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TABIIY USULDA GRADUIRLANGAN FILIFORM NILRADIKALLI YECHILUVCHI LEYBNITS ALGEBRALARINING $\frac{1}{2}$-DIFFERENSIALLASHLARI

Mamadaliev O‘ktamjon Xasanboyevich
Namangan davlat universiteti dotsenti
ORCID: 0009-0001-2003-2320
E-mail: mamadaliyevuktamjon@mail.ru
Maxmudova Shoxsanam Shavkatjon qizi
Namangan davlat universiteti magistranti
E-mail: shohsanammahmudova092@gmail.com

Annotatsiya: *ushbu maqolada nilradikali tabiiy usulda graduirlangan filiform Leybnits algebrasi bo‘lgan yechiluvchan Leybnits algebralarining* $\frac{1}{2}$*-differensiallashlari o‘rganilgan.*

Kalit so‘zlar: *Leybnits algebrasi, yechiluvchan Leybnits algebrasi, nilradikal, algebraning* $\frac{1}{2}$*-differensiallashi.*

$\frac{1}{2}$-ДИФФЕРЕНЦИРОВАНИЯ РАЗРЕШИМЫХ АЛГЕБР ЛЕЙБНИЦА С ЕСТЕСТВЕННЫМ ОБРАЗОМ ГРАДУИРОВАННЫМ ФИЛИФОРМНЫМ НИЛЬРАДИКАЛОМ

Аннотация: *В данной статье исследуются* $\frac{1}{2}$*-дифференцирования разрешимых алгебр Лейбница, нильрадикал которых является естественным образом градуированной филиформной алгеброй Лейбница.*

Ключевые слова: *алгебра Лейбница, разрешимая алгебра Лейбница, нильрадикал,* $\frac{1}{2}$*-дифференцирование алгебры.*

$\frac{1}{2}$-DERIVATIONS OF SOLVABLE LEIBNIZ ALGEBRAS WITH A NATURALLY GRADED FILIFORM NILRADICAL

Abstract: *In this article we study* $\frac{1}{2}$*-derivations of solvable Leibniz algebras whose nilradical is a naturally graded filiform Leibniz algebra.*

Keywords: *Leibniz algebra, solvable Leibniz algebra, nilradical,* $\frac{1}{2}$*-derivation of algebra.*

Hozirgi vaqtda zamonaviy algebraning muhim yo‘nalishlaridan biri Leybnits algebralari nazariyasini tadqiq qilishdir. Li algebralarini tabiiy ravishda umumlashtiradigan va Leybnits ayniyati bilan beriladigan ushbu algebralar sinfi birinchi marta Blox [15] tomonidan 1965 yilda D-algebralar nomi bilan taklif qilingan. Ta’kidlash joizki, Leybnits ayniyatining bajarilishi, algebra elementiga o‘ngdan ko‘paytirish operatorining algebra uchun differensiallash bo‘lishiga teng kuchlidir. Shuning uchun D-algebralarni, har bir algebra elementiga o‘ngdan ko‘paytirish operatori differensiallash

bo‘lishining sharti bilan ham aniqlash mumkin. O‘tgan asrning 90-yillari boshlarida D-algebralar J.-L. Lode va T. Pirashvilining [3-5] ishlarida Leybnits algebralari deb nomlanib o‘rganila boshlandi. Shundan so‘ng Leybnits algebralariga qiziqish keskin ortib ketdi [1-2], [6-10], [13-14].

Ta’rif 1. F maydon ustida aniqlangan *L* algebraning ixtiyoriy *x*, *y*, *z* elementlari uchun quyidagi Leybnits ayniyati bajarilsa,

$$[x,[y,z]]=[[x,y],z]-[[x,z],y],$$

L algebra Leybnits algebrasi deyiladi, bu yerda [-,-] – *L* da aniqlangan ko‘paytirish amali.

Ixtiyoriy *L* Leybnits algebrasi uchun quyidagi quyi markaziy va hosilaviy ketma-ketliklarni mos ravishda aniqlaymiz:

L1 = L, Lk+1 = [Lk, L1]; L[1] = L, L[k+1] = [L[k], L[k]], k1.≥

Ta’rif 2. *L* Leybnits algebrasi uchun shunday $s\in \mathrm{N}$ son mavjud bo‘lib, (mos ravishda,) bo‘lsa, u holda L yechiluvchan (mos ravishda, nilpotent) Leybnits algebrasi deyiladi.$L^{[\mathrm{s}]}=0L^{s}=0$

Ta’rif 3. Ixtiyoriy Leybnits algebrasining maksimal nilpotent ideali uning nilradikali deyiladi.

