^n(x+z^2)+\sum_{k=0}^{n-1}\left(P_n^k(x)\right)'z^k+\sum_{k=1}^{n-1}P_n^k(x)kz^{k-1}(x+z^2)=$$

$$=(n+1)!z^{n+2}+(n+1)!xz^n+\sum_{k=0}^{n-1}\left(P_n^k(x)\right)'z^k+\sum_{k=1}^{n-1}kxP_n^k(x)z^{k-1}+\sum_{k=1}^{n-1}kP_n^k(x)z^{k+1}.$$

So‘ngi ikki yig‘indini

$$\sum_{k=1}^{n-1}kxP_n^k(x)z^{k-1}=\sum_{k=0}^{n-2}(k+1)xP_n^{k+1}(x)z^k, \quad \sum_{k=1}^{n-1}kP_n^k(x)z^{k+1}=\sum_{k=2}^{n}(k-1)P_n^{k-1}(x)z^k$$

ko‘rinishiga keltirib olamiz.

Demak,

$$z^{(n+1)}=(n+1)!z^{n+2}+\left[(n+1)!x+(n-1)P_n^{n-1}(x)\right]z^n+\left[\left(P_n^{n-1}(x)\right)'+(n-2)P_n^{n-2}(x)\right]z^{n-1}+$$

$$+\sum_{k=2}^{n-2}\left[\left(P_n^k(x)\right)'+(k+1)xP_n^{k+1}(x)+(k-1)P_n^{k-1}(x)\right]z^k+\left[\left(P_n^1(x)\right)'+2xP_n^2(x)\right]z+$$

$$+\left[\left(P_n^0(x)\right)'+xP_n^1(x)\right]=(n+1)!z^{n+2}+P_{n+1}^n(x)z^n+P_{n+1}^{n-1}(x)z^{n-1}+\sum_{k=2}^{n-2}P_{n+1}^k(x)z^k+$$

$$+P_{n+1}^1(x)z+P_{n+1}^0(x)=(n+1)!z^{n+2}+\sum_{k=0}^{n}P_{n+1}^k(x)z^k.$$

Xulosa. Matematikaning bir qator masalalari cheksiz noma’lumli differensial tenglamalar sistemasiga ketiriladi. Bunday sistema rekurrent ko‘rinishdagina bo‘lsagina yechiladi. Maqolada shu turga mansub sistema namunasi o‘rganilgan. Mulohazalar cheksiz differensial tenglamalarni o‘rganishga asos bo‘lib hizmat qiladi.

Foydalanilgan adabiyotlar.

1. I. Kaplansky. *An Introduction to differential algebra*. Hermann. Paris.1957.

2. J. F. Ritt. *Integration in finite terms, Liouville theory of elementary methods.* Columbia university press.1948.
3. Richard Andre-Jeannin. *A note on a general class of polynomials.* IUT GEA, Route de Romain, 54400 Longwy, France (Submitted April 1993).
4. В. Н. Сачков. *Введение в комбинаторные методы дискретной математики*. Москва Издательство МЦНМО 2004.
5. А. О. Гельфонд. *Исчичление конечных разност.* Государственное издптелбство физико-математической литературы. Москва 1959.