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AHOLI SONI O‘SISHINI MATEMATIK MODELLASHTIRISH: O‘ZBEKISTON AHOLISI UCHUN MALTUS VA VERHULST MODELLARI TADBIQI

Annotatsiya: Aholining o‘sishi ijtimoiy-iqtisodiy va ekologik omillarga sezilarli ta’sir ko‘rsatadi, bu esa samarali siyosatni ishlab chiqish uchun aniq prognozlash modellarini zarur qiladi. Ushbu tadqiqot O‘zbekiston aholisining o‘sishini eksponensial va logistik modellar yordamida baholaydi. Eksponensial model 2030 yilga kelib aholi soni 46 million atrofida bo‘lishini prognoz qilmoqda, biroq so‘nggi ma’lumotlar bilan nomuvofiqlikni ko‘rsatdi. Bundan farqli o‘laroq, logistik model 2030-yilga kelib taxminan 43,9 millionni bashorat qiladi va uzoq muddatli tendentsiyalarga yaxshiroq mos keladi. Ushbu natijalarlar yanada ishonchli bashorat qilish uchun bir nechta modellarni birlashtirish zarurligini ta’kidlaydi. Kelajakdagi tadqiqotlar modelning aniqligini oshirish uchun dinamik o‘zgaruvchilar va ijtimoiy-iqtisodiy omillarni birlashtirishga qaratilishi kerak.

Kalit so‘zlar: Aholi o‘sishi, matematik modellashtirish, Maltus modeli, Verhulst modeli, O‘zbekiston, demografik o‘zgarishlar, eksponensial o‘sish, logistik model

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ РОСТА НАСЕЛЕНИЯ: ПРИМЕНЕНИЕ МАЛЬТУЗИАНСКОЙ И МОДЕЛИ ВЕРХЮЛЬСТА В УЗБЕКИСТАНЕ

Аннотация: *Рост населения существенно влияет на социально-экономические и экологические факторы, поэтому точные модели* *прогнозирования необходимы для эффективной разработки политики. В этом исследовании оценивается рост населения Узбекистана с использованием экспоненциальных и логистических моделей. Экспоненциальная модель прогнозирует, что к 2030 году численность населения составит около 46 миллионов человек, но она показывает расхождения с недавними данными. Напротив, логистическая модель, учитывающая пропускную способность, прогнозирует примерно 43,9 миллиона к 2030 году и лучше согласуется с долгосрочными тенденциями. Эти результаты подчеркивают необходимость интеграции нескольких моделей для более надежных прогнозов. Будущие исследования должны быть сосредоточены на включении динамических переменных и социально-экономических факторов для повышения точности модели.*

Ключевые слова: *рост населения, математическое моделирование, мальтузианская модель, модель Ферхюльста, Узбекистан, демографические изменения, экспоненциальный рост, логистическая модель.*

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MATHEMATICAL MODELING OF POPULATION GROWTH: APPLICATION OF THE MALTHUSIAN AND VERHULST MODELS TO UZBEKISTAN

Abstract: *Population growth significantly affects socio-economic and environmental factors, making accurate forecasting models essential for effective policy-making. This study evaluates Uzbekistan's population growth using exponential and logistic models. The exponential model forecasts a population of around 46 million by 2030, but has shown discrepancies with recent data. In contrast, the logistic model, accounting for carrying capacity, predicts approximately 43.9 million by 2030 and aligns better with long-term trends. These* *findings highlight the need for integrating multiple models for more reliable predictions. Future research should focus on incorporating dynamic variables and socio-economic factors to enhance model accuracy.*

Keywords: *Population growth, mathematical modeling, Malthusian model, Verhulst model, Uzbekistan, demographic changes, exponential growth, logistic model*

INTRODUCTION

Population growth is a critical phenomenon that has significant implications for a range of socio-economic and environmental factors. Understanding the patterns and dynamics of population growth is crucial for informed policy-making and planning in various domains, such as healthcare, education, housing, employment, and resource management. In this research, we investigate the population growth patterns of Uzbekistan, a Central Asian country that has experienced significant demographic changes in recent decades. Specifically, we use two different mathematical models - exponential growth model, and logistic model - to analyze the historical and projected trends in population growth in Uzbekistan. The primary aim of this research is to compare the accuracy and utility of these models in predicting population growth in Uzbekistan and to discuss their implications for various facets of life in the country. In this section, we provide a brief overview of the research methods and materials used in this study.

Population is a crucial factor in the development of any country. Understanding the dynamics of population growth is essential for policymakers to make informed decisions regarding healthcare, education, and other social services. To understand the growth pattern of a population, we use population growth models. These models help us predict the future population growth based on historical data.

