Usmonov Doniyor Abdumutolib o‘g‘li
Farg‘ona davlat universiteti
Matematik analiz va differensial tenglamalar
kafedrasi katta o‘qituvchisi
E-mail: usmonov-doniyor@inbox.ru
Fozilov Muhammaddiyor Nurmatjon o‘g‘li
Farg‘ona davlat universiteti magistranti
E-mail: fozilovmuhammaddiyor@gmail.com

UCHINCHI TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMA UCHUN CHEGARAVIY MASALA

Annotatsiya: *Ushbu maqolada uchinchi tartibli xususiy hosilali bir jinsli bo‘lmagan differensial tenglama uchun chegaraviy masalaning yechimi o‘rganilgan. Yechim usuli integral tenglamalarga ketma-ket yaqinlashish metodini qo‘llashga asoslangan bo‘lib, Riman-Liuvil integrali va Bessel-Klifford funksiyalaridan foydalanilgan. Olingan yechimning to‘g‘riligi chegaraviy shartlar tekshiruvi orqali isbotlangan va mavjudlik teoremasi shakllantirilgan. Ishlab chiqilgan natijalar matematik fizika va tegishli sohalarda yuqori tartibli tenglamalarni tahlil qilish vositalarini kengaytiradi.*

Kalit so‘zlar: *xususiy hosilali differensial tenglama, Riman-Liuvil integrali, Bessel-Klifford funksiyasi, Reyt tipidagi funksiya, Poxgammer simvoli ketma-ket yaqinlashish usuli, chegaraviy masala.*

Усмонов Дониёр Абдумутолиб угли
Ферганский государственный университет
Старший преподаватель кафедры математического
анализа и дифференциальных уравнений
E-mail: usmonov-doniyor@inbox.ru
Фозилов Мухаммаддиёр Нурматжон оглы
Магистрант Ферганского государственного университета
E-mail: fozilovmuhammaddiyor@gmail.com

КРАЕВАЯ ЗАДАЧА ДЛЯ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ С ЧАСТНЫМИ ПРОИЗВОДНЫМИ ТРЕТЬЕГО ПОРЯДКА

Аннотация: *В данной статье исследуется решение краевой задачи для неоднородного дифференциального уравнения с частными производными третьего порядка. Метод решения основан на применении последовательных* *приближений к интегральным уравнениям, включая использование интегралов Римана-Лиувилля и функций Бесселя-Клиффорда. Корректность полученного решения проверена через граничные условия, а также сформулирована теорема существования. Результаты работы расширяют инструментарий для анализа уравнений высших порядков в математической физике и смежных областях.*

Ключевые слова: *дифференциальное уравнение с частными производными, интеграл Римана-Лиувилля, функция Бесселя-Клиффорда, функция типа Рэлея, символ Похгаммера, метод последовательных приближений, краевая задача.*

Analysis and Differential Equations
usmonov-doniyor@inbox.ru
Fozilov Muhammaddiyor Nurmatjon ugli
Master's Student at Fergana State University
fozilovmuhammaddiyor@gmail.com

BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER PARTIAL DIFFERENTIAL EQUATION

Annotation: *This article investigates the solution to a boundary value problem for a nonhomogeneous third-order partial differential equation. The solution method is based on applying successive approximations to integral equations, utilizing Riemann-Liouville integrals and Bessel-Clifford functions. The correctness of the obtained solution is verified through boundary conditions, and an existence theorem is formulated. The developed results extend the toolkit for analyzing higher-order equations in mathematical physics and related fields.*

Keywords: *partial differential equation, Riemann-Liouville integral, Bessel-Clifford function, Rayleigh-type function, Pochhammer symbol, method of successive approximations, boundary value problem.*

Kirish

Ma'lumki, ko'plab jarayonlarning matematik modellashtirilishi, real hayotda sodir bo'ladigan jarayonlar, xususiy hosilalarga ega tenglamalar uchun nostandart boshlang'ich-chegara, to'g'ri va teskari masalalarni o'rganishga olib keladi, bunday tenglamalar klassik matematik-fizikada an'analari bo'lmagan analoglarga ega.