Ta’rif 4. L Leybnits algebrasi, agar bo‘lsa, filiform Leybnits algebrasi deyiladi, bu yerda.$\dim L^{i}=n-i,\ 2\leq i\leq nn=\dim L$

Tabiiy usulda graduirlangan Leybnits algebrasi ta’rifini keltiramiz.

L chekli o‘lchamli nilpotent Leybnits algebrasi bo‘lsin. $\mathrm{gr}(L)_i:=L^i/L^{i+1}$, $1\leq i\leq s-1$ deb olamiz, bu yerda s – *L* algebraning nilindeksi va

$\mathrm{gr}L=\mathrm{gr}(L)_1\oplus\mathrm{gr}(L)_2\oplus\ \dots\ \oplus\mathrm{gr}(L)_{s-1}$

deb belgilab olamiz. gr*L* da ko‘paytmani quyidagicha aniqlaymiz:

$[x+L^i,\ y+L^j]_{\mathrm{gr}L}=[x,\ y]+L^{i+j-1}$, bu yerda $x\in L^{i-1}$, $y\in L^{j-1}$.

U holda $[\mathrm{gr}(L)_i,\ \mathrm{gr}(L)_j]_{\mathrm{gr}L}\subseteq\mathrm{gr}(L)_{i+j}$ bo‘ladi va biz gr*L* graduirlangan algebrani hosil qilamiz.

Ta’rif 5. Yuqorida qurilgan graduirovkani tabiiy graduirovka deb ataymiz. Agar *L* Leybnits algebrasi grL algebraga izomorf bo‘lsa, u holda *L tabiiy usulda graduirlangan Leybnits algebrasi* deb ataladi.

Quyidagi teoremada [2]-ishda keltirilgan, tabiiy usulda graduirlangan Li bo‘lmagan filiform Leybnits algebralarining tasnifini eslatib o‘tamiz.

Teorema 1. [2] Har qanday *n*-o‘lchamli kompleks tabiiy usulda graduirlangan Li bo‘lmagan filiform Leybnits algebrasi quyidagi o‘zaro izomorf bo‘lmagan algebralardan biriga izomorf:

$\mathrm{F}_{\mathrm{n}}^{1}$: **$[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, bu yerda $2\leq i\leq n-1$,**

$\mathrm{F}_{\mathrm{n}}^{2}$: $[e_1,e_1]=e_3$, $[e_i,e_1]=e_{i+1}$, bu yerda $3\leq i\leq n-1$.

Nilradikali N va Q to‘ldiruvchi fazosining o‘lchami s ga teng bo‘lgan barcha yechiluvchan Leybnits algebralari oilasini R(N, s) bilan belgilaymiz.

Quyidagi teoremada nilradikali tabiiy usulda graduirlangan filiform Leybnits algebrasi bo‘lgan va Q to‘ldiruvchi fazosining o‘lchami ikkiga teng bo‘lgan yechiluvchan Leybnits algebralarini tasnifi keltiriladi.

Teorema 2. [12] Nilradikali $\mathrm{F}_{\mathrm{n}}^{1}$ algebra bo‘lgan ixtiyoriy (*n*+2)-o‘lchamli yechiluvchan Leybnits algebrasi quyidagi algebraga izomorf:

$$R(\mathrm{F}_n^1,2):\begin{cases}[e_i,e_1]=e_{i+1}, & 2\leq i\leq n-1,\\ [x_1,e_1]=-e_1,\\ [e_1,x_1]=e_1,\\ [e_i,x_1]=(i-1)e_i, & 2\leq i\leq n,\\ [e_i,x_2]=e_i, & 2\leq i\leq n.\end{cases}$$

