BIR JINSLI BO‘LMAGAN TOQ TARTIBLI TENGLAMA UCHUN CHEGARAVIY MASALA

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Annotatsiya: *Ushbu maqolada yuqori toq tartibli differensial tenglamalar uchun chegaraviy masala o‘rganilgan. Masala uchun yechimning mavjudligi va yagonaligi teoremalari isbotlangan. Yechimni qurishda Grin funksiyasi va Fur’e qatori usullari qo‘llanilgan. Shuningdek, fundamental yechimlarning asimptotik xossalari hamda Grin funksiyasining baholari keltirilgan. Natijalar yuqori tartibli tenglamalar uchun chegaraviy masalalarni tahlil qilishda muhim ahamiyat kasb etadi.*

Kalit so‘zlar: *yuqori tartibli tenglama, chegaraviy masala, Grin funksiyasi, asimptotik baho, determinant, yaqinlashuv.*

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КРАЕВАЯ ЗАДАЧА ДЛЯ НЕОДНОРОДНОГО УРАВНЕНИЯ НЕЧЁТНОГО ПОРЯДКА

Аннотация: *В данной статье исследуется краевая задача для дифференциальных уравнений высокого нечётного порядка. Доказаны* *теоремы существования и единственности решения. При построении решения использованы методы функции Грина и ряда Фурье. Приведены асимптотические свойства фундаментальных решений и оценки функции Грина. Полученные результаты имеют важное значение для анализа краевых задач для уравнений высокого порядка.*

Ключевые слова: *дифференциальное уравнение высокого порядка, краевая задача, функция Грина, асимптотическая оценка, определитель, сходимость.*

BOUNDARY VALUE PROBLEM FOR A NONHOMOGENEOUS EQUATION OF ODD ORDER

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Abstract: *This article studies a boundary value problem for higher odd-order differential equations. The existence and uniqueness theorems for the solution are proved. The Green's function and Fourier series methods are applied to construct the solution. Asymptotic properties of the fundamental solutions and estimates for the Green's function are presented. The obtained results are of significant importance for the analysis of boundary value problems for higher-order equations.*

Keywords: *higher-order differential equation, boundary value problem, Green's function, asymptotic estimate, determinant, convergence.*

1.Kirish

Hozirgi kunda yuqori tartibli differensial tenglamalar uchun chegaraviy masalalarni o‘rganishga katta e’tibor qaratilmoqda. Bunday tenglamalar ko‘plab fizik jarayonlarning, jumladan, elastik tizimlar dinamikasi, plastinkalar va sterjenlarning tebranishi, shuningdek, bir xil bo‘lmagan muhitlardagi issiqlik o‘tkazuvchanlik jarayonlarining matematik modeli sifatida namoyon bo‘ladi [1], [2].

Ayniqsa, singulyar va o‘zgaruvchan koeffitsiyentli yuqori tartibli tenglamalarni o‘rganish dolzarb masala hisoblanadi. Chunki ular yordamida chegaraviy sharoitlar ta’sirida yechimlarning xatti-harakatini yanada aniqroq tavsiflash mumkin [3], [4].

So‘nggi yillarda ko‘plab olimlar tomonidan yuqori tartibli aralash tipdagi hamda kasr tartibli hosilali operatorli tenglamalar uchun boshlang‘ich-chegaraviy masalalar keng o‘rganilmoqda [5], [6]. Bunday masalalarda Grin funksiyasini qurish, yechimlarning mavjudligi va ularning yagona yechimga ega bo‘lishi kabi nazariy muammolar alohida ahamiyat kasb etadi [7], [8].

Ba’zi tadqiqotlarda [9], [10] toq tartibli chiziqli differensial tenglamalar uchun yechimlarning asimptotik xossalari tahlil qilingan bo‘lib, bunday natijalar yuqori tartibli tenglamalar uchun umumlashtirilgan yechimlarni qurishda muhim nazariy asos vazifasini bajaradi.

Shunday qilib, yuqori toq tartibli, ayniqsa, singulyar va buzilishga ega koeffitsiyentli tenglamalar uchun chegaraviy masalalarni o‘rganish matematika va amaliy fizika sohasida dolzarb ilmiy yo‘nalishlardan biri hisoblanadi. Ushbu ishda aynan shunday turdagi tenglamalar uchun yechimlarning mavjudligi, ularning asimptotik xossalari hamda Grin funksiyasi yordamida yechim qurish usullari ko‘rib chiqiladi.