In this article, we will apply a population growth models to Uzbekistan's population. Uzbekistan is a landlocked country in Central Asia, with a population of more than 36 million. The country has undergone significant demographic changes in recent decades, making it an interesting case study for population growth models.

LITERATURE REVIEW

Population growth has been a topic of concern for many decades due to its potential impact on the environment, economy, and society. Mathematical models have been developed to help understand and predict population growth. This literature review discusses various population growth models, their applications, and limitations. The paper “Doomsday: Friday, 13 November, A.D. 2026” by (Heinz Von Foerster, 1960), introduced the idea of the ultimate limit to population growth. The authors predicted that if current population growth rates continued, the world population would reach infinity on November 13, 2026. This paper highlights the importance of considering the limitations of growth models and their potential consequences. Thomas Hillen’s article “Applications and Limitations of the Verhulst Model for Populations” discusses the Verhulst model, which is a commonly used model for biological population growth. Hillen explains the assumptions and limitations of the model and provides examples of its applications in ecology and epidemiology. (Hillen, 2003)

D. Joyce’s paper “The Logistic Population Model” provides a proof and application of the logistic model, which is another widely used population growth model. The paper explains how the logistic model accounts for environmental factors that limit population growth and provides examples of its applications in ecology and economics. (Joyce, 2013) “The Limits to Growth” by Donella et al. is a book that discusses the factors that limit population growth, including resource depletion and environmental degradation. The book uses computer simulations to predict the potential consequences of unchecked population growth and resource consumption. (Donella H.Meadows, 1972) In “Human Population Growth: Stability or Explosion” David A. Smith discusses the different types of population growth models, including exponential, logistic, and oscillatory models. The paper highlights the importance of understanding the assumptions and limitations of these models and their potential implications. (Smith, 1977)

Rezaul Karim et al.'s paper “Modeling on Population Growth and its Adaptation: A Comparative Analysis between Bangladesh and India” compares population growth models and their applications in Bangladesh and India. The paper highlights the importance of considering the unique social, economic, and environmental factors that affect population growth in different regions (Rezaul Karim, 2020). Michael Anatoly Golosovsky provides in “Models of the World Human Population Growth - Critical Analysis” a critical analysis of various population growth models, including the logistic model, the Malthusian model, and the Kuznets curve model. The paper highlights the limitations of these models and the challenges of predicting future population growth. (Golosovsky, 2009) Venkatesha's paper “Mathematical Modelling of Population Growth” discusses various mathematical models of population growth, including the exponential, logistic, and Gompertz models. The paper provides examples of their applications in ecology, epidemiology, and economics and highlights the importance of understanding the assumptions and limitations of these models. (Venkatesha P., 2017). In the article “Mathematical Modelling of Population Growth: The Case of Logistic and Von Bertalanffy Models” Mohammed Dawed et al. discusses the theoretical mathematical aspects of population growth models, specifically the logistic and von Bertalanffy models. The paper explains the underlying equations and assumptions of these models and provides examples of their applications in ecology and economics. (Mohammed Dawed, 2014)

Abdunazarov's article “Demographic Characteristics of Population Growth in Uzbekistan” provides a comprehensive overview of the factors that contribute to population growth in Uzbekistan. The article analyzes both natural and mechanical population growth in the country and its regions, including the impact of factors such as fertility, mortality, migration, and urbanization. The author notes that Uzbekistan has experienced significant population growth in recent years, due in part to high fertility rates and low mortality rates. However, the country also faces challenges related to urbanization and migration, with many people leaving rural areas in search of employment and better living conditions in urban centers. (Abdunazarov Husan Menglievich, 2021). “The Application of Differential Equation of Verhulst Population Model on Estimation of Bandar Lampung Population” by Mujib et al. is a research article that applies the Verhulst population model, which is a logistic growth model, to estimate the population of Bandar Lampung in Indonesia. The authors used data from the Indonesian Central Bureau of Statistics to estimate the parameters of the model and validate the model’s accuracy.

In conclusion, mathematical models have been developed to help understand and predict population growth. However, it is essential to consider the assumptions and limitations of these models and the unique social, economic, and environmental factors that affect population growth in different regions. Future research should focus on improving the accuracy of these models and developing more comprehensive models that consider the complex interactions between population growth, resource consumption, and environmental degradation.

