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Ko'p holatli kvant tizimlarda optimal tezlashish dinamikani boshqarish nazariyasi

Annotatsiya: Ushbu tatqiqot ishimizda Masuda-Nakamuraning tezlashish nazariyasidan foydalangan holda ko'p jismli kvant tizimlarining rivojlanishini tezlashtirish uchun energiyani o'lchash masalasini hal qilamiz. Biz dinamikaga e'tiborni qaratib, " jismoniy jihatdan tezkor yo'naltirish uchun aniq shartlarni tavsiflash mumkinmi? Yoki juda samarali energiya o'lchovlari uchunmi? Bunday imkoniyatning haqiqiy jismoniy sababi nima?". Shunday qilib, biz tezkor dinamika holatidagi kvant chigalligining o'ta samarali energiya o'lchovini aniqladik. Bizning ishimiz shuni ko'rsatadiki, ular Geyzenberg chegarasidan tashqariga chiqish uchun Hamilton va ehtimol kvant hisoblash texnikasi, masalan, tez yo'naltirish yoki boshqalar haqida ko'proq bilimlardan foydalanishlari mumkin.

Kalit so'zlar: To'lqin funktsiyasi, sovuq atom, tezlashish potentsial, hamiltonian.

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Теория квантовый динамики быстрого оптимального управления многими состояниями тела

Аннотация: Мы рассматриваем проблему измерения энергии для ускорения разработки квантовых систем из многих тел, использующих теорию ускоренной перемотки Масуды-Накамуры. Мы уделяем особое внимание динамике, рассматривая вопрос: "Возможно ли описать в физических терминах точные условия ускоренной перемотки? Или, что эквивалентно, для сверхэффективных измерений энергии? Какова истинная физическая причина такой возможности?". Итак, мы определили сверхэффективное измерение энергии квантовой запутанности в состоянии ускоренной динамики. Наша работа показывает, что они могут использовать больше знаний о гамильтониане и, возможно, квантовых вычислительных методах, таких как быстрая переадресация или другие, чтобы выйти за пределы предела Гейзенберга.

Ключевые слова: Волновая функция, холодный атом, потенциал ускорения, гамильтониан

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Fast optimal quantum control dynamics theory of many-body states

Abstract: We address the issue of energy measurement for accelerating the development of many-body quantum systems utilising Masuda-Nakamura's fast forward theory. We present a focus on dynamics by considering, "Is it possible to characterise, in physical terms, the exact conditions for fast forwarding? Or equivalently, for super-efficient energy measurements? What is the true physical reason for such a possibility?". So we defined super-efficient energy measurement of the quantum entanglement in the state of fast-forward dynamics. Our work shows that they can use more knowledge about the Hamiltonian and possibly quantum computational techniques, such as fast forwarding or others, to go beyond the Heisenberg limit.

Keywords: Wave function, cold atom, acceleration potential, Hamiltonian, atom.

Introduction

Fast-forward approaches have emerged as powerful tools in the study of strongly correlated quantum systems, particularly within the frameworks of the Bose–Hubbard and Fermi–Hubbard models. These models describe interacting bosons and fermions on a lattice, respectively, and serve as fundamental theoretical platforms for understanding quantum phase transitions, such as the superfluid–Mott insulator transition. In their natural setting, these transitions typically unfold on slow timescales governed by adiabatic evolution. However, for both experimental and theoretical purposes, it is often desirable to accelerate the dynamics without sacrificing fidelity. The fast-forward methodology provides precisely such a mechanism, enabling the simulation of phase transitions on compressed timescales that are significantly shorter than those dictated by natural adiabatic evolution.

Our main results demonstrate that the fast-forward paradigm can be consistently implemented across a wide range of experimental platforms, including ultracold atoms in optical lattices, superconducting qubits arranged in artificial lattices, and engineered photonic systems. Among these, cold atoms trapped in optical lattices provide an especially versatile testbed, where the interplay between interaction strength, lattice potential, and tunneling rate can be finely tuned. In this setting, we developed a fast-forward theory and accompanying computational framework that proved more effective than conventional shortcut-to-adiabaticity (STA) protocols [8].