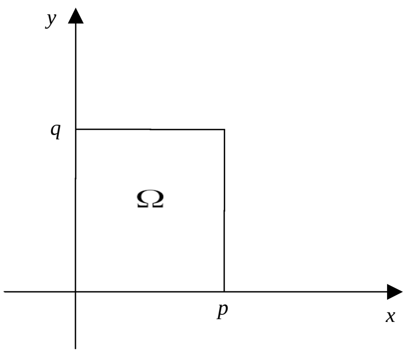
Ilovalar natijasida paydo bo'lgan muammolar, xususan, tovush va yuqori tezlikdagi gaz dinamikasi muammolari [1, 2, 3], moment nazariyasisiz qobiq nazariyasi va boshqalar [4], aralash tipdagi tenglamalarni tizimli ravishda o'rganishga olib keldi. Bu tenglamalar ko'rilayotgan sohalarning turli qismlarida turli tiplarga mansub bo'lib, ajratuvchi chiziqlarda ulash shartlari bajariladi.

Uchinchi tartibli qismli hosilali differensial tenglamalar gidrodinamika, elastiklik nazariyasi va matematik biologiya kabi turli sohalarda qo'llaniladi. Biroq, ularni tahlil qilish bir jinsli bo‘lmaganlik (noyelilik) va chegaraviy shartlarning murakkabligi bilan bog‘liq qiyinchiliklarni tug‘diradi. Ikkinchi tartibli tenglamalardan farqli o‘laroq, uchinchi tartibli masalalar uchun yechim usullari kamroq ishlab chiqilgan, bu esa ushbu sohadagi tadqiqotlarning dolzarbligini belgilaydi.

Mazkur ishda uchinchi tartibli bir jinsli bo‘lmagan qismli hosilali differensial tenglama ko‘rib chiqiladi, unga integral tenglamalar asosidagi yondashuv taklif etilgan. Riman-Liuvil operatorlari va Bessel-Klifford funksiyalaridan foydalanish asosiy masalani Volterra tipidagi ikkinchi tur integral tenglamaga keltirish imkonini beradi. Asosiy vosita — yechimga yaqinlashuvni ta’minlovchi ketma-ket yaqinlashish usulidir. Tadqiqotning yangiligi klassik usullarning yuqori tartibli tenglamalarga moslashtirilishi va berilgan funksional sinfda yechim mavjudligining isbotidadir.

Maqola quyidagi tuzilishga ega: birinchi qismda masala va asosiy farazlar shakllantiriladi; ikkinchi qismda yechimning integral ifodasi quriladi; uchinchi qism metodning to‘g‘riligini isbotlashga bag‘ishlangan; yakuniy qismda mavjudlik teoremasi va yaqinlashuv tahlili keltirilgan. Olingan natijalar murakkab chegaraviy shartlarni hisobga oladigan matematik modellashtirish masalalarida qo‘llanishi mumkin.

Tahlillar va natijalar



$\Omega = \{(x,y): 0 < x < p,\ 0 < y < q\}$ sohada ushbu

$$u_{xyy} + \gamma^{2} u_x + \lambda u = f(x,y) \tag{1}$$

xususiy hosilali differensial tenglamani qaraylik, $u(x,y)$ - noma’lum funksiya, $f(x,y)$ - berilgan funksiya, γ, λ, p, q - berilgan haqiqiy sonlar bo‘lib, $p > 0$, $q > 0$.

A **masala.** Ω sohada (1) tenglamani, soha chegarasida

$$u(x,0) = \tau(x),\quad u_y(x,0) = \nu(x),\quad 0 \le x \le p, \tag{2}$$

$$u(0,y) = \varphi(y),\quad 0 \le y \le q, \tag{3}$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,y)\in C_{x,y}^{0,1}(\overline{\Omega})$, $u_x(x,y)$, $u_{xyy}(x,y)\in C(\Omega)$ sinfga tegishli yechimi topilsin, bu yerda $\tau(x)$, $\nu(x)$ va $\varphi(y)$ lar berilgan funksiyalar bo‘lib, $\tau(0)=\varphi(0)$, $\varphi'(0)=\nu(0)$ shartlarini qanoatlantiradi.

Dastlab, (1) tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{\partial}{\partial x}\left(u_{yy}+\gamma^2 u\right)+\lambda u=f(x,y). \tag{4}$$

$I_{0y}^{\alpha,\gamma}$ Riman-Liuvil operatorini qaraylik:

$$I_{0y}^{\alpha,\gamma}a(y)=\frac{1}{\Gamma(\alpha)}\int_0^y (y-z)^{\alpha-1}\overline{J}_{\frac{\alpha-1}{2}}\left[\gamma(y-z)\right]a(z)dz, \quad \alpha>0,$$

bu yerda $\overline{J}_\nu(z)$-Bessel-Klifford funksiyasi

$$\overline{J}_\nu(z)=\Gamma(\nu+1)\left(\frac{z}{2}\right)^{-\nu}J_\nu(z)=\sum_{k=0}^{\infty}\frac{(-1)^k(z/2)^{2k}}{k!(\nu+1)_k},$$

$J_\nu(z)$-esa birinchi turdagi Bessel funksiyasi, $(a)_k$ -Poxgamber simvoli, $\Gamma(z)$-Eylerning gamma funksiyasi[6]. Agar $\alpha=2$ bo‘lsa, u holda $I_{0y}^{\alpha,\gamma}$ operatorning ko‘rinishi

$$I_{0y}^{2,\gamma}f(y)=\frac{1}{\Gamma(2)}\int_0^y (y-z)\overline{J}_{\frac{1}{2}}\left[\gamma(y-z)\right]f(z)dz$$

ko‘rinishga keladi.