2.Masala qo‘yilishi

$\Omega = \{ (x, y): 0 < x < 1,\ 0 < y < 1 \}$ sohada quyidagi chegaraviy masalani o‘rganamiz

$$L[u] = (-1)^n \frac{\partial^{2n+1} u}{\partial x^{2n+1}} + (-1)^m \frac{\partial^{2m} u}{\partial y^{2m}} = f(x, y),\ m, n \in N, \tag{1}$$

A masala. (1) tenglamaning $u(x, y) \in C_{x,y}^{2n+1,2m}(\Omega) \cap C_{x,y}^{2n,2m-1}(\overline{\Omega})$, sinfga tegishli, shunday yechimi topilsinki, u quyidagi shartlarni qanoatlantirsin:

$$\frac{\partial^{2j} u}{\partial y^{2j}}(x, 0) = \frac{\partial^{2j} u}{\partial y^{2j}}(x, 1) = 0,\ j = 0, 1, ..., m-1, \tag{2}$$

$$\frac{\partial^{i} u}{\partial x^{i}}(0, y) = 0, i = 0, 1, 2, ..., n,$$
$$\frac{\partial^{j} u}{\partial x^{j}}(1, y) = 0,\ j = 0, 1, 2, ..., n-1. \tag{3}$$

bu yerda $f(x, y)$ - yetarlicha silliq funksiya.

3.Yechimning yagonaligi

Teorema 1. Agar A masala yechimga ega bo‘lsa, u holda u yechim yagona bo‘ladi.

Isbot. Teskaridan faraz qilaylik. A masalaning yechimi ikkita $u_1(x, y)$ va $u_2(x, y)$ bo‘lsin. U holda $u(x, y) = u_1(x, y) - u_2(x, y)$ ayirma ham (1) tenglamaning bir jinsli chegaraviy shartlarini qanoatlantiradi. $\overline{\Omega}$ da $u(x, y) \equiv 0$ ekanligini isbotlaymiz.

Ω sohada quyidagi ayniyat o‘rinli:

$$uL[u] = (-1)^n \frac{\partial}{\partial x}\left(u \frac{\partial^{2n} u}{\partial x^{2n}} - u_x \frac{\partial^{2n-1} u}{\partial x^{2n-1}} + ... + \frac{(-1)^n}{2}\left(\frac{\partial^n u}{\partial x^n} \right)^2 \right) +$$
$$+ (-1)^m \frac{\partial}{\partial y}\left(u \frac{\partial^{2m-1} u}{\partial y^{2m-1}} - u_y \frac{\partial^{2m-2} u}{\partial y^{2m-2}} + ... + (-1)^{m-1}\left(\frac{\partial^{m-1} u}{\partial y^{m-1}} \frac{\partial^m u}{\partial y^m} \right) \right) + \left(\frac{\partial^m u}{\partial y^m} \right)^2$$

(4)

(4) ayniyatni Ω sohada integrallab (2) va (3) shartlardan quyidagiga ega bo‘lamiz

$$\int_0^1\int_0^1 uL[u]dxdy = \int_0^1\int_0^1(-1)^n\left(\frac{\partial}{\partial x}\left(u\frac{\partial^{2n}u}{\partial x^{2n}}\right)-\frac{\partial}{\partial x}\left(u_x\frac{\partial^{2n-1}u}{\partial x^{2n-1}}\right)+\ldots+\frac{(-1)^n}{2}\frac{\partial}{\partial x}\left(\frac{\partial^n u}{\partial x^n}\right)^2\right)dxdy+$$

$$+\int_0^1\int_0^1\left((-1)^m\frac{\partial}{\partial y}\left(u\frac{\partial^{2m-1}u}{\partial y^{2m-1}}-u_y\frac{\partial^{2m-2}u}{\partial y^{2m-2}}+\ldots+(-1)^{m-1}\left(\frac{\partial^{m-1}u}{\partial y^{m-1}}\frac{\partial^m u}{\partial y^m}\right)\right)+\left(\frac{\partial^m u}{\partial y^m}\right)^2\right)dxdy,$$

$$\int_0^1\int_0^1 uL[u]\,dxdy = \frac{1}{2}\int_0^1\left(\frac{\partial^n u(1,y)}{\partial x^n}\right)^2 dy + \int_0^1\int_0^1\left(\frac{\partial^m u(x,y)}{\partial y^m}\right)^2 dxdy = 0,$$

bundan

$\frac{\partial^m u(x,y)}{\partial y^m}=0$ yoki $u(x,y)=y^{m-1}c_1(x)+y^{m-2}c_2(x)+\ldots+yc_{m-1}(x)+c_m(x),$

(2) shartdan $u(x,y)\equiv 0,$ kelib chiqadi.

