CIRCULAR ORBITS NEAR FIVE-DIMENSIONAL NON-EXTREMAL REISSNER-NORDSTRÖM BLACK HOLE

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Abstract. We focused on the following two things in this work first, predict the processes in the black hole attribute depending on the properties of the metric function and their variables, the movement of particles in circular motion. In the second part, we calculated the movement of particles around the black hole using the metric function, their ISCO parameters and how much they change depending on the change of Q and M . To date, these features of this work have not been fully studied. We tried to fill these gaps in this work.

Key words: *Black hole, ISCO, Circular orbits.*

КРУГОВЫЕ ОРБИТЫ ВБЛИЗИ ПЯТИМЕРНОЙ НЕЭКСТРЕМАЛЬНОЙ ЧЕРНОЙ ДЫРЫ РЕЙССНЕРА-НОРДСТРЁМА

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Аннотация. В этой работе мы сосредоточились на следующих двух вещах: во-первых, предсказать процессы в атрибуте черной дыры в зависимости от свойств метрической функции и их переменных, движение частиц по круговой траектории. Во второй части мы рассчитали движение частиц вокруг черной дыры с использованием метрической функции, их параметры ISCO и то, насколько они изменяются в зависимости от изменения Q и M. На сегодняшний день эти особенности этой работы изучены не полностью. Мы попытались заполнить эти пробелы в этой работе.

Ключевые слова: *Черная дыра, ISCO, Круговые орбиты.*

BESH O'LCHAMLI EKSTREMAL BO'LMAGAN REISSNER-NORDSTRÖM QORA TUYUK YAQINIDAGI AYLANA ORBITALAR

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Annotatsiya. Biz bu ishda birinchi navbatda quyidagi ikkita narsaga e‘tibor qaratdik, metrik funktsiya va ularning o‘zgaruvchilari xususiyatlariga, zarrachalarning

aylana harakatida harakatiga qarab qora tuynuk atrofidagi jarayonlarni bashorat qilamiz. Ikkinchi qismda metrik funktsiyadan foydalanib, qora tuynuk atrofidagi zarrachalarning harakatini, ularning ISCO parametrlarini va Q va M ning o‘zgarishiga qarab qanchalik o‘zgarishini hisoblab chiqdik. Bugungi kunga qadar ishning ushbu xususiyatlari to‘liq o‘rganilmagan. Biz bu ishda ana shu kamchiliklarni to‘ldirishga harakat qilindi.

Kalit so‘zlar: *Qora tuynuk, ISCO, Sirkular orbitalar.*

INTRODUCTION

In the last decade, black hole research has emerged as a thrilling field in physics. It has been suggested that black holes should be considered physical systems instead of merely mathematical constructs [1]. This perspective uses information theory to understand their properties. By integrating classical gravity, mechanics, and quantum theory within the framework of black hole thermodynamics, this approach aims to advance the study of quantum gravity [2].

Additionally, when combined with a negative cosmological constant, the Reissner-Nordström black hole (RNblack hole) has proven to be a valuable tool for exploring the thermodynamic properties of gravity [3,4] and for examining the impact of modified radiation equations of state at the Planck scale on the Reissner-Nordström-Anti-de Sitter black hole (RNAdSblack hole) [5]. Despite the RNblack hole in AdS spacetime being an effective solution for many gravity field theories, it still presents several perplexing issues. It fails to satisfy the Bootstrap bound, even in extreme conditions or at absolute zero, unlike in asymptotically Minkowski spacetime. While its classical instability has not been explicitly demonstrated without additional charged matter, this may suggest that the extremal RN solution in AdS spacetime is unstable [6−8].

It is known that under large charged scalar perturbations, superradiant stability applies to all non-extremal ReissnerNordström black holes (RNblack holes) in five dimensions. This suggests that all RNblack holes in higher dimensions may be superradiantly stable when subjected to charged massive scalar perturbations [9]. Conversely, the relaxation rate of the four-dimensional RNblack hole associated with the massless scalar field was analyzed in Ref. [10].

