HAR BIR O‘YINCHINING TEZLIGI SO‘NUVCHI BO‘LGAN HOLDA QOCHISH DIFFERENSIAL O‘YINI

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Annotatsiya: Maqolada obyektlarning dinamikasi birinchi tartibli chiziqli differensial tenglama bo‘yicha harakatlanganda va tezliklari vaqt o‘tishi bilan sekinlanuvchi bo‘lgan hol uchun qochish masalasi o‘rganiladi. Bunda qochuvchi uchun strategiya taklif qilinadi va qochishning yangi yetarli shartlari topiladi.

Kalit so‘zlar: Differensial o'yin, geometrik chegaralanish, qochuvchi, quvlovchi, strategiya.

THE EVASION DIFFERENTIAL GAME FOR THE CASE WHERE THE SPEED OF EACH PLAYER IS DAMPING

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Annotation: In this paper, it is considered the evasion problem for the case when the dynamics of objects is given according to the first-order linear differential equations and their velocities are decelerating over time. Here, a evasion strategy for the evader is proposed and new sufficient conditions for escape are found.

Keywords: Differential game, geometrical constraint, evader, pursuer, strategy.

ДИФФЕРЕНЦИАЛЬНАЯ ИГРА С УБЕГАНИЕМ ДЛЯ СЛУЧАЯ, КОГДА СКОРОСТЬ КАЖДОГО ИГРОКА ЗАТУХАЕТ

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Аннотация: В статье исследуется задача убегания для случая, когда динамика обьектов данное по линейному дифференциальному уравнению первого порядка и их скорости замедляются с течением времени. При этом предлагается стратегия убегания и находятся новые достаточные условия для побега.

Ключевые слова: Дифференциальная игра, геометрическое ограничение, убегающий, преследователь, стратегия.

Let a controlled object P called the Pursuer, chase another object E called the Evader in the space $\mathbb{R}^n$. Denote by x a state of the Pursuer and denote by y a state of the Evader in $\mathbb{R}^n$. In the present we study the Pursuit problem when the objects move in accordance with equations

$$P: \ \dot{x} + \lambda x = u, \quad x(0) = x_0, \tag{1}$$

$$E: \ \dot{y} + \lambda y = v, \quad y(0) = y_0, \tag{2}$$

where $\lambda \neq 0$, $x, y, u, v \in \mathbb{R}^n, n \geq 2;$ x_0, y_0 are the initial positions of the objects P and E, respectively; it is assumed that $x_0 \neq y_0;$ the parameter u is the velocity vector of the Pursuer and here the temporal variation of u is considered to be a Lebesgue measurable function $u(\cdot):[0, +\infty) \to \mathbb{R}^n$. We denote by U^{-}_{exp} the set of all measurable functions $u(\cdot)$ satisfying the geometric constraint (briefly, the G^{-}_{exp} - constraint)

$$|u(t)| \leq \rho e^{-kt} \ \text{ for almost every } t \geq 0, \tag{3}$$

where ρ, k are positive numbers.

Similarly, the parameter v is the velocity vector of the Evader and here the temporal variation of v is considered to be a Lebesgue measurable function $v(\cdot):[0, +\infty) \to \mathbb{R}^n$. We denote by V^{-}_{exp} the set of all measurable functions $v(\cdot)$ such that

$$|v(t)| \leq \sigma e^{-kt} \ \text{for almost every } t \geq 0, \tag{4}$$

where σ is a positive number.

Since the problem (1)−(4) for the case $k = 0$ was studied, here we will study the case where k is the positive number.

From the physical viewpoint, the constraints (3) and (4) indicate that the maximal values of velocities of both objects will damp in duration time of the considered game.

Definition 1. The measurable function $u(\cdot)\in \mathrm{U}^{-}_{exp}$ is called *the admissible control function* of the Pursuer, respectively in the game (1)−(4), or in brief, the G^{-}_{exp} - Game.

Definition 2. For the pair of $(x_0, u(\cdot))$, $u(\cdot)\in \mathrm{U}^{-}_{exp}$ the solution of equation (1), that is

$$x(t) = x_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} u(s)ds \tag{5}$$

is called *a trajectory of the Pursuer* in the interval $[0, +\infty)$.

Definition 3. For the pair of $(y_0, v(\cdot))$, $v(\cdot)\in \mathrm{V}^{-}_{exp}$ the solution of equation (2), that is

$$y(t) = y_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} v(s)ds \tag{6}$$

is called *a trajectory of the Evader* in the interval $[0, +\infty)$.

In the G^{-}_{exp} - Game, Pursuer aims to catch Evader, i.e. to realize equality $x(t) = y(t)$ for some $t > 0$, and the aim of Evader is to keep relation $x(t) \neq y(t)$ for all t, $t \geq 0$.

Let $z = x - y$, $z_0 = x_0 - y_0$. Then from (1) and (2) we have the unique equation

$$\dot z = u - v, \quad z(0) = z_0.$$

To solve the Problem of Evasion, assume that at each current time t, the Pursuer is allowed to know the initial states x_0, y_0, the constants λ, ρ, σ, k, the current time t.

Definition 4. We call the control function

$$v^*(t) = -\sigma e^{-kt}\xi_0, \quad t \geq 0, \text{ where } \xi_0 = \frac{z_0}{|z_0|} \tag{7}$$

the strategy of the Evader in the (1)-(4) Game.

Definition 5. We say that the control (7) guarantees winning for E if, for any $u(\cdot)\in \mathrm{U}^{-}_{exp}$, the relation $x(t) \neq y(t)$ holds for every $t \geq 0$.

Theorem. If $\sigma \geq \rho$, then $v^*(t)$ strategy for Evader is winning in the interval $[0,+\infty)$.

Proof. Let $\sigma \geq \rho$. Then assume that P uses any control $u(\cdot) \in U^{-}_{exp}$ and E employs the control (7). As a consequence of (5), (6) and $z(t) = x(t) - y(t)$, it proceeds

$$z(t) = z_0 e^{-\lambda t} + \int_0^t e^{-ks} \sigma e^{-\lambda(t-s)} \xi_0 ds + \int_0^t e^{-\lambda(t-s)} u(s) ds \tag{8}$$

Using the properties of module and considering (3) we evaluate the absolute value of (8) from below, i.e.

$$|z(t)| \geq e^{-\lambda t} \left| z_0 + \sigma \xi_0 \int_0^t e^{(\lambda-k)s} ds \right| - \left| \rho \int_0^t e^{(\lambda-k)s} ds \right| \tag{9}$$

I. Let $\lambda = k$, then from (9) we have

$$|z(t)| \geq h(t),$$

where $h(t) = e^{-\lambda t}(|z_0| + (\sigma - \rho)t)$.

From $\sigma \geq \rho$ and the last estimations we get $|z(t)| > 0$.

II. Let $\lambda \neq k$, then from (9) we have

$$|z(t)| \geq e^{-\lambda t}(|z_0| + \frac{\sigma}{\lambda - k}(e^{(\lambda-k)t} - 1) - \frac{\rho}{\lambda - k}(e^{(\lambda-k)t} - 1)) \Rightarrow$$

$$|z(t)| \geq e^{-\lambda t} g(t), \text{ where}$$

$g(t) = |z_0| + \frac{\sigma - \rho}{\lambda - k}(e^{(\lambda-k)t} - 1)$. From $\sigma \geq \rho$ and $\lambda \neq k$ we check the function g(t) and have $g(t) \geq |z_0|$ and this we get

$$|z(t)| > 0$$

The proof is complete.

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