Teorema 3. [11] Nilradikali F_{n}^{2} algebra bo‘lgan ixtiyoriy (*n*+2)-o‘lchamli yechiluvchan Leybnits algebrasi quyidagi algebralardan biriga izomorf:

$$R_1(\mathrm{F}_n^2,2):\begin{cases}[e_1,e_1]=e_3, & [e_i,e_1]=e_{i+1}, \quad 3\le i\le n-1,\\ [e_1,x_1]=e_1, & [x_1,e_1]=-e_1,\\ [e_2,x_2]=-[x_2,e_2]=e_2, & [e_i,x_1]=(i-1)e_i, \quad 3\le i\le n,\end{cases}$$

$$R_2(\mathrm{F}_n^2,2):\begin{cases}[e_1,e_1]=e_3, & [e_i,e_1]=e_{i+1}, \quad 3\le i\le n-1,\\ [e_1,x_1]=e_1, & [x_1,e_1]=-e_1,\\ [e_2,x_2]=e_2, & [e_i,x_1]=(i-1)e_i, \quad 3\le i\le n.\end{cases}$$

Ta'rif 6. Aytaylik d– L Leybnits algebrasining chiziqli almashtirishi bo'lsin. Agar ixtiyoriy $x, y \in L$ lar uchun quyidagi tenglik bajrilsa

$d([x, y])= \frac{1}{2}$ $([d(x), y]+[x, d(y)])$,

u holda d chiziqli almashtirishga L Leybnits algebrasining $\frac{1}{2}$-differensiallashi deyiladi.

Quyidagi tasdiqda $R(\mathrm{F}_n^1,2)$ algebrani $\frac{1}{2}$-differensiallashi topilgan.

Tasdiq 1. $R(\mathrm{F}_n^1,2)$ algebraning ixtiyoriy $\frac{1}{2}$-differensiallashi quyidagi ko'rinishda bo'ladi

$$d(e_1)=a_1e_1, \qquad d(e_2)=b_2e_2,$$
$$d(e_i)=\frac{b_2+(2^{i-2}-1)a_1}{2^{i-2}}e_i, \quad 3\le i\le n,$$
$$d(x_1)=a_1x_1+(b_2-a_1)x_2, \quad d(x_2)=b_2x_2.$$

Isbot. Aytaylik d – $R(\mathrm{F}_n^1,2)$ algebraning $\frac{1}{2}$-differensiallashi bo'lsin. $\frac{1}{2}$-differensiallashni hosil qiluvchi bazis elementlari uchun berish yetarli, shuning uchun aytaylik,

$$d(e_1)=\sum_{i=1}^{n}a_ie_i+a_{n+1}x_1+a_{n+2}x_2,$$
$$d(e_2)=\sum_{i=1}^{n}b_ie_i+b_{n+1}x_1+b_{n+2}x_2,$$
$$d(x_1)=\sum_{i=1}^{n}c_ie_i+c_{n+1}x_1+c_{n+2}x_2,$$
$$d(x_2)=\sum_{i=1}^{n}f_ie_i+f_{n+1}x_1+f_{n+2}x_2$$

bo'lsin.

$\frac{1}{2}$-differensiallashni ta'rifidan foydalanib quyidagi tengliklarni qarab chiqamiz.

1. $d([e_1,e_1])=\frac{1}{2}([d(e_1),e_1]+[e_1,d(e_1)])$ tenglikdan $a_i=0$, $2\le i\le n$ bo'ladi. Bundan $d(e_1)=a_1e_1+a_{n+1}x_1+a_{n+2}x_2$ tenglikka ega bo'lamiz.

2. $d([e_2,e_2])=\frac{1}{2}([d(e_2),e_2]+[e_2,d(e_2)])$ tenglikdan $b_1=0$, $b_{n+1}=-b_{n+2}$ tengliklarni olamiz.

3. $d([e_1,e_2])=\frac{1}{2}([d(e_1),e_2]+[e_1,d(e_2)])$ tenglikni tekshirib $b_{n+1}=0$ ga tengligini aniqlaymiz, bundan $d(e_2)=\sum_{i=1}^{n}b_ie_i$ ko‘rinishda bo‘ladi.

4. $d([x_1,x_1])=\frac{1}{2}([d(x_1),x_1]+[x_1,d(x_1)])$ endi bu ko‘paytmani differensiallashini hisoblaymiz va $c_i=0,\quad 3\le i\le n$ ekanligini topamiz. U holda $d(x_1)=c_1e_1+c_2e_2+c_{n+1}x_1+c_{n+2}x_2$ **bo‘ladi.**

5. $d([x_2,x_2])=\frac{1}{2}([d(x_2),x_2]+[x_2,d(x_2)])$ endi shu tenglikni tekshirib $f_i=0,\quad 2\le i\le n$ ekanligidan $d(x_2)=f_1e_1+f_{n+1}x_1+f_{n+2}x_2$ bo‘ladi.