Poxgammer simvolining qiymati $(a)_k=\dfrac{\Gamma(a+k)}{\Gamma(a)}$ ko‘rinishda aniqlanadi.

Integral ichida quyidagi hisoblashlarni bajaramiz, bunda $J_{\frac{1}{2}}[x]=\left(\dfrac{2}{\pi x}\right)^{\frac{1}{2}}\sin x$ ekanligini[6] inobat olamiz:

$$(y-z)\overline{J}_{\frac{1}{2}}\left[\gamma(y-z)\right]=(y-z)\Gamma\left(\frac{3}{2}\right)\left[\frac{\gamma(y-z)}{2}\right]^{-\frac{1}{2}}J_{\frac{1}{2}}\left[\gamma(y-z)\right]=$$

$$=\frac{1}{\gamma}\sin(\gamma(y-z)).$$

Endi $I_{0y}^{2,\gamma}$ ni

$$I_{0y}^{2,\gamma} f(y) = \frac{1}{\gamma}\int_0^y \sin(\gamma(y-z)) f(z)dz \tag{5}$$

ko‘rinishda yozib olamiz.

(4) tenglamaga $I_{0x}^{2,\gamma}$ ni ta’sir ettiramiz:

$$\frac{\partial}{\partial x}\left(I_{0y}^{2,\gamma}u_{yy} + I_{0y}^{2,\gamma}\gamma^2 u\right) + \lambda I_{0,y}^{2,\gamma}u = I_{0,y}^{2,\gamma} f(x,y). \tag{6}$$

Avval $I_{0y}^{2,\gamma}u_{yy}$ ni soddalashtirib olamiz, bunda $u(x,0)=\tau(x)$, $u_y(x,0)=\nu(x)$ shartlardan foydalanib,

$$I_{0y}^{2,\gamma}u_{yy}(x,y) = \frac{1}{\gamma}\int_0^y \sin(\gamma(y-z))u_{zz}(x,z)dz =$$

$$= -\frac{1}{\gamma}\sin(\gamma y)\nu(x) + u(x,y) - \cos(\gamma y)\tau(x) - \gamma^2 I_{0y}^{2,\gamma}u(x,y)$$

ni hosil qilamiz. Bu tenglikni (6) ga qo‘yib,

$$\frac{\partial}{\partial x}\left(-\frac{1}{\gamma}\sin(\gamma y)\nu(x) + u(x,y) - \cos(\gamma y)\tau(x)\right) + \lambda I_{0y}^{2,\gamma}u = I_{0,y}^{2,\gamma} f(x,y)$$

tenglamani hosil qilamiz. Bu tenglikka I_{0x}^{1} operatorni ta’sir ettiramiz va (3) shartdan foydalanib, Volterraning 2-tur integral tenglamasini hosil qilamiz:

$$u(x,y) + \lambda I_{0x}^{1} I_{0y}^{2,\gamma}u(x,y) = g(x,y), \tag{7}$$

bu yerda I_{0x}^{β} - Riman-Liuvill ma’nosidagi β tartibli integral operator[6] bo‘lib,

$$I_{0x}^{\beta} g(x) = \frac{1}{\Gamma(\beta)}\int_0^x (x-z)^{\beta-1} g(z)dz, \quad \beta > 0.$$

$$g(x,y) = I_{0x}^{1} I_{0,y}^{2,\gamma} f(x,y) + \varphi(y) - \sin(\gamma y)\nu(0) + \frac{1}{\gamma}\sin(\gamma y)\nu(x) -$$

$$- \cos(\gamma y)\tau(0) + \cos(\gamma y)\tau(x). \tag{8}$$

(7) integral tenglamaning yechimini ketma-ket yaqinlashish usulidan[5] foydalanib topamiz. Buning uchun quyidagi rekurent formulani qaraymiz:

$$u_m(x,y) = g(x,y) - \lambda I_{0x}^{1} I_{0y}^{2,\gamma}u_{m-1}(x,y), \quad m \in N.$$