Teorema 1 isbotlandi.

4.Yechimning mavjudligi

A masala yechimini quyidagi ko‘rinishda izlaymiz:

$$u(x,y)=\sum_{k=1}^{\infty}X_k(x)\sin\pi ky. \tag{5}$$

(5) ni (1) tenglamaga qo‘yib quyidagiga ega bo‘lamiz

$$\begin{cases}(-1)^n X_k^{(2n+1)}(x)+(\pi k)^{2m}X_k(x)=f_k(x),\\ X_k^{(i)}(0)=0, i=1,2,\ldots,n,\\ X_k^{(j)}(1)=0,\ j=1,2,\ldots,n-1,\end{cases} \tag{6}$$

bu yerda $f_k(x)$ Fur’e koeffisienti bo‘lib, u quyidagicha aniqlanadi

$$f_k(x) = 2\int_0^1 f(x,y)\sin \pi k y dy,$$

bunda quyidagi baho o‘rinli

$$\left| f_k(x) \right| \le M, 0 < M = const.$$

Xarakteristik tenglamani tuzamiz

$$(-1)^n r^{2n+1} + (\pi k)^{2m} = 0,$$
$$r^{2n+1} = (-1)^{n+1} (\pi k)^{2m},$$
$$(-1)^{n+1} = \cos \pi(n+1) + i \sin \pi(n+1) = e^{i\pi(n+1)+2\pi i s},$$

$$r_s = (\pi k)^{\frac{2m}{2n+1}} e^{\frac{\pi i(n+1+2s)}{2n+1}}, s = 0,1,...,2n-1,$$

r_s ildizlar tartib raqamlarini $0 < \operatorname{Re} r_1 < \operatorname{Re} r_2 < ... < \operatorname{Re} r_n$ belgilab olamiz.

$G(x,\xi)$ Grin funksiyasini quyidagi ko‘rinishda quramiz

$$G(x,\xi) = \begin{cases} G_1(x,\xi) = a_1(\xi)e^{r_1 x} + a_2(\xi)e^{r_2 x} + ... + a_{2n+1}(\xi)e^{r_{2n+1} x}; 0 \le \xi \le x \le 1, \\ G_2(x,\xi) = b_1(\xi)e^{r_1 x} + b_2(\xi)e^{r_2 x} + ... + b_{2n+1}(\xi)e^{r_{2n+1} x}; 0 \le x \le \xi \le 1. \end{cases}$$

Grin funksiyasi quyidagi shartlarni qanoatlantiradi

$$\begin{cases} G^{(i)}(0,\xi) = 0, i = 0,1,2,...,n, \\ G^{(i)}(1,\xi) = 0, i = 1,2,...,n, \\ \frac{\partial^j G_1(\xi,\xi)}{\partial x^j} - \frac{\partial^j G_2(\xi,\xi)}{\partial x^j} = 0, j = 0,1,2,...,2n-1, \\ \frac{\partial^{2n} G_1(\xi,\xi)}{\partial x^{2n}} - \frac{\partial^{2n} G_2(\xi,\xi)}{\partial x^{2n}} = 1. \end{cases}$$

(7)

$a_j(\xi), b_j(\xi)$ noma’lum funksiyalarni topib olamiz. Faraz qilaylik $d_i(\xi) = a_i(\xi) - b_i(\xi)$ bo‘lsin, u holda Grin funksiyasining xossasiga ko‘ra, $x = \xi,$ bo‘lganda quyidagi sistemaga ega bo‘lamiz

$$\begin{cases} d_1(\xi)e^{r_1\xi} + d_2(\xi)e^{r_2\xi} + \ldots + d_{2n+1}(\xi)e^{r_{2n+1}\xi} = 0, \\ r_1 d_1(\xi)e^{r_1\xi} + r_2 d_2(\xi)e^{r_2\xi} + \ldots + r_{2n+1} d_{2n+1}(\xi)e^{r_{2n+1}\xi} = 0, \\ \ldots \\ r_1^{2n} d_1(\xi)e^{r_1\xi} + r_2^{2n} d_2(\xi)e^{r_2\xi} + \ldots + r_k^{2k} d_{2n+1}(\xi)e^{r_{2n+1}\xi} = 1. \end{cases} \qquad (8)$$