6. $d([x_1,x_2])=\frac{1}{2}\left([d(x_1),x_2]+[x_1,d(x_2)]\right)$ **tenglikni hisoblab** f_1 **koeffetsientni 0 ga teng bo‘lishini topdik.**

7. $d([e_1,x_1])=\frac{1}{2}\left([d(e_1),x_1]+[e_1,d(x_1)]\right)$ **tenglikdan** $a_1=c_{n+1}, a_{n+1}=0, a_{n+2}=0$ **ekanligini topdik. U holda** e_1 **va** x_1 **bazislarning differensiallashlarini ko‘rinishi** $d(e_1)=a_1e_1,\ d(x_1)=c_1e_1+a_1x_1+c_{n+2}x_2$ **ko‘rinishga keladi.**

8. $d([x_2,e_1])=\frac{1}{2}\left([d(x_2),e_1]+[x_2,d(e_1)]\right)$ **tengliklardan** $f_{n+1}=0$ **ekanligini** x_2 **bazisning ko‘rinishi** $d(x_2)=f_{n+2}x_2$ **ekanligini topdik.**

9. $d([e_2,x_2])=\frac{1}{2}\left([d(e_2),x_2]+[e_2,d(x_2)]\right)$ **tenglikni tekshirib** $f_{n+2}=b_2$ va $b_i=0,\ 3\le i\le n$ bunda bazislarning differensiallari $d(e_2)=b_2e_2,\ d(x_2)=b_2x_2$ ko‘rinishda bo‘ladi.

10. $d([e_2,x_1])=\frac{1}{2}\left([d(e_2),x_1]+[e_2,d(x_1)]\right)$ **differensialni hisoblab** $c_1=0,\ c_{n+2}=b_2-a_1$ tengliklarni olamiz, bundan $d(x_1)=a_1x_1+(b_2-a_1)x_2$ bo‘lishini olamiz.

Yuqoridagi differensiallashlardan foydalanib, $d(e_1)=a_1e_1,\quad d(e_2)=b_2e_2,\quad d(x_2)=b_2x_2,\quad d(x_1)=a_1x_1+(b_2-a_1)x_2$ **tengliklarni topdik. Endi qolgan bazis elementlari ko‘rinishlarini ham differensiallash formulalaridan foydalanib topamiz.**

11. $d([e_i,e_1])=\frac{1}{2}([d(e_i),e_1]+[e_i,d(e_1)]),\ 2\le i\le n-1$ tengliklarni ko‘rib chiqib va matematik induksiyadan foydalanib,

$$d(e_i)=\frac{b_2+(2^{i-2}-1)a_1}{2^{i-2}}e_i,\quad 3\le i\le n$$

tengliklarni olamiz.

$\frac{1}{2}$-differensiallash xossasini qolgan ko‘paytmalar uchun tekshirish ayniyatni beradi.

Tasdiq isbotlandi.

Tasdiq 2. $R_1(\mathrm{F}_n^2,2)$ algebraning ixtiyoriy $\frac{1}{2}$-differensiallashi quyidagi ko‘rinishda bo‘ladi:

$$d(e_1)=a_1e_1,\qquad d(e_2)=b_2e_2,\qquad d(x_1)=a_1x_1$$
$$d(x_2)=f_2e_2+b_2x_2,\quad d(e_i)=a_1e_i,\qquad 3\le i\le n.$$

Isbot. Aytaylik d – $R_1(\mathrm{F}_n^2,2)$ algebraning $\frac{1}{2}$-differensiallashi bo‘lsin, bu differensiallashni $R_1(\mathrm{F}_n^2,2)$ algebraning hosil qiluvchi elementlari uchun aniqlanish yetarli bo‘lganligi sababli, faraz qilaylik:

$$d(e_1)=\sum_{i=1}^{n}a_ie_i+a_{n+1}x_1+a_{n+2}x_2,$$

$$d(e_2)=\sum_{i=1}^{n}b_ie_i+b_{n+1}x_1+b_{n+2}x_2,$$

$$d(x_1)=\sum_{i=1}^{n}c_ie_i+c_{n+1}x_1+c_{n+2}x_2,$$

$$d(x_2)=\sum_{i=1}^{n}f_ie_i+f_{n+1}x_1+f_{n+2}x_2$$

bo‘lsin. $\frac{1}{2}$ -differensiallashning ta’riflaridan foydalanib quyidagi tengliklarni ko‘rib chiqamiz.