$$u_0(x,y) = g(x,y).$$

Dastlab, $u_1(x,y)$ ni topamiz:

$$u_1(x,y)=g(x,y)-\lambda I_{0x}^{1}I_{0y}^{2,\gamma}u_0(x,y).$$

Endi $m=2$ deb olib $u_2(x,y)$ni

$$u_2(x,y)=g(x,y)-\lambda I_{0x}^{1}I_{0y}^{2,\gamma}u_1(x,y)$$

ko‘rinishda topamiz. So‘ngra $I_{0y}^{\alpha,\gamma}I_{0,y}^{\beta,\gamma}=I_{0y}^{\alpha+\beta,\gamma}$ va $I_{0y}^{\alpha}I_{0,y}^{\beta}=I_{0y}^{\alpha+\beta}$ tengliklardan foydalanib, $u_2(x,y)$ni

$$u_2(x,y)=u_0(x,y)-\lambda I_{0x}^{1}I_{0y}^{2,\gamma}u_0(x,y)+\lambda^2 I_{0x}^{2}I_{0y}^{4,\gamma}u_0(x,y)$$

ko‘rinishda topamiz. Ketma-ketlikni davom ettirib, $u_m(x,y)$ni

$$u_m(x,y)=u_0(x,y)-\lambda I_{0x}^{1}I_{0y}^{2,\gamma}u_0(x,y)+\ldots+(-1)^m\lambda^m I_{0x}^{m}I_{0y}^{2m,\gamma}u_0(x,y)$$

ko‘rinishida topamiz. Integral tenglamalar nazariyasiga ko‘ra, $\lim\limits_{m\to\infty}u_m(x,y)$ limit mavjud bo‘lsa, u holda uning limit funksiyasi (7) integral tenglamaning yechimidir.

$m\to\infty$ limitga o‘tib, $u(x,y)$ ning qiymatini quyidagi ko‘rinishda topamiz:

$$u(x,y)=\lim_{m\to\infty}u_m(x,y)=u_0(x,y)+\sum_{n=1}^{\infty}(-\lambda)^n I_{0x}^{n}I_{0y}^{2n,\gamma}u_0(x,y).$$

$u_0(x,y)$ni o‘rniga (8) ni qo‘yamiz va ba’zi hisoblashlarni bajarib $u(x,y)$ ni

$$u(x,y)=\varphi(y)-\frac{1}{\gamma}\sin(\gamma y)\nu(0)+\frac{1}{\gamma}\sin(\gamma y)\nu(x)-\cos(\gamma y)\tau(0)+\cos(\gamma y)\tau(x)-$$

$$-\lambda x\int_0^y(y-\eta)\varphi(\eta)\mathrm{E}_{(1,2),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta-$$

$$-\lambda y^2\int_0^x\tau(\xi)\mathrm{E}_{(1,1),(2,3,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi-$$

$$-\lambda y^3\int_0^x\nu(\xi)\mathrm{E}_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\tau(0)xy^2\mathrm{E}_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]-$$

$$-\lambda\nu(0)xy^3\mathrm{E}_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right]+$$

$$+\int_0^x\int_0^y (y-\eta)E_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta \tag{9}$$

ko‘rinishida topamiz, bu yerda

$$E_{(\alpha_1,\beta_1),(\alpha_2,\beta_2,\theta)}[x;y]=\sum_{n=0}^{+\infty}\frac{x^n}{\Gamma(\alpha_1 n+\beta_1)\Gamma(\alpha_2 n+\beta_2)}\bar{J}_{\frac{\alpha_2 n}{2}+\theta}[y] \tag{10}$$

ko‘rinib turibdiki, (10) funksiya (11) -Reyt tipidagi funksiya.

$$e^{\beta_1,\beta_2}_{\alpha_1,-\alpha_2}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{\Gamma(\alpha_1 n+\beta_1)\Gamma(\alpha_2 n+\beta_2)} \tag{11}$$

(10) va (11) qatorlarning yaqinlashuvchi ekanidan, shuningdek, ν ning $-\frac{1}{2}$ dan kichik bo‘lmagan qiymatlari uchun o‘rinli bo‘lgan $\bar{J}_\nu(z)\leq 1$ tengsizligidan foydalanib, $a_1>0,\ a_2>0$ uchun (10) qatorning $-\infty<x,\ y<+\infty$ qiymatlar uchun absolyut va tekis yaqinlashishi ma’lum.

