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ANALYSIS-CHOICE SENSITIVITY IN RED-GIANT POPULATION STUDIES: A MULTIVERSE APPROACH

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Abstract: *Population-level conclusions about red giants are usually reported from a single analysis path, although decisions about selection boundaries, evolutionary-stage composition and classification method can change the result. Here the author's earlier results – an APOGEE–GALAH age comparison, HST chromosome maps of globular clusters, cross-survey chemical-label transportability and a Kepler asteroseismic sample – are re-analysed within one multiverse framework. Across the 28 possible contiguous T_{eff} windows, the reported mean age offset ranges from −0.66 to +2.71 Gyr, and from +0.42 to +1.21 Gyr for the 13 windows that retain at least half of the sample. Varying the red-clump fraction from 0 to 100 % lowers the expected offset from 1.23 to 0.19 Gyr. With a star-by-star scatter of 2.43 Gyr, a 0.5 Gyr systematic floor is already reached at ≈24 stars. Catalogue-level quantities should therefore be published together with the selection window and stage composition that produced them.*

Keywords: *red giants, stellar ages, APOGEE, GALAH, globular clusters, systematic errors, multiverse analysis, selection effects.*

QIZIL GIGANT POPULYATSIYALARI TADQIQOTLARIDA TAHLIL TANLOVLARIGA SEZGIRLIK: KO‘P VARIANTLI TAHLIL

Annotatsiya: *Qizil gigantlar bo‘yicha populyatsion xulosalar ko‘pincha bitta tahlil yo‘li bilan olinadi, holbuki tanlov chegaralari, evolyutsion bosqich tarkibi va tasniflash usuli bo‘yicha qabul qilingan qarorlar natijani o‘zgartirishi mumkin. Ushbu ishda muallifning APOGEE–GALAH yoshlarini solishtirish, sharsimon to‘dalarning HST xromosoma xaritalari, kimyoviy belgilarning so‘rovlararo ko‘chuvchanligi va Kepler asteroseysmik namunasi bo‘yicha oldingi natijalari yagona ko‘p variantli yondashuvda qayta tahlil qilindi. T_{eff} oralig‘ining 28 ta mumkin bo‘lgan tanlovi uchun o‘rtacha yosh farqi −0,66 dan +2,71 Gyr gacha, namunaning kamida yarmini saqlovchi 13 ta oraliq uchun esa +0,42 dan +1,21 Gyr gacha o‘zgarishi ko‘rsatildi. RC ulushi 0 dan 100 % gacha o‘zgarganda kutilgan siljish 1,23 dan 0,19 Gyr gacha kamayadi. Yulduzma-yulduz tarqoqlik 2,43 Gyr bo‘lganda 0,5 Gyr sistematik chegaraga atigi ≈24 yulduzda erishiladi. Natijalar katalog darajasidagi miqdorlarni tanlov oynasi va bosqich tarkibi bilan birga e'lon qilish zarurligini ko‘rsatadi.*

Kalit so‘zlar: *qizil gigantlar, yulduz yoshlari, APOGEE, GALAH, sharsimon to‘dalar, sistematik xatoliklar, ko‘p variantli tahlil, tanlov effektlari.*

ЧУВСТВИТЕЛЬНОСТЬ К ВЫБОРУ МЕТОДИКИ АНАЛИЗА В ИССЛЕДОВАНИЯХ ПОПУЛЯЦИЙ КРАСНЫХ ГИГАНТОВ: МУЛЬТИВЕРСНЫЙ АНАЛИЗ

Аннотация: *Популяционные выводы о красных гигантах обычно получают одним путём анализа, хотя решения о границах выборки, составе по эволюционным стадиям и способе классификации могут менять результат. В работе прежние результаты автора – сравнение возрастов APOGEE и GALAH, хромосомные карты шаровых скоплений по данным HST, переносимость химических меток между обзорами и астросейсмическая выборка Kepler – повторно проанализированы в рамках единого мультиверсного подхода. Показано, что для 28 возможных интервалов T_{eff} среднее различие возрастов меняется от −0,66 до +2,71 млрд лет, а для 13 интервалов, сохраняющих не менее половины выборки, – от +0,42 до +1,21 млрд лет. При изменении доли красного сгущения от 0 до 100 % ожидаемое смещение уменьшается с 1,23 до 0,19 млрд лет. При разбросе 2,43 млрд лет систематический предел 0,5 млрд лет достигается уже при ≈24 звёздах. Результаты показывают, что величины каталожного уровня следует публиковать вместе с окном отбора и составом по стадиям.*

Ключевые слова: *красные гиганты, возрасты звёзд, APOGEE, GALAH, шаровые скопления, систематические ошибки, мультиверсный анализ, эффекты отбора.*

Introduction

Red giants are the principal tracers of Galactic archaeology. They are luminous enough to be observed across the disc, they dominate the target lists of large spectroscopic surveys, and, unlike most other tracers, they can be assigned individual masses and ages [1]. Over the past decade the APOGEE and GALAH surveys have released stellar parameters, abundances and value-added age catalogues for hundreds of thousands of giants [2–4], placed on the astrometric framework of Gaia DR3 [5], while the data-driven astroNN and the Bayesian BSTEP pipelines have turned these labels into ages [6–8]. Asteroseismic samples from Kepler have meanwhile become the reference against which such ages are judged [9].

A result derived from these data is never the output of the data alone. Between the public catalogue and the published number lies a chain of analysis decisions: which effective-temperature window counts as a «clean» giant sample, whether red-clump (RC) stars are kept or removed, whether stars are assigned hard population labels or probabilistic weights, and which quality flags are applied. In statistics and the social sciences this problem is known as the «garden of forking paths» [10] or «researcher degrees of freedom» [11], and two remedies have been proposed: multiverse analysis, in which a result is computed for every reasonable combination of choices and the whole distribution is reported [12], and many-analyst studies, in which independent teams analyse the same data [13]. Both remain rare in stellar astrophysics.

In a series of recent studies the author and co-authors have, each time for a different scientific question, encountered the same phenomenon: the headline number depended strongly on one analysis choice. The aim of this paper is to turn these scattered observations into a single quantitative question that none of the earlier works addressed: how large is the spread of reported results that is generated purely by defensible analysis choices, and how does it compare with the statistical uncertainty that is normally quoted? The paper does not re-derive the earlier measurements; it uses their published summary statistics as inputs and computes new quantities – a full selection-window multiverse, a stage-mixing model, a statistical-versus-systematic error budget and a cross-study survival audit – that are presented here for the first time.

Literature review

Cross-survey systematics. Hegedűs et al. [14] compared APOGEE and GALAH atmospheric parameters for common stars and found offsets that change with the parameter values rather than a fixed zero-point. Ages, which are derived from these parameters through different pipelines, are expected to inherit and amplify such trends. The astroNN ages [6, 7] are produced by a neural network trained on asteroseismic ages, whereas BSTEP [8] fits isochrones to spectroscopic, astrometric and photometric inputs; any comparison of the two catalogues therefore mixes survey and method effects.

Evolutionary-stage classification. Bedding et al. [15] showed that gravity-mode period spacings separate hydrogen-shell-burning RGB stars from helium-core-burning RC stars that occupy nearly the same place in the Hertzsprung–Russell diagram. The APOKASC catalogue [16] provided seismic masses and stage flags for Kepler giants, and spectroscopic mass and age estimates for giants calibrated on such samples were developed by Martig et al. [17]. Miglio et al. [9] demonstrated that age precision depends on the stage and on the mass proxy used. A practical consequence is that the RGB/RC composition of a sample is itself a variable that can shift population averages.

Multiple populations in globular clusters. The HST UV Legacy Survey [18] and the chromosome-map technique [19] made it possible to separate first- and second-generation stars on the RGB of dozens of clusters; the HUGS catalogues [20] provide homogeneous photometry for this purpose. Radial differences between populations are interpreted as dynamical fossils, but the partition of overlapping sequences into discrete populations is itself a modelling choice.

The author's earlier results. Turaev, Arslonova and Nuritdinov [21] analysed 56 Galactic globular clusters and showed that strict two-family Gaussian-mixture labels are stable in only 11 of 55 eligible clusters, while a posterior-weighted radial contrast is reproducible in 34 of 55; a label-free extreme-tail statistic increases with compactness ($\varrho = 0.500$, $p = 0.0026$). Arslonova, Nuritdinov and Turaev [22] found that cross-calibrating APOGEE and GALAH abundances is necessary but insufficient: only [Ni/Fe] satisfied both radial and vertical equivalence on a balanced common footprint of 34,177 stars. Conference contributions [23, 24] and a submitted article [25] compared astroNN and BSTEP ages for 29,809 common giants and found a mean offset of +0.97 Gyr, a scatter of 2.43 Gyr and a monotonic dependence on T_{eff}; contribution [24] noted qualitatively that the level of agreement depends on where the temperature cut is placed. A further contribution [26] discussed asteroseismic separation of RGB and RC stars. No earlier work has quantified the sensitivity of these results to analysis choices across all possible alternatives, which is the gap addressed here.

Research methodology

Inputs. The analysis uses only published or submitted summary statistics from the studies listed in Table 1, together with the author's compilation of 1,850 Kepler giants with seismic parameters built from the APOKASC catalogue [16]. No new observations are used; the novelty lies in the computations described below. For the age comparison the basic quantity is

$$\Delta t = t_{APOGEE} - t_{GALAH}, \quad (1)$$

so that a positive value means that the astroNN catalogue assigns the older age. Its statistics in seven T_{eff} bins between 3500 and 5500 K (mean m_i, standard deviation σ_i, star count N_i) and separately for RGB and RC stars are taken from [25].

Table 1

Input studies and the analysis choices examined

Study	Data	Sample	Choice examined
APOGEE–GALAH ages [23–25]	APOGEE DR17 astroNN, GALAH DR4 BSTEP, Gaia DR3	29,809 giants (of 68,289)	T_{eff} window; RGB/RC mix; quality cuts
Chromosome maps [21]	HST HUGS photometry	55 GCs	hard vs soft labels; statistic definition
Chemical transport [22]	Gaia–APOGEE–GALAH atlas	34,177 stars (balanced)	element choice; footprint
Seismic stages [16, 26]	APOKASC Kepler giants	1,850 stars	requirement of a stage label

Selection-window multiverse. Every contiguous T_{eff} window $W = [T_{min}, T_{max}]$ that can be built from the eight bin edges is a defensible «clean sample» definition; there are $7 \cdot 8/2 = 28$ such windows. For each window the reported mean, the pooled standard deviation and the retained fraction are computed exactly from the bin statistics:

$$M_W = \Sigma N_i m_i / \Sigma N_i, \quad (2)$$

$$SW2 = \Sigma N_i [\sigma i2 + (m_i - M_W)2] / \Sigma N_i, \quad (3)$$

where the sums run over the bins inside W. Because a small mean can arise from positive and negative bins that cancel, a cancellation index is introduced:

$$K_W = \Sigma N_i |m_i| / \Sigma N_i - |M_W|. \quad (4)$$

$K_W = 0$ when all bins have the same sign; a large K_W flags «agreement» that is produced by averaging opposite systematic errors.

Stage-mixing model. If RGB and RC stars have mean offsets m_{RGB} and m_{RC}, a sample with RC fraction f reports

$$M(f) = (1 - f)\, m_{RGB} + f\, m_{RC}. \quad (5)$$

This assumes that the per-stage offsets are transferable between samples, which is a simplification discussed below.

Statistical versus systematic