1. $d([e_1,e_2])=\frac{1}{2}([d(e_1),e_2]+[e_1,d(e_2)])$ **tenglikdan** $a_{n+2}=0$, $b_1=0$ ekanligini aniqladik.

2. $d([e_2,e_1])=\frac{1}{2}([d(e_2),e_1]+[e_2,d(e_1)])$ **endi bu tenglikni ishlab** $b_i=0,\quad 3\le i\le n-1$ ekanligini, ya’ni koeffetsiyentlar nolga tengligini aniqlaymiz. Bu holatda e_2 bazisning differensiali quyidagi ko‘rinishda bo‘ladi: $d(e_2)=b_2e_2+b_ne_n+b_{n+2}x_2$

3. Endi $d([x_1,x_1])=\frac{1}{2}([d(x_1),x_1]+[x_1,d(x_1)])$ tenglikni qarab, $c_i=0$, $3\le i\le n$ $b_i=0,\quad 3\le i\le n-1$ ekanligini topamiz. U holda $d(x_1)=c_1e_1+c_2e_2+c_{n+1}x_1+c_{n+2}x_2$

bo‘ladi.

4. $d([x_1,x_2])=\left([d(x_1),x_2]+[x_1,d(x_2)]\right)$ **tenglikni hisoblab** c_2 **va** f_1 **koeffetsientlarni 0 ga teng bo‘lishini topdik.**

5. $d([x_2,x_1])=\frac{1}{2}([d(x_2),x_1]+[x_2,d(x_1)])$ **dan** $f_i=0,\ 3\le i\le n$ **ekanligini topdik,** x_2 **bazisning differensiallashini ko‘rinishi** $d(x_2)=f_2e_2+f_{n+1}x_1+f_{n+2}x_2$ **ga keladi.**

6. $d([e_1,x_1])=\frac{1}{2}\left([d(e_1),x_1]+[e_1,d(x_1)]\right)$ **tenglikdan** $a_1=c_{n+1}, c_1=0$, $a_2=0, a_i=0, 4\le i\le n$ **ekanligini topdik. U holda** e_1 **va** x_1 **bazislarning differensiallashlarini ko‘rinishi** $d(e_1)=a_1e_1+a_3e_3, d(x_1)=a_1x_1+c_{n+2}x_2$ **ko‘rinishga keladi.**

7. $d([e_2,x_1])=\frac{1}{2}([d(e_2),x_1]+[e_2,d(x_1)])$ **tenglikdan** $b_n=0$ **va** $c_{n+2}=0$ **ekanligini topdik.** $d(e_2)=b_2e_2+b_{n+2}x_2$, $d(x_1)=a_1x_1$ **ko‘rinishlarini topdik.**

8. $d([x_1,e_1])=\frac{1}{2}([d(x_1),e_1]+[x_1,d(e_1)])$ **differensiallashni hisoblab** $a_3=0$ **ligini topib** e_1 **bazisning** $d(e_1)=a_1e_1$ **ko‘rinishini topdik.**

9. $d([e_2,x_2])=\frac{1}{2}\left([d(e_2),x_2]+[e_2,d(x_2)]\right)$ **differensiallashni hisoblab** $b_{n+2}=0$, $b_2=f_{n+2}$ **ekanliginidan** $d(e_2)=b_2e_2$, $d(x_2)=f_2e_2+f_{n+1}x_1+b_2x_2$ **tengliklarga ega bo‘lamiz.**

10. $d([x_2,e_1])=\frac{1}{2}\left([d(x_2),e_1]+[x_2,d(e_1)]\right)$ **tengliklardan** $f_{n+1}=0$ **ekanligini** x_2 **bazisning ko‘rinishi** $d(x_2)=f_2e_2+b_2x_2$ **ekanligini topdik.**

11. $d([e_1,e_1])=\frac{1}{2}([d(e_1),e_1]+[e_1,d(e_1)])$ **differensiallashdan** $d(e_3)=a_1e_3$ **tenglikni aniqladik.**

So‘ngra $\frac{1}{2}$**-differensiallash xossasini** $[e_i, e_1] = e_{i+1}, \quad 3 \le i \le n-1$, **ko‘paytmalar uchun ko‘rib va matematik induksiyadan foydalanib,** $d(e_i) = a_1 e_i$, $3 \le i \le n$ **ekanligini olamiz.** $\frac{1}{2}$**-differensiallash xossasini qolgan ko‘paytmalar uchun tekshirish ayniyatni beradi.**

Tasdiq isbotlandi.

Tasdiq 3. $R_2(\mathrm{F}_n^2, 2)$ algebraning ixtiyoriy $\frac{1}{2}$-differensiallashi quyidagi ko‘rinishda bo‘ladi:

$$d(e_1) = a_1 e_1, \quad d(e_2) = b_2 e_2,$$
$$d(e_i) = a_1 e_i, \quad 3 \le i \le n,$$
$$d(x_1) = a_1 x_1, \quad d(x_2) = b_2 x_2.$$

Isbot. Aytaylik d – $R_2(\mathrm{F}_n^2, 2)$ algebraning $\frac{1}{2}$-differensiallashi bo‘lsin, bu differensiallashni hosil qiluvchi bazis elementlari uchun berish yetarli. Bazis elementlarini differensiallashlari quyidagicha bo‘lsin:

$$d(e_1) = \sum_{i=1}^{n} a_i e_i + a_{n+1} x_1 + a_{n+2} x_2,$$
$$d(e_2) = \sum_{i=1}^{n} b_i e_i + b_{n+1} x_1 + b_{n+2} x_2,$$
$$d(x_1) = \sum_{i=1}^{n} c_i e_i + c_{n+1} x_1 + c_{n+2} x_2,$$
$$d(x_2) = \sum_{i=1}^{n} f_i e_i + f_{n+1} x_1 + f_{n+2} x_2.$$

$\frac{1}{2}$-differensiallashning ta’riflaridan foydalanib quyidagi tengliklarni ko‘rib chiqamiz.

1. $d([e_1, e_2]) = \frac{1}{2}([d(e_1), e_2] + [e_1, d(e_2)])$ **tenglikni qarab chiqib,** $b_1 = 0$, $b_{n+1} = 0$ ekanligini, ya’ni koeffetsiyentlar nolga tengligini aniqlaymiz. Bu holatda e_2 bazisning ko‘rinishi $d(e_2) = \sum_{i=2}^{n} b_i e_i + b_{n+2} x_2$ bo‘ladi.

2. $d([e_2, e_1]) = \frac{1}{2}([d(e_2), e_1] + [e_2, d(e_1)])$ **endi bu tenglikni ishlab** $a_{n+2} = 0$, $b_i = 0$, $3 \le i \le n-1$ ekanligini, ya’ni koeffetsiyentlar nolga tengligini aniqlaymiz. Bu holatda e_2 bazisning differensiali quyidagi ko‘rinishda bo‘ladi: $d(e_2) = b_2 e_2 + b_n e_n + b_{n+2} x_2$

3. $d([x_1, x_1]) = \frac{1}{2}([d(x_1), x_1] + [x_1, d(x_1)])$ endi bu tenglikni tekshirib, $c_i = 0$, $3 \le i \le n$ ekanligini topamiz. U holda $d(x_1) = c_1 e_1 + c_2 e_2 + c_{n+1} x_1 + c_{n+2} x_2$ bo‘ladi.

4. $d([e_2, e_2]) = \frac{1}{2}([d(e_2), e_2] + [e_2, d(e_2)])$ **differensiallashni hisoblab** $b_{n+2} = 0$ ekanligini topamiz va $d(e_2) = b_2 e_2 + b_n e_n$ ko‘rinishda bo‘ladi.

5. $d([e_2, x_1]) = \frac{1}{2}([d(e_2), x_1] + [e_2, d(x_1)])$ **tenglikdan** $b_n = 0$ **va** $c_{n+2} = 0$ **ekanligini topdik.** $d(e_2) = b_2 e_2$ **va** $d(x_1) = c_1 e_1 + c_2 e_2 + c_{n+1} x_1$ **ko‘rinishlarini topdik.**

6. $d([x_1, x_2]) = \frac{1}{2}([d(x_1), x_2] + [x_1, d(x_2)])$ tenglikni hisoblab, $c_2 = 0$, $f_1 = 0$ ga tengligini aniqladik.

7. $d([x_2, x_2]) = \frac{1}{2}([d(x_2), x_2] + [x_2, d(x_2)])$ **differensiallash xossasidan** $f_2 = 0$ ekanligini aniqladik.

8. $d\left(\left[x_2, x_1\right]\right)=\frac{1}{2}\left(\left[d(x_2), x_1\right]+\left[x_2, d(x_1)\right]\right)$ tenglikdan differensiallashni hisoblab ushbu natijalarni olamiz. $f_i=0,\ 3\le i\le n$ va x_2 bazisning ko‘rinishi $d(x_2)=f_{n+1}x_1+f_{n+2}x_2$ bo‘ladi.

9. $d\left(\left[e_1, x_2\right]\right)=\frac{1}{2}\left(\left[d(e_1), x_2\right]+\left[e_1, d\left(x_2\right)\right]\right)$ **tenglikni tekshirib** a_2, f_{n+1} koeffetsiyentlarni 0 ga teng ekanligini aniqladik. Bu holda $d(x_2)=f_{n+2}x_2$ **bo‘ladi.**

10. $d\left(\left[e_1, x_1\right]\right)=\frac{1}{2}\left(\left[d\left(e_1\right), x_1\right]+\left[e_1, d(x_1)\right]\right)$ **tenglikdan** $a_1=c_{n+1}$, $a_{n+1}=0$, $a_i=0,\ 4\le i\le n$, $c_1=0$ **ekanligini topdik. U holda** e_1 **va** x_1 **bazislarning differensiallashlarini ko‘rinishi** $d(e_1)=a_1e_1+a_3e_3$, $d(x_1)=a_1x_1$ **ko‘rinishga keladi.**

11. $d\left(\left[x_1, e_1\right]\right)=\frac{1}{2}\left(\left[d(x_1), e_1\right]+\left[x_1, d(e_1)\right]\right)$ **differensiallashni hisoblab** $a_3=0$ **ligini topib** e_1 **bazisning** $d(e_1)=a_1e_1$ **ko‘rinishini topdik.**

12. $d\left(\left[e_2, x_2\right]\right)=\frac{1}{2}\left(\left[d\left(e_2\right), x_2\right]+\left[e_2, d\left(x_2\right)\right]\right)$ **differensiallashni hisoblab** $b_2=f_{n+2}$ **ekanliginidan** $d(x_2)=b_2x_2$ **tenglikga ega bo‘lamiz.**

13. $d\left(\left[e_1, e_1\right]\right)=\frac{1}{2}\left(\left[d(e_1), e_1\right]+\left[e_1, d(e_1)\right]\right)$ **differensiallashdan** $d(e_3)=a_1e_3$ **tenglikni aniqladik. Xuddi shu kabi qolgan bazis elementlarini ham ko‘rinishini differensiallash hisoblashlardan foydalanib** $d(e_i)=a_1e_i$, $3\le i\le n$ **ekanligini aniqladik.**

Tasdiq isbotlandi.

XULOSA

Ushbu maqolada nilradikali tabiiy usulda graduirlangan filiform Leybnits algebrasi bo‘lgan yechiluvchan Leybnits algebralarining $\frac{1}{2}$-differensiallashlari o‘rganildi. Tadqiqot davomida qaralayotgan algebralar oilalarining tuzilish xususiyatlaridan foydalanib, ularning $\frac{1}{2}$-differensiallashlarini aniqlash masalasi ko‘rib chiqildi.

Olingan natijalar nilradikali tabiiy usulda graduirlangan filiform Leybnits algebrasidan iborat bo‘lgan yechiluvchan Leybnits algebralarining chiziqli almashtirishlarini o‘rganishda muhim ahamiyatga ega. Xususan, ular mazkur algebralarning $\frac{1}{2}$-differensiallashlarini aniqlash, ularning algebraik tuzilishga bog‘liq xususiyatlarini tahlil qilish hamda Leybnits algebralarida differensiallashlarning boshqa turlarini o‘rganish uchun nazariy asos bo‘lib xizmat qilishi mumkin.

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