(10) funksiya uchun

$$E_{(\alpha_1,\beta_1)(\alpha_2,\beta_2,\theta)}[x;y]=e^{\beta_1,\beta_2}_{\alpha_1,-\alpha_2}(x),\ E_{(\alpha_1,\beta_1)(\alpha_2,\beta_2,\theta)}[0;y]=\frac{1}{\Gamma(\beta_1)\Gamma(\beta_2)}\bar{J}_\theta(y), \tag{12}$$

$$E_{(\alpha_1,\beta_1)(\alpha_2,\beta_2,\theta)}[0;0]=\frac{1}{\Gamma(\beta_1)\Gamma(\beta_2)} \tag{13}$$

tengliklar, hamda quyidagi differensiallash formulalari o‘rinlidir:

$$\frac{\partial}{\partial y}\left[y^{\alpha_2}E_{(\alpha_1,\beta_1)\left(\alpha_2,\alpha_2+1,\frac{\alpha_2-1}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]\right]=$$

$$=y^{\alpha_2-1}E_{(\alpha_1,\beta_1)\left(\alpha_2,\alpha_2,\frac{\alpha_2-1}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]-\gamma^2y^{\alpha_2+1}E_{(\alpha_1,\beta_1)\left(\alpha_2,\alpha_2+2,\frac{\alpha_2+1}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right] \tag{14}$$

$$\frac{\partial}{\partial y}\left[y^{\beta_2-1}E_{(\alpha_1,\beta_1)\left(\alpha_2,\beta_2,\frac{\beta_2-1}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]\right]=y^{\beta_2-2}E_{(\alpha_1,\beta_1)\left(\alpha_2,\beta_2-1,\frac{\beta_2-3}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right],\beta_2\neq 1 \tag{15}$$

$$\frac{\partial}{\partial y}\left[E_{(\alpha_1,\beta_1)(\alpha_2,1,-1/2)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]\right]=$$

$$=-\lambda x^{\alpha_1}y^{\alpha_2-1}E_{(\alpha_1,\alpha_1+\beta_1)\left(\alpha_2,\alpha_2,\frac{\alpha_2-1}{2}\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]-\gamma^2yE_{(\alpha_1,\beta_1)(\alpha_2,2,1/2)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right] \tag{16}$$

$$\frac{\partial}{\partial x}\left\{x^{\beta_1-1}\mathrm{E}_{(\alpha_1,\beta_1),(\alpha_2,\beta_2,\theta)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]\right\}=x^{\beta_1-2}E_{(\alpha_1,\beta_1-1),(\alpha_2,\beta_2,\theta)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right],\beta_1\neq 1, \tag{17}$$

$$\frac{\partial}{\partial x}\left\{E_{(\alpha_1,1),(\alpha_2,\beta_2,\theta)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]\right\}=-\lambda x^{\alpha_1-1}y^{\alpha_2}E_{(\alpha_1,\alpha_1),\left(\alpha_2,\alpha_2+\beta_2,\frac{\alpha_2}{2}+\theta\right)}\left[-\lambda x^{\alpha_1}y^{\alpha_2};\gamma y\right]. \tag{18}$$

Teorema. *Agar* $\tau(x)\in C[0,p]\cap C^1(0,p)$, $\nu(x)\in C[0,p]\cap C^1(0,p)$, $\varphi(y)\in C^1[0,q]\cap C^2(0,q)$, *va* $\tau(0)=\varphi(0)$, $\varphi'(0)=\nu(0)$ $f(x,y)=(xy)^{-1}f_1(x,y)$ *bo‘lsa, u holda A masalaning yechimini* (9) *ko‘rinishda bo‘ladi.*

Isbot. Dastlab (1) tenglamani (2) va (3) chegaraviy shartlarni qanoatlantirishini ko‘rsatamiz. Buning uchun (9) ifodaga $y=0$ qo‘yib, $\tau(x)\in C[0,p]$, $\nu(x)\in C[0,p]$ va $\varphi(y)\in C[0,q]$ ekanligini e’tiborga olib quyidagi

$$u(x,0)=\varphi(0)-\tau(0)+\tau(x)$$

ifodani hosil qilamiz. $\tau(0)=\varphi(0)$ ekanidan foydalansak (9) ni (2) shartning bir qismini qanoatlantirishi kelib chiqadi. $u_y(x,0)=\nu(x)$ ekani isbotlash uchun (14) va (15) formulalardan foydalanib, (9) umumiy yechimning *y* bo‘yicha xususiy hosilasini

$$u_y(x,y)=\varphi'(y)+\cos(\gamma y)(\nu(x)-\nu(0))+\gamma\sin(\gamma y)(\tau(0)-\tau(x))-$$

$$-\lambda x\int_0^y\varphi(\eta)\mathrm{E}_{(1,2),(2,1,-1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta-$$