The present paper has two broad aims. First, to present some important results on both geometrical and topological aspects of the five-dimensional RNblack hole spacetime, by constructing different types of strong retractions and strong homotopy retracts on the background under consideration, which will be mathematically denoted, for simplicity, by N^5 black hole. In addition, we will use it to prove the existence of apparent horizons, which are crucial to the studies of wave phenomena on black hole spacetimes. Furthermore, we will also show that the end limit of N^5 black hole is a 0-dimensional non-extremal black hole. Second, to discuss the interaction between quantum charged massive scalar particles and the background under consideration, by obtaining the spectrum of quasibound states, which are related to the boundary conditions imposed on the radial wave solution. In addition, we will also calculate the corresponding angular and radial wave eigenfunctions. Indeed, these are two very important, interesting features from a mathematical physics point of view.

The paper is organized as follows. In section II, we talk about the field equation, its solution, and the analysis of the metric function. Section III describes the movement of particles around a static black hole. Section III C describes the innermost stable circular orbit (ISCO) and how it depends on the parameters.

Field Equation In Higher-Dimensional Reissner-Nordström Black Holes

The standard four-dimensional black hole solutions to Einstein's equations were generalized to higher dimensions by Myers and Perry [11]. The higher-dimensional action, which is a generalization of the Einstein-Hilbert action, is given by:

$$S=\int \sqrt{-g}\, d^D x \left(\frac{1}{16\pi G_D} R + L_{\text{matter}} \right)$$

(1)

where D is the number of dimensions, g is the determinant of the metric tensor, G is the gravitational constant, R is the Ricci scalar, and L_{matter} represents the matter Lagrangian and $g \equiv det\left(g_{\mu\nu}\right)$. By differentiating this action with respect to the space-time metric $\left(g_{\mu\nu}\right)$, the Einstein equations in higher dimensions are derived:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G_D T_{\mu\nu}$$

(2)

where $R_{\mu\nu}$ is the Ricci tensor. Now, for simplicity and without loss of generality, we may adopt the natural units where $G_D = 1$. Next, by imposing a harmonic gauge condition, a static, asymptotically flat and spherically-symmetric vacuum solution of Eq. (2), which generalizes the RNblack hole solution in D-dimensions, has the following form

$$d s^2 = -f(r)\, d t^2 + f ¿$$

¿

where the metric function, $f(r)$. In eq. (3) find metric function read as [12]:

$$d s^2 = -f(r)\, d t^2 + f ¿$$

¿

with

$$f(r)=1-\frac{2M}{r^2}+\frac{Q^2}{r^4}, M=\frac{4M}{3\pi}, Q=\frac{2Q}{\sqrt{3}\pi} \quad (5)$$

where θ, ϕ run over the range 0 to π, and χ from 0 to 2π.

Analysing of metric function. The metric function is a size division representing space-time, in our case its expression is represented by Eq.(4). Now to find the event horizon, we wing the following conditions:

$$f(r)=0, f'(r)=0 \quad (6)$$

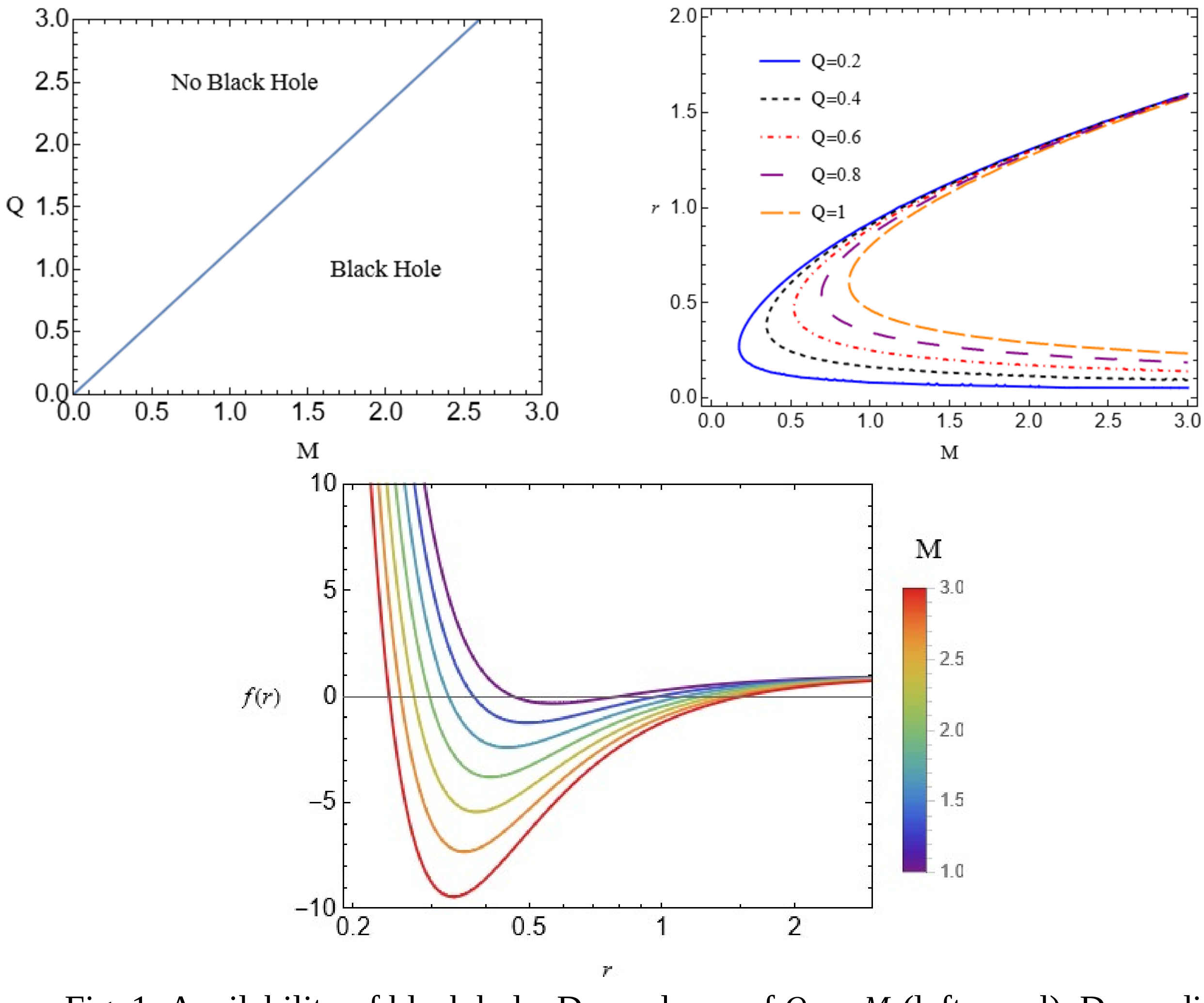

Fig. 1: Availability of black hole. Dependence of Q on M (left panel). Depending of r to M at different values of Q (right panel).

For the parameters M and Q , we can obtain the existence of BN and the non-existence of black hole.Eq.(6) above can be described as Figure 1. All the following theoretical calculations and results obtained from them were selected for the $(1,3)$ range of M and the $(0,1)$ range for Q. The reason for this is that the existence condition of black hole is winged in all fields of large values. As we increase the value of Q, the value of the horizon increases. Consequently, as the value of Q increases in the given interval, the mass of black hole also increases (Figure 1 right panel). Now let's look at the metric function (Figure 1 lower panel) as a function of M. With $Q=1, M$ has two (inner and outer) horizons at all values $(1,3)$. As we increase the mass of the black hole in the given interval, the minimum of the metric function moves towards the black hole and its value decreases.

Motion Of Test Particles Around Static Black Holes

Effective potential. In this section, we briefly look at the motion of chargeless test particles around static phantom black holes, using the following Lagrangian formulation:

$$L_p=\frac{1}{2}m\, g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \tag{7}$$

with the conserved quantities provided as:

$$g_{tt}\dot{t}=-E\,,\ g_{\phi\phi}\dot{\phi}=L \tag{8}$$

where $E=E/m$ and $L=L/m$ represent the particle's specific energy and angular momentum, respectively. The equations governing the motion of the particles are derived employing the normalization condition:

$$g_{\mu\nu}u^{\mu}u^{\nu}=\epsilon \tag{9}$$

with ϵ being assigned the value of 0 for massless particles and -1 for massive particles.