(8) sistemaning asosiy determinanti Δ' quyidagi ko‘rinishga ega

$$\Delta' = \begin{vmatrix} e^{r_1 x} & e^{r_2 x} & \ldots & e^{r_{2n+1} x} \\ r_1 e^{r_1 x} & r_2 e^{r_2 x} & \ldots & r_{2n+1} e^{r_{2n+1} x} \\ . & . & . & . \\ r_1^{2n} e^{r_1 x} & . & . & r_{2n+1}^{2n} e^{(2n+1)x} \end{vmatrix} = \prod_{i<j} (r_i - r_j).$$

Kramer usuli yordamida yechimni quyidagi ko‘rinishda topamiz

$$d_s(\xi) = \frac{1}{e^{sx} \prod\limits_{s>j} (r_s - r_j) \prod\limits_{j>s} (r_j - r_s)} = \frac{M_1}{k^{\frac{2n}{2n+1}} e^{r_s x}}, 0 \neq M_1 = const, s = 1, 2, \ldots, 2n+1.$$

Endi $a_j(\xi)$ topish uchun quyidagi sistemaga ega bo‘lamiz (x=0 va x=1da):

$$\begin{cases} a_1 + a_2 + \ldots + a_{2n+1} = d_1 + d_2 + \ldots + d_{2n+1}, \\ r_1 a_1 + r_2 a_2 + \ldots + r_{2n+1} a_{2n+1} = r_1 d_1 + r_2 d_2 + \ldots + r_{2n+1} d_{2n+1}, \\ \ldots \\ r_1^n a_1 + r_2^n a_2 + \ldots + r_{2n+1}^n a_{2n+1} = r_1^n d_1 + r_2^n d_2 + \ldots + r_{2n+1}^n d_{2n+1}, \\ a_1 e^{r_1} + a_2 e^{r_2} + \ldots + a_{2n+1} e^{2n+1} = 0, \\ r_1 a_1 e^{r_1} + r_2 a_2 e^{r_2} + \ldots + r_{2n+1} a_{2n+1} e^{2n+1} = 0, \\ \ldots \\ r_1^{n-1} a_1 e^{r_1} + r_2^{n-1} a_2 e^{r_2} + \ldots + r_{2n+1}^{n-1} a_{2n+1} e^{r_{2n+1}} = 0, \end{cases} \qquad (9)$$

(9) sistemaning asosiy determinanti Δ', quyidagi ko‘rinishga ega

$$\begin{vmatrix} 1 & 1 & \dots & 1 \\ r_1 & r_2 & \dots & r_{2n+1} \\ . & . & . & . \\ r_1^n & r_2^n & \dots & r_{2n+1}^n \end{vmatrix} \times \begin{vmatrix} e^{r_1} & e^{r_2} & \dots & e^{r_{2n+1}} \\ r_1 e^{r_1} & r_2 e^{r_2} & \dots & r_{n+1} e^{r_{2n+1}} \\ . & . & . & . \\ r_1^{n-1} e^{r_1} & r_2^{n-1} e^{r_2} & \dots & r_{2n+1}^{n-1} e^{r_{2n+1}} \end{vmatrix} + O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right) =$$

$$= M_2 k^{\frac{n-1}{2}} e^{\sum_{j=1}^{n} \operatorname{Re} r_j} + O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right), 0 \neq M_2 = const, k \to \infty.$$

Shundan song quyidagini hosil qilamiz

$$a_j = \frac{\Delta_j}{\Delta} = \frac{d_j M_2 k^{\frac{n-1}{2}} e^{\sum_{j=1}^{n} \operatorname{Re} r_j} + O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right)}{M_2 k^{\frac{n-1}{2}} e^{\sum_{j=1}^{n} \operatorname{Re} r_j} + O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right)} =$$

$$= d_j + o(1) = \frac{1}{k^{\frac{2n}{2n+1}} e^{r_j x}} + o(1), \ j = 1, 2, \dots, n; k \to \infty,$$

$$b_j = a_j - d_j = \frac{O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right)}{M_2 k^{\frac{n-1}{2}} e^{\sum_{j=1}^{n} \operatorname{Re} r_j} + O\left(e^{\sum_{j=1}^{n} \operatorname{Re} r_j - \operatorname{Re} r_1} \right)},$$

Grin funksiyasi uchun keltirilgan baholar o‘rinli

$$\left| \frac{\partial^j G(x,\xi)}{\partial x^j} \right| \leq M k^{\frac{j-2n}{2n+1}} x^{n-j} \xi^n, \ j = 0, 1, 2, \dots, n-1,$$

$$\left| \frac{\partial^j G(x,\xi)}{\partial x^j} \right| \leq M k^{\frac{j-2n}{2n+1}} \xi^n, \ j = n, n+1, \dots, 2n+1.$$