RESEARCH METHODS AND MATERIALS

Exponential growth model (Malthusian model), sometimes called a simple exponential growth model, is essentially exponential growth based on the idea of the function being proportional to the speed to which the function grows. The model is named after Thomas Robert Malthus, who wrote An Essay on the Principle of Population (Malthus, 1798), one of the earliest and most influential books on population[1]

We suppose the size N of the population under consideration is a function of time t, and that its rate of change dN/dt is a known function of N and t. To be more precise, we suppose that the population has a birth rate B and a mortality rate M,

[1] https://en.wikipedia.org/wiki/Malthusian_growth_model

both of which may be functions of N and t, expressed as functions of the total population, so that the net growth rate may be written as

(1) $$\frac{dN}{dt}=BN-MN=(B-M)N=PN$$ XE "$\dot{N}=AN-BN$:1"

where $P=B-M$ is the production rate, that is, the net rate at which individuals in the populations are reproducing.

(2) $$\frac{dN}{dt}=PN$$

Equation (2) is a differential equation and we'll solve it using by separating variables method.

(3) $$\frac{dN}{N}=Pdt$$

Integrating both sides of the equation (3). We'll have equation (4) below

(4) $$lnN=Pt+C$$

We'll come to the result below after solving the equation (4) respect to N

(5) $N=e^{Pt+C}$ or $N=Ce^{Pt}$

We've initial condition which in the previous period $t=0$ the number of population was N_0. Thus,

(6) $$N(0)=N_0$$

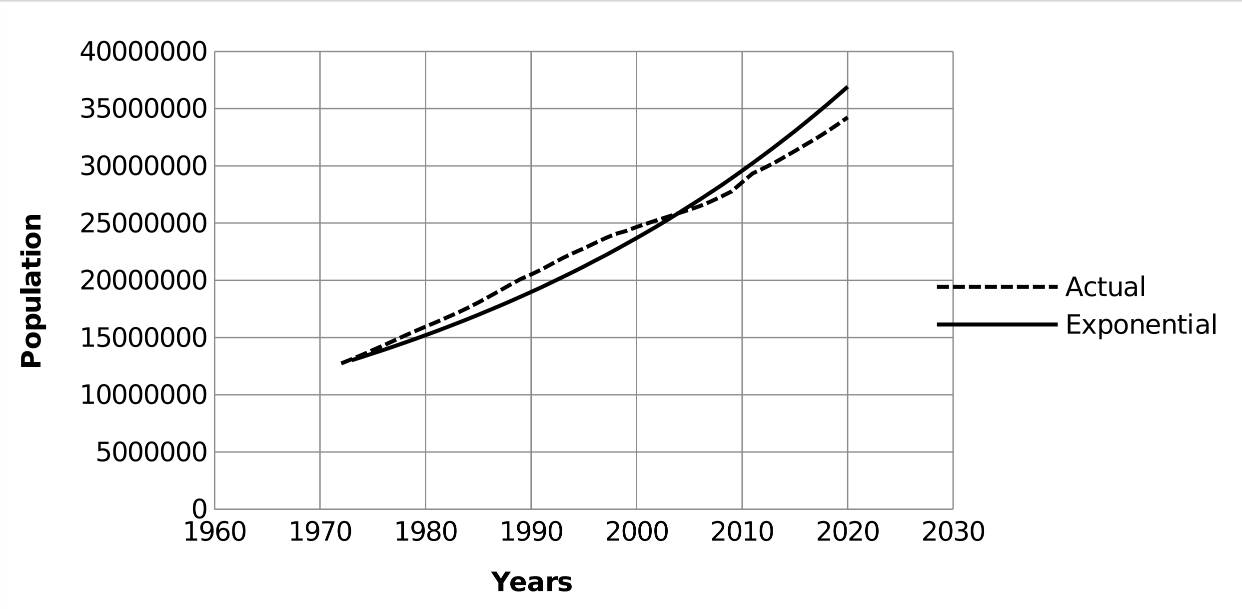
From equation (6) it'll be found C $N_0=Ce^{P\cdot 0}=Ce^0=C \rightarrow C=N_0$

After replacing C with N_0 in equation (5) we'll have final point of the model

(7) $$N=N_0e^{Pt}$$ **(Smith, 1977)**

The given information illustrates a comparison between the actual population of Uzbekistan in the years 1972 and 2020, as well as an exponential model for population growth, displayed in **Graph 1**. The exponential line, representing the projected population growth based on a mathematical model, was observed to be below the actual population from 1972 until 2004. However, in subsequent years, the model indicates a greater trend than the actual population numbers.

Furthermore, it is important to consider the potential implications of population growth for social, economic, and environmental sustainability. Rapid population growth can strain resources and infrastructure, while declining populations can lead to labor shortages and economic decline. Therefore, it is crucial for policymakers and individuals to carefully monitor population trends and plan for a sustainable future.