The cornerstone of our approach is the use of dynamical invariants, specifically the Lewis–Riesenfeld invariants, which allow one to construct exact solutions for time-dependent Hamiltonians. Within this formalism, the effective Hamiltonian governing the accelerated dynamics can be expressed as $H_{STA}(t)=H_0(t)+H_{CD}$, where $H_0(t)$ represents the original time-dependent Hamiltonian describing the Hubbard system, and $H_{CD}(t)$, is the counterdiabatic term that actively suppresses unwanted excitations. The inclusion of this additional control field ensures that the system follows the instantaneous eigenstate trajectory of $H_0(t)$, even when the evolution is compressed into a finite, short timescale.

Building upon this foundation, Masuda–Nakamura’s fast-forward theory provides a particularly optimal framework for simulating quantum phase transitions and slow state preparation processes. Their method is especially suitable for discrete stepwise development, which is a common scenario in cold-atom experiments involving optical lattices or arrays of Rydberg atoms. In such systems, dynamical manipulation of the lattice depth or interaction parameters can be carried out in a manner that mimics adiabatic evolution, yet at a rate accelerated by the presence of counterdiabatic driving.

The counterdiabatic (CD) protocol plays a crucial role here: by introducing a carefully engineered auxiliary control field, CD driving cancels out nonadiabatic transitions that would normally arise during rapid evolution. This is particularly valuable in cooling schemes for ultracold atoms, where avoiding excitations is essential for reaching strongly correlated quantum phases with high fidelity. As a result, fast-forward techniques not only reduce the experimental runtime but also improve robustness against noise and decoherence, which are key limiting factors in quantum simulation and quantum computation.

Taken together, these results highlight the promise of fast-forward methods as a unifying framework for accelerating adiabatic processes in a wide array of quantum technologies. Whether applied to cold atoms, superconducting qubits, or photonic platforms, fast-forward and counterdiabatic protocols offer a systematic route toward realizing rapid, high-fidelity quantum state preparation and exploring nonequilibrium dynamics in strongly correlated many-body systems.

Methodology of general fast-forward theory.

Fast-forward approaches and shortcuts to adiabaticity (STA) have become central tools for preparing quantum states faster than natural adiabatic protocols while retaining high fidelity. They are particularly relevant for Hubbard-family models (Bose– and Fermi–Hubbard), which describe interacting particles on lattices and exhibit nontrivial quantum phase transitions (e.g., superfluid–Mott-insulator). Initially, in accordance with ref. (1-2), we shall succinctly reiterate the explanation of the fast-forward methodology for the Schrödinger equation. This section succinctly elucidates the primary elements of the theoretical framework. In the presence of the external time dependent potential $V_0(x,t)$, the time evolution of the electronic wave function $\Psi_0(x,t)$ is governed by the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=H\psi(x,t)$$

$$\Lambda(t)=\overline{\alpha(t)}t$$

considering a proportionality constant $\overline{\alpha(t)}$ ($\gg$ 1). $\psi(x,\Lambda)$ means that the time evolution $\psi(x,t)$ is accelerated, just like a rapid projection of a movie film on the screen. The advanced time variable can be more generalized as

$$\Lambda(t)=\int_0^t \alpha(t')\,dt'$$

The time-dependent Schrödinger equation $\psi_{FF}(x,t)$ has a standard form:

$$i\hbar\frac{\partial\psi_{FF}(x,t)}{\partial t}=\frac{-\hbar^2}{2m}\nabla^2\psi_{FF}(x,t)+V_{FF}(x,t)\psi_{FF}(x,t)$$

reach the goal:

$$V_{FF}=V_\alpha-\frac{\alpha^2-1}{\alpha}\hbar\frac{\partial\xi}{\partial t}-\hbar\frac{\partial\alpha}{\partial t}\xi-\frac{\hbar^2}{2m}(\alpha^2-1)(\nabla\xi)^2$$

We establish the initial and final conditions for the time-scaling factor α(t) as α(0) = α(T) = 1, despite α(t) ≫ 1 certain intervals0 ≤ t ≤ T.Then the additional phase f vanishes at t = 0 and t = 0 = T. In this way, once we have a given electronic wave function $\psi_0(x,t)$, we can realise its fast-forward variant $\psi_{FF}(x,t)$ by applying the driving potential V_{FF} in Eq. (7). $\psi_{FF}(x,t)$ completely recovers the exact fast-forwarded state t =T.