For test particles, the equations governing the motion of time-like geodesics can be derived by applying the condition specified in Eq. (9) along with Eq. (8), taking the following form [13-15]:

$$\dot{r}^2=E^2+g_{tt}\left(1+\frac{k}{r^2}\right) \tag{10}$$

$$\dot{\theta}^2=\frac{1}{g_{\theta\theta}^2}\left(k-\frac{L^2}{\sin^2\theta}\right) \tag{11}$$

$$\dot{\phi}=\frac{L}{g_{\phi\phi}} \tag{12}$$

$$\dot{t}=\frac{-E}{g_{tt}} \tag{13}$$

where k represents the constant for the angular motion, also known as the Carter constant, corresponding to the total angular momentum of the test particle. In this context, we consider particle motion to be confined to the equatorial plane, where: $\theta=\pi/2, \dot{\theta}=0$. Furthermore, we determine the Carter constant as $k=L^2$. As a result, the equation governing the radial motion of the particles transforms into

$$\dot{r}^2=E^2-V_{eff} \tag{14}$$

where the effective potential for the radial motion of test particles is expressed as:

$$V_{eff}=f(r)\left(1+\frac{L^2}{r^2}\right) \tag{15}$$

Let's briefly look at V_{eff} for this case. V_{eff} is directly related to the metric function $f(r)$. The function $f(r)$ immediately depends on parameters M and Q. Choosing $Q=1$ from these parameters, the change of V_{eff} was studied in the range $M\in(1,3)$ (Fig. 2). As we increase the value of M, it can be observed that the value of the maximum of V_{eff} decreases and moves further away from the black hole.

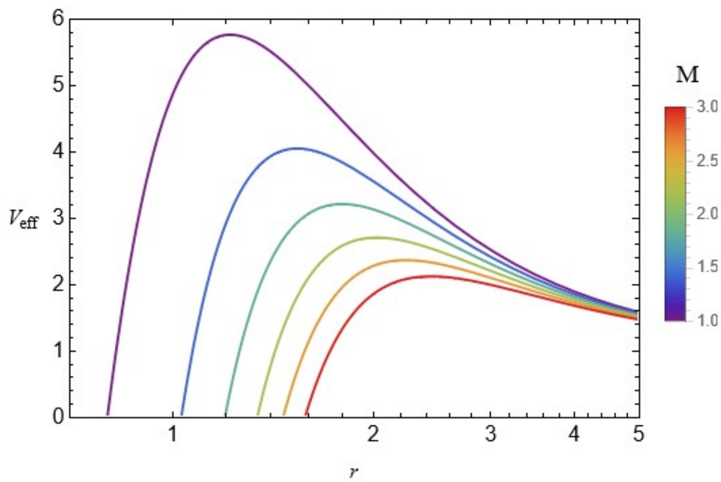

Fig. 2: Different lines of the effective potential at different values of M, in cases where $Q=1, L^2=16$.

Circular motion. To explore the circular movement of particles, we use the following criteria:

$$V_{eff}=E, V'_{eff}=0 \tag{16}$$

The stability of circular orbits for test particles orbiting a central point-like gravitating compact object is determined by the equation $\partial_{rr}V_{\text{eff}}=0$, which ensures that the effective potential is at a minimum. Typically, this condition is