$$-\lambda\int_0^x\tau(\xi)\left[y\mathrm{E}_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]-\gamma^2y^3\mathrm{E}_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]\right]d\xi-$$

$$-\lambda\int_0^x\nu(\xi)y^2\mathrm{E}_{(1,1),(2,3,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\tau(0)x\left[y\mathrm{E}_{(1,2),(2,2,1/2)}\left[(-\lambda)xy^2;\gamma y\right]-\gamma^2y^3\mathrm{E}_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right]\right]-$$

$$-\lambda\nu(0)y^2\mathrm{E}_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]+$$

$$+\int_0^x\int_0^y\mathrm{E}_{(1,2),(2,1,-1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta \tag{19}$$

ko‘rinishda topib olamiz. Hosil qilgan (19) tenglikka $y=0$ ni qo‘yib,

$$u_y(x,0)=\varphi'(0)+\nu(x)-\nu(0)$$

ifodani hosil qilamiz. $\varphi'(0)=\nu(0)$ ekanligini hisobga olib,

$$u_y(x,0)=\nu(x)$$

(9) tenglama (2) shartning ikkinchi qismini ham qanoatlantirishini ko‘rishimiz mumkin.

Endilikda (9) ifodaga $x=0$ ni qo‘yib,

$$u(0,y)=\varphi(y)$$

ekanligini topamiz. Demak (9) ifoda (2) va (3) shartlarni qanoatlantirar ekan.

(9) funksiya (1) tenglamani qanoatlantirishini isbotlaymiz. Buning uchun (9) ni ushbu

$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

ko’rinishda yozib olamiz, bu yerda

$$u_1(x,y)=\varphi(y)-\lambda x\int_0^y(y-\eta)\varphi(\eta)\mathrm{E}_{(1,2),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta \quad . \qquad (20)$$

$$u_2(x,y)=\cos(\gamma y)\tau(x)-\cos(\gamma y)\tau(0)-$$

$$-\lambda y^2\int_0^x\tau(\xi)\mathrm{E}_{(1,1),(2,3,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi-$$

$$+\lambda\tau(0)xy^2\mathrm{E}_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]. \qquad (21)$$

$$u_3(x,y)=\frac{1}{\gamma}\sin(\gamma y)\nu(x)-\frac{1}{\gamma}\sin(\gamma y)\nu(0)-$$

$$-\lambda y^3\int_0^x\nu(\xi)\mathrm{E}_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\nu(0)xy^3\mathrm{E}_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right] \qquad (22)$$

$$u_4(x,y)=\int_0^x\int_0^y(y-\eta)\mathrm{E}_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta \quad . \qquad (23)$$

$u_1(x,y)$, $u_2(x,y)$, $u_3(x,y)$ va $u_4(x,y)$ larni birma-bir (1) tenglamaga qo‘yamiz.

Dastlab, (20) ning *y* bo‘yicha xususiy hosilasini (15) formuladan foydalanib,

$$\frac{\partial u_1(x,y)}{\partial y}=\varphi'(y)-\lambda x\int_0^y \varphi(\eta)E_{(1,2),(2,1,-1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta \tag{25}$$

tenglikni hosil qilamiz va (13) tenglikdan hamda (14) formuladan foydalanib quyidagi tenglikni hosil qilamiz:

$$\frac{\partial^2 u_1(x,y)}{\partial y^2}=\varphi''(y)-\frac{\lambda x\varphi(y)}{\Gamma(2)\Gamma(1)}+$$

$$+\lambda^2x^2\int_0^y (y-\eta)\varphi(\eta)E_{(1,3),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta+$$

$$+\lambda x\gamma^2\int_0^y (y-\eta)\varphi(\eta)E_{(1,2),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta \tag{26}$$

(17) formuladan hamda topib olgan (25) va (26) tengliklardan foydalanib,

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_1(x,y)}{\partial y^2}+\gamma^2 u_1(x,y)\right]=-\lambda\varphi(y)-$$

$$-\lambda^2\int_0^y (y-\eta)\varphi(\eta)\frac{\partial}{\partial x}\left[x^2E_{(1,3),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]\right]d\eta=$$

$$-\lambda\left[\varphi(y)-\lambda x\int_0^y (y-\eta)\varphi(\eta)E_{(1,2),(2,2,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]d\eta\right]=$$

$$=-\lambda u_1(x,y) \tag{27}$$

(27) tenglikni hosil qilamiz. Bu esa $u_1(x,y)$ funksiyamiz (1) tenglamani qanoatlantirishini isbotlaydi.