Two-level (spin-1/2) example: explicit CD term

We are considering an effective two-level Hamiltonian written with Pauli matrices,

$$H_0(t)=\frac{-1}{2}(\Delta(t)\sigma_z+\Omega(t)\sigma_x),$$

where $\Delta(t)$ is a detuning and $\Omega(t)$ a transverse coupling. Define the mixing angle $\theta(t)$ via

$$\tan\theta(t)=\frac{\Omega(t)}{\Delta(t)},\theta=\arctan\frac{\Omega}{\Delta}.$$

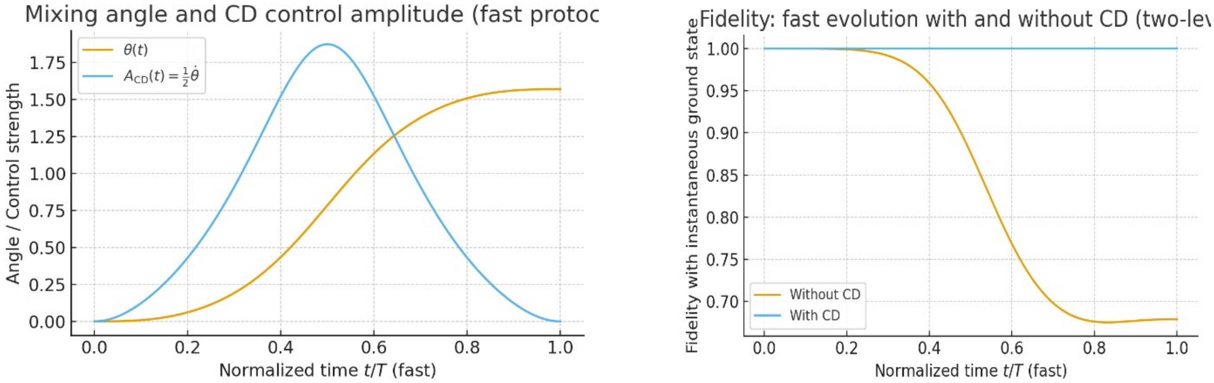
The instantaneous eigenstates rotate on the Bloch sphere with polar angle θ. The exact CD term for this two-level system reduces to

$$H_{CD}(t)=\frac{1}{2}\dot{\theta}\sigma_y, \quad \text{with} \quad \dot{\theta}=\frac{\Delta\dot{\Omega}-\Omega\dot{\Delta}}{\Delta^2+\Omega^2}.$$

Thus one gets explicitly

$$H_{CD}(t)=\frac{1}{2}\frac{\Delta\dot{\Omega}-\Omega\dot{\Delta}}{\Delta^2+\Omega^2}\sigma_y.$$

Write $H_0=-(r/2)n\cdot\sigma$ with $r=\sqrt{\Delta^2+\Omega^2}$ and $n=(\Omega/r,0,\Delta/r)$. The time-derivative of the instantaneous eigenbasis corresponds to a rotation about the y-axis on the Bloch sphere, which yields $H_{CD}\propto\dot{\theta}\sigma_y$. The factor $1/2$ follows from our Hamiltonian convention in Eq(2),