Teorema 2. $k \to +\infty$ boʻlganda $X_k(x)$ funksiyalar uchun quyidagi baholar oʻrinli:

$$\left|X_k^{(j)}(x)\right| \le \frac{M}{(k)^{\frac{j-2n}{2n+1}}}\left|f_k(\tau)\right|,\ j=0,1,2,\ldots,2n,$$

bu yerda

$$\left|f_k(\tau)\right| = \max_{0\le\xi\le1}\left|f_k(\xi)\right|.$$

Demak (6) masalaning yechimi quyidagi koʻrinishga ega:

$$X_k(x) = \int_0^1 G_k(x,\xi)\, f_k(\xi)\, d\xi.$$

Teorema 3. Agar quyidagi shartlar bajarilsa:

1) $f(x,y), \dfrac{\partial^{2m+1} f}{\partial y^{2m+1}} \in C[0,1];$

2) $f_y^{(i)}(x,0) = f_y^{(i)}(x,1) = 0, i = 0,1,2,\ldots,2m,$

U holda (5) qator A masalaning yechimi boʻladi.

Isbot. (5) qatorni y o’zgaruvchi bo’yicha hadma-had differensiallash mumkinligini koʻrsatish yetarlidir.

$$\left|\frac{\partial^{2m+1}u}{\partial y^{2m+1}}\right| \le M\sum_{k=1}^{\infty} k^{2m}\left|X_k(x)\right| \le M\sum_{k=1}^{\infty} k^{2m}\frac{\left|f_k(\tau)\right|}{k^{\frac{2n}{2n+1}}} \le M\sum_{k=1}^{\infty} k^{\frac{4mn+2m-2n}{2n+1}}\left|f_k(\tau)\right|,$$

$$f_k(\tau) = 2\int_0^1 f(\tau,y)\sin\pi ky\,dy = -\frac{2}{\pi k}\left[f(\tau,y)\cos\pi ky\Big|_0^1 -\right.$$

$$\left. -\int_0^1 f_y(\tau,y)\cos\pi ky\,dy\right] = \frac{2}{\pi k}\int_0^1 f_y(\tau,y)\cos\pi ky\,dy =$$

$$= \frac{2}{(\pi k)^2}\left[f_y(\tau,y)\sin\pi ky\Big|_0^1 - \int_0^1 f_{yy}(\tau,y)\sin\pi ky\,dy\right] =$$

$$= -\frac{2}{(\pi k)^2}\int_0^1 f_{yy}(\tau,y)\sin\pi ky\,dy = -\frac{2}{(\pi k)^3}\left[f_{yy}(\tau,y)\cos\pi ky\Big|_0^1 -\right.$$

$$\left. -\int_0^1 f_{yyy}(\tau,y)\cos\pi ky\,dy\right] = \frac{2}{(\pi k)^3}\int_0^1 f_{yyy}(\tau,y)\cos\pi ky\,dy,$$

Shu jarayonni 2m+1 marta takrorlasak, quyidagiga ega bo’lamiz

$$f_k(\tau) = \frac{2}{(\pi k)^{2m+1}}\int_0^1 f_y^{(2m+1)}(\tau,y)\sin\pi ky\,dy,$$

Demak

$$\left|f_k(\tau)\right| \le \frac{M}{k^{2m+1}},$$

bu yerdan

$$\left|\frac{\partial^{2m+1}u}{\partial y^{2m+1}}\right| \le M\sum_{k}^{\infty} k^{\frac{4mn+2m-2n}{2n+1}}\frac{1}{k^{2m+1}} \le M\sum_{k=1}^{\infty}\frac{1}{k^{\frac{4n+1}{2n+1}}} < \infty.$$

Teorema 3 isbotlandi.

Xulosa

Maqolada yuqori toq tartibli tenglama uchun chegaraviy masalaning yechimlarini topish usuli ishlab chiqildi. Yechimning mavjudligi va yagonaligi teoremalari isbotlandi, hamda Grin funksiyasi yordamida yechimning aniq ko‘rinishi olindi. Shuningdek, Grin funksiyasi uchun asimptotik baholar va funksional cheklanishlar keltirilib, yechimning barqarorligi ko‘rsatildi. Olingan natijalar yuqori tartibli differensial tenglamalar uchun analitik yechimlarni qurishda nazariy asos bo‘lib xizmat qiladi

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