Graph 1:Actual Population and Exponential model

According to the exponential model, the population of Uzbekistan is projected to be around 46 million by 2030. However, this prediction may not align with reality and may deviate from the actual population size. It is important to note that there are various factors that may affect population growth, including social, economic, and environmental factors. Additionally, unforeseen events such as natural disasters, pandemics, and political instability can significantly impact population growth.

Therefore, while the exponential model may provide valuable insights into population growth trends, it is essential to use it with caution and not rely solely on its predictions. A combination of various models and expert knowledge can provide a more accurate picture of the population growth trajectory, enabling policymakers and individuals to plan for a sustainable future.

Logistic model of population growth, (Verhulst model) The logistic function was introduced in a series of three papers by Pierre François Verhulst between 1838 and 1847, who devised it as a model of population growth by adjusting the exponential growth model, under the guidance of Adolphe Quetelet. Verhulst first devised the function in the mid 1830 s, publishing a brief note in 1838, (Verhulst, 1838) then presented an expanded analysis and named the function in 1844 the third paper adjusted the correction term in his model of Belgian population growth.[2]

Assume the birth rate Bconstant, but the mortality rate M is proportional to the population, say $M=mN$ for some constant m. Then the growth model (1) becomes

$$\frac{dN}{dt}=BN-mN^2=(B-mN)N \tag{8}$$

Equation (8) raises a possibility that does not occur with Malthusian model, namely that there might be a non-zero equilibrium population, one for which $N'=0$. Specifically, this would be the case if $N=B/m$. The possibility of a non-zero equilibrium population is very important, as we shall see, so we introduce the abbreviation $\lambda=B/m$ for this special number. Observe from equation (8) that, if $0<N<\lambda$, than $N'>0$, so if the population is increasing, and if $N>\lambda$, then $N'<0$, so the population is decreasing. This is an example of a stable equilibrium, it is moving toward it.

By factoring out the birth rate B, the Verhulst may be written as

$$\frac{dN}{dt}=B\left(1-\frac{N}{\lambda}\right)N \tag{9}$$

[2] https://en.wikipedia.org/wiki/Logistic_function

Equation (9) is a differential equation and we'll solve it using by separating variables method.

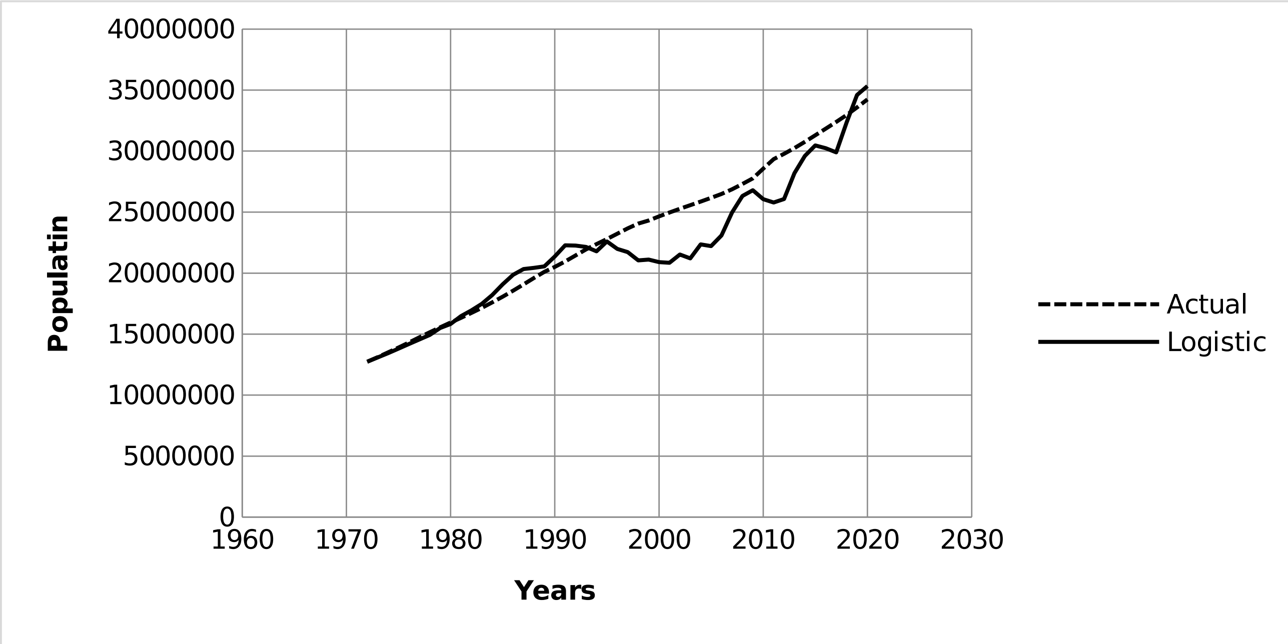
- $\frac{dN}{N\left(1-\frac{N}{\lambda}\right)}=Bdt$
- $\frac{\lambda\, dN}{N(\lambda-N)}=Bdt$
- $\left(\frac{1}{N}+\frac{1}{\lambda-N}\right)dN=Bdt$
- $lnN-\ln(\lambda-N)=Bt+C$
- $\ln\frac{N}{\lambda-N}=Bt+C$
- $\frac{N}{\lambda-N}=e^{\alpha t+C}$
- $\frac{N}{\lambda-N}=Ce^{\alpha t}$
- $N=\lambda Ce^{\alpha t}-NCe^{\alpha t}$
- $N+NCe^{\alpha t}=\lambda Ce^{\alpha t}$
- $N(1+Ce^{\alpha t})=\lambda Ce^{\alpha t}$ by finding N we'll reach general equation