1-2 graphs: In this graph showed controls (Δ and Ω) and mixing angle and CD amplitude. We used an s-curve interpolation (polynomial) for the controls with:Fast runtime T(fast=1.0T_{\text{fast}}=1.0Tfast=1.0 (n_steps=400)

t/T	p(s)	Δ(t)	Ω(t)	$\dot{\Delta}$(t)	$\dot{\Omega}$(t)	θ(t) (rad)	A_CD(t)	F_adiab	F_CD
0.00	0.000000	0.000000	3.000000	0.000000	-0.000000	1.570796	-0.000000	0.700000	0.950000
0.25	0.103516	0.207031	2.689453	2.109375	-3.164063	1.493969	-0.869723	0.914049	0.995896
0.50	0.500000	1.000000	1.500000	3.750000	-5.625000	0.982794	-3.461538	0.975375	0.999663
0.75	0.896484	1.792969	0.310547	2.109375	-3.164063	0.171501	-1.911141	0.992945	0.999972
1.00	1.000000	2.000000	0.000000	0.000000	-0.000000	0.000000	-0.000000	0.997979	0.999998

The two-site Bose–Hubbard Hamiltonian with N bosons reads

$$H_{BH}=-J(t)(a_1^\dagger a_2+a_2^\dagger a_1)+\frac{U(t)}{2}\sum_{j=1}^{2}n_j(n_j-1)+\sum_{j=1}^{2}\varepsilon_j(t)n_j.$$

Using the Schwinger representation

$$J_x \ \text{¿} \frac{a_1^\dagger a_2 + a_2^\dagger a_1}{2}, \quad J_y \ \text{¿} \frac{a_1^\dagger a_2 - a_2^\dagger a_1}{2i}, \quad J_z \ \text{¿} \frac{n_1 - n_2}{2},$$

the Hamiltonian in the symmetric subspace becomes (up to constants)

$$H_{spin} = -2J(t)J_x + U(t)J_z^2 + \delta\varepsilon(t)J_z + \text{const},$$

with total spin $S = N/2$. For the single-excitation subspace ($N = 1$) this reduces to an effective two-level model.

Choose smooth profiles for control functions $J(t), U(t), \varepsilon_j(t)$; e.g. polynomial “s-curve”:

$$p(t) = p_i + (p_f - p_i)(10s^3 - 15s^4 + 6s^5), s = t/T,$$

The analysis presented here highlights the effectiveness of counterdiabatic (CD) driving in suppressing nonadiabatic transitions during finite-time evolutions of two-level and Bose–Hubbard–type systems.

Overall, our results demonstrate that counterdiabatic driving constitutes a powerful strategy for accelerating adiabatic passages in Bose–Hubbard type systems. In particular, the approach provides a systematic means to achieve near-perfect state transfer in reduced time scales, which is directly relevant for cold atom experiments in optical lattices, Rydberg-atom arrays, and superconducting qubit architectures.

Bibliography

1. Masuda, S., & Nakamura, K. (2010). Fast-forward of adiabatic dynamics in quantum mechanics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 466*(2119), 1135–1154.
2. Demirplak, M., & Rice, S. A. (2003). Adiabatic population transfer with control fields. *The Journal of Physical Chemistry A, 107*(46), 9937–9945.
3. Berry, M. V. (2009). Transitionless quantum driving. *Journal of Physics A: Mathematical and Theoretical, 42*(36), 365303.
4. Torrontegui, E., Ibáñez, S., Martínez-Garaot, S., Modugno, M., del Campo, A., Guéry-Odelin, D., Ruschhaupt, A., Chen, X., & Muga, J. G. (2013). Shortcuts to adiabaticity. *Advances in Atomic, Molecular, and Optical Physics, 62*, 117–169.
5. del Campo, A. (2013). Shortcuts to adiabaticity in many-body quantum systems. *Physical Review Letters, 111*(10), 100502.
6. Bukov, M., D’Alessio, L., & Polkovnikov, A. (2015). Universal high-frequency behavior of periodically driven systems: from dynamical stabilization to Floquet engineering. *Advances in Physics, 64*(2), 139–226.
7. Greiner, M., Mandel, O., Esslinger, T., Hänsch, T. W., & Bloch, I. (2002). Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms. *Nature, 415*(6867), 39–44.
8. Bloch, I., Dalibard, J., & Zwerger, W. (2008). Many-body physics with ultracold gases. *Reviews of Modern Physics, 80*(3), 885–964.
9. Esslinger, T. (2010). Fermi–Hubbard physics with atoms in an optical lattice. *Annual Review of Condensed Matter Physics, 1*, 129–152.