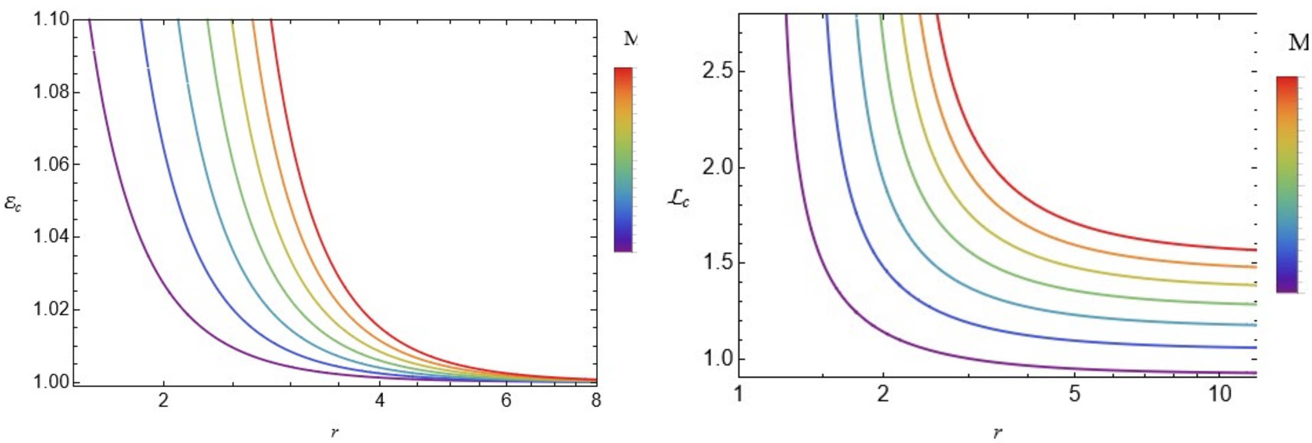

Fig. 3: The radial dependence of specific energy (left panel) and angular momentum (right panel) of the test particle around the extreme black holes.

used alongside the circularity condition provided in Eq.(16), known as the equation for the Innermost Stable Circular Orbit (ISCO). Consequently, the equation for the effective potential presented in Eq.(15) takes on a modified form to satisfy these conditions: [13]

$$f^{'}(r)\left(2r\frac{f^{'}(r)}{f(r)}-3\right)-rf^{''}(r)\geq 0 \tag{17}$$

The observations can be made about E_c according to Fig. 3 (left panel). We can see that E_c min increases with initial values of M (the case from purple to the red line), initially decreasing and then increasing at values corresponding to M. Also, as the value of γ increases, the value of E_c minimum moves to the right on the horizontal axis. We can see in Fig. 3 (right hand) that the angular momentum of the particle L_c increases monotonically with increasing values of γ.

$$L_c^2=\frac{8r^2\left(\pi M r^2-Q^2\right)}{\pi r^2\left(3\pi r^2-16M\right)+12Q^2} \tag{18}$$

(18) into to (15), we consider that:

$$E_c^2=\frac{\left(\pi r^2\left(3\pi r^2-8M\right)+4Q^2\right)^2}{3\pi^2 r^4\left(\pi r^2\left(3\pi r^2-16M\right)+12Q^2\right)} \tag{19}$$

ISCO. In the study of a black hole, its ISCO is important. It can be observed from Fig. (4) that between the values of $Q=1$ and $M\in(0,1)$, we can see that the values of its ISCO slowly rise with different areas. It can be concluded from the figure that ISCO accepts only one value for certain values of M, while some values have 2 values. We can find the ISCO parameter from the following expression:

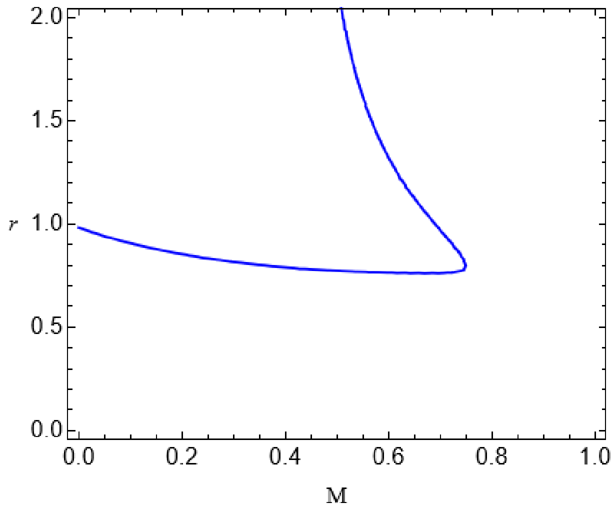

Fig. 4: ISCO parameter corresponded to r and M.

$$45\pi^6 r^{12}\left(16M^2-3Q^2\right)512\pi^3 Q^2 r^6\left(37M^3+15MQ^2-864\pi^5 M r^{10}\left(4M^2+Q^2\right)\right)$$
$$+48\pi^2 Q^4 r^4\left(512M^2+45Q^2\right)+12\pi^4 r^8\left(512M^4+624M^2Q^2+81Q^4\right)$$
$$-13824\pi M Q^6 r^2+2880Q^8=0 \qquad (20)$$

The radius of the ISCO of the test particles can be calculated numerically by solving Eq.(20).

CONCLUSION

This article concluded that: All selected intervals of Q and M have inner and outer horizons. The greater the charge of the black hole, the greater the observation distance of the horizon. The maximum of the effective potential is inversely proportional to the mass of the black hole. At certain values of M, the value of ISCO decreases non-linearly, and at certain values it increases sharply.

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