(10) $$N=\frac{\lambda Ce^{\alpha t}}{1+Ce^{\alpha t}}$$

In equation (10) we have unknown parameter C. Using by initial condition $N(0)=N_0$ we'll find the value of C.

- $N_0=\frac{\lambda C}{1+C}$
- $N_0+N_0C=\lambda C$
- $\lambda C-N_0C=N_0$
- $C(\lambda-N_0)=N_0$
- $C=\frac{N_0}{\lambda-N_0}$
- $N=\frac{\frac{N_0}{\lambda-N_0}\lambda e^{\alpha t}}{1+\frac{N_0}{\lambda-N_0}e^{\alpha t}}$ by simplifying this expression we'll came to our final equation

(11) $$N = \frac{N_0 \lambda e^{\alpha t}}{\lambda - N_0 + N_0 e^{\alpha t}}$$ (Smith, 1977)



Graph 2: Actual population and Logistic model

The logistic model assumes that the population growth will slow down as the population approaches its carrying capacity, which the maximum number of individuals that the environment can support. The model is based on the idea that resources are limited and there is no a limit to how many individuals can be sustained in a given area. As the population approaches this limit, competition for resources increases and the growth rate slows down, eventually reaching zero.

One possible reason why the logistic model was not very suitable for predicting the growth of Uzbekistan's population is that the carrying capacity of the environment may not have been accurately estimated. The carrying capacity is affected by a variety of factors, including food availability, disease, natural disasters, and environmental degradation. In Uzbekistan, it is possible that the carrying capacity of the environment was overestimated, leading to an underestimation of the population growth rate.

In conclusion, while the logistic model is a useful tool for predicting population growth, its assumptions may not always be well-suited for particular situations. In the case of Uzbekistan’s population growth, it appears that alternative models may better capture the population dynamics and provide more accurate predictions.

RESULTS

The exponential growth model predicted the population growth trends in Uzbekistan but showed discrepancies when compared with the actual population data. From 1972 to 2004, the actual population was higher than the model's predictions, but in subsequent years, the model indicated a greater trend than the actual population. By 2030, the model projects the population to be around 46 million, though this prediction may not align with real-world complexities and uncertainties.

The logistic model, which accounts for environmental carrying capacity, demonstrated limitations in predicting Uzbekistan’s population growth. Factors such as overestimation of the carrying capacity and variable growth rates contributed to inaccuracies. Despite these issues, the logistic model's projection for 2030 (approximately 43.9 million) is closer to the expected real population, indicating some utility in longer-term predictions.

DISCUSSION

The research underscores the importance of using multiple models to understand population growth dynamics. The exponential growth model, while useful for identifying trends, may not fully capture the complexities of real-world population changes, especially in the face of unforeseen events such as natural disasters and policy shifts. On the other hand, the logistic model's consideration of environmental limits offers a more nuanced perspective but is also limited by assumptions about carrying capacity and constant growth rates.

For Uzbekistan, the demographic changes observed in recent decades highlight the need for adaptive and multifaceted approaches to population modeling. Accurate population predictions are essential for effective planning and resource management. Policymakers should combine insights from different models, adjusting for real-world factors, to develop sustainable strategies for healthcare, education, housing, and employment.

Future research should focus on refining these models by incorporating more dynamic variables and improving the estimation of carrying capacity. Additionally, integrating socio-economic and environmental data will enhance the accuracy and relevance of population growth predictions, ensuring that they more effectively inform policy and planning efforts in Uzbekistan and similar contexts.

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