2) $u_2(x,y)$ funksiyaning y bo‘yicha xususiy hosilasini (14) formuladan topamiz:

$$\frac{\partial u_2(x,y)}{\partial y}=\gamma\sin(\gamma y)(\tau(0)-\tau(x))+$$

$$+\lambda\tau(0)xyE_{(1,2),(2,2,1/2)}\left[(-\lambda)xy^2;\gamma y\right]-$$

$$-\lambda\tau(0)x\gamma^2y^3E_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right]-$$

$$-\lambda y\int_0^x \tau(\xi)E_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\gamma^2 y^3\int_0^x \tau(\xi)E_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi \tag{28}$$

$u_2(x,y)$ ning ikkinchi tartibli xususiy hosilasini (15) dan foydalanib quyidagi ko‘rinishda topamiz:

$$\frac{\partial^2 u_2(x,y)}{\partial y^2}=\gamma^2\cos(\gamma y)(\tau(0)-\tau(x))+$$

$$+\lambda\tau(0)xE_{(1,2),(2,1,-1/2)}\left[(-\lambda)xy^2;\gamma y\right]-$$

$$-\lambda\tau(0)x\gamma^2 y^2E_{(1,2),(2,3,1/2)}\left[(-\lambda)x(y-\eta)^2;\gamma(y-\eta)\right]-$$

$$-\lambda\int_0^x \tau(\xi)E_{(1,1),(2,1,-1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\gamma^2 y^2\int_0^x \tau(\xi)E_{(1,1),(2,3,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi \tag{29}$$

(28) va (29) tengliklardan foydalanib,

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_2(x,y)}{\partial y^2}+\gamma^2 u_2(x,y)\right]=$$

$$+\lambda\tau(0)\bar{J}_{-\frac{1}{2}}\left[\gamma y\right]+\lambda\tau(0)\sum_{n=1}^{\infty}\frac{(-\lambda)^n x^n y^{2n}}{\Gamma(2n+1)\Gamma(n+1)}\bar{J}_{\frac{2n-1}{2}}\left[\gamma y\right]-$$

$$-\lambda\tau(x)\bar{J}_{-\frac{1}{2}}\left[\gamma y\right]-\lambda\int_0^x \tau(\xi)\sum_{n=1}^{\infty}\frac{(-\lambda)^n (x-\xi)^{n-1} y^{2n}}{\Gamma(2n+1)\Gamma(n)}\bar{J}_{\frac{2n-1}{2}}\left[\gamma y\right]d\xi \tag{30}$$

tenglikni hosil qilamiz. $\bar{J}_{-\frac{1}{2}}\left[\gamma y\right]=\cos(\gamma y)$ ekani hisobga olib, (30) tenglikni

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_2(x,y)}{\partial y^2}+\gamma^2 u_2(x,y)\right]=\lambda\cos(\gamma y)(\tau(0)-\tau(x))-$$

$$-\lambda^2\tau(0)xy^2E_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]-$$

$$+\lambda^2 y^2\int_0^x \tau(\xi)E_{(1,1),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]d\xi=-\lambda u_2(x,y) \tag{31}$$

ko‘rinishida topamiz. Demak, $u_2(x,y)$ funksiyamiz ham (1) tenglamaning yechimi ekan.

3) Endilikda $u_3(x,y)$ funksiyamiz (1) tenglamani qanoatlantirishini tekshiraylik. Buning uchun (22) dan *y* bo‘yicha birinchi va ikkinchi tartibli xususiy hosila olaylik, bunda uchun avval (15), so‘ngra (14) formulalardan foydalanamiz:

$$\frac{\partial}{\partial y}u_3(x,y)=\cos(\gamma y)(\nu(x)-\nu(0))-$$

$$-\lambda y^2\int_0^x \nu(\xi)E_{(1,1),(2,3,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda xy^2\nu(0)E_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right] \qquad (32)$$

$$\frac{\partial^2}{\partial y^2}u_3(x,y)=-\gamma\sin(\gamma y)(\nu(x)-\nu(0))-$$

$$-\lambda y\int_0^x \nu(\xi)E_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi-$$

$$-\lambda(-\gamma^2)y^3\int_0^x \nu(\xi)E_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda\nu(0)xyE_{(1,2),(2,3,1/2)}\left[(-\lambda)xy^2;\gamma y\right]+$$

$$+\lambda(-\gamma^2)\nu(0)xy^3E_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right]. \qquad (33)$$

(32) va (33) tengliklardan foydalanib,

$$\frac{\partial^2 u_3(x,y)}{\partial y^2}+\gamma^2 u_3(x,y)=$$

$$-\lambda y\int_0^x \nu(\xi)E_{(1,2),(2,2,1/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi+$$

$$+\lambda xy\nu(0)E_{(1,2),(2,2,1/2)}\left[(-\lambda)xy^2;\gamma y\right]$$

ni hosil qilamiz va uning *x* bo‘yicha xususiy hosilasini (17) dan foydalanib,

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_3(x,y)}{\partial y^2}+\gamma^2 u_3(x,y)\right]=-\lambda y\bar{J}_{\frac{1}{2}}\left[\gamma y\right](\nu(0)-\nu(x))+$$

$$+\lambda^2 y^3\int_0^x (x-\xi)\nu(\xi)E_{(1,1),(2,4,3/2)}\left[(-\lambda)(x-\xi)y^2;\gamma y\right]d\xi-$$

$$-\lambda^2 xy^3\nu(0)E_{(1,2),(2,4,3/2)}\left[(-\lambda)xy^2;\gamma y\right]$$

ko‘rinishda topamiz. $\bar{J}_{\frac{1}{2}}\left[\gamma y\right]=\frac{\sin(\gamma y)}{\gamma y}$ ekanligidan

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_3(x,y)}{\partial y^2}+\gamma^2 u_3(x,y)\right]=-\lambda u_3(x,y) \quad (34)$$

4) (23)ning y bo‘yicha birinchi va ikkinchi tartibli xususiy hosilalarini (15) va (16) formulalardan foydalanib quyidagi ko‘rinishda topamiz:

$$\frac{\partial u_4(x,y)}{\partial y}=\int_0^x\int_0^y E_{(1,1),(2,1,-1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta$$

$$\frac{\partial^2 u_4(x,y)}{\partial y^2}=-\lambda xy\int_0^x\int_0^y E_{(1,2),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta-$$

$$-\gamma^2\int_0^x\int_0^y(y-\eta)E_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta+$$

$$+\int_0^x f(\xi,y)dx\,.$$

Endilikda (17) dan foydalanib, $\frac{\partial}{\partial x}\left[\frac{\partial^2 u_4(x,y)}{\partial y^2}+\gamma^2 u_4(x,y)\right]$ ifodani

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 u_4(x,y)}{\partial y^2}+\gamma^2 u_4(x,y)\right]=$$

$$=\frac{\partial}{\partial x}\left[-\lambda x\int_0^x\int_0^y(y-\eta)E_{(1,2),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta\right]+$$

$$+\frac{\partial}{\partial x}\left[\int_0^x f(\xi,y)dx\right]$$

$$=-\lambda\int_0^x\int_0^y(y-\eta)E_{(1,1),(2,2,1/2)}\left[(-\lambda)(x-\xi)(y-\eta)^2;\gamma(y-\eta)\right]f(\xi,\eta)d\xi d\eta+f(x,y)=$$

$$=-\lambda u(x,y)+f(x,y) \quad (35)$$

ko‘rinishda topamiz. (27), (31), (34) va(35) ifodalarlardan ma’lumki, haqiqatdan ham (9) funksiya (1) tenglamaning umumiy yechimi ekan.

Foydalanilgan adabiyotlar

1. Бере Л. Математические вопросы до звуковой и около звуковой газовой динамики. - М.: ИЛ., 1961. - 208 с.
2. Франкль Ф.И. О задаче Чаплыгина для дифференциального уравнения во II сверхзвуковой области // Известия АН СССР. Серия матем. – 1945. – Т. 9. – № 2. – С. 121–142.
3. Френкель И.Б. Избранные труды по газовой динамике. – М.: Наука, 1973. – 712 с.
4. Кузьмин А.Г. Неклассические уравнения смешанного типа и их приложения к газодинамике. - Ленинград: Изд-во ЛГУ, 1990. - 208 с.
5. M.Salohiddinov. Integral tenglamalar: Oliy o'quv yurtlari talabalari uchun darslik. Mas’ul muharrir A.Oripov. O‘zbekiston Respublikasi Oliy va o'rta maxsus ta’lim vazirligi.— T.: Yangiyul poligraph service, 2007-256 b
6. А.Қ.Ўринов. Махсус функциялар ва махсус операторлар: ўқув услубий қўлланма - Фаргона: "Фаргона"нашриёти, 2012. - 112 бет.
7. A.O’rinov. D.Usmonov. “О задаче Коши для одного обыкновенного дифференциального уравнения, содержащего интегро - дифференциальный оператор с функцией Бесселя в ядре” // MATEMATIKA INSTITUTI BYULLETENI. – 2023 6 (1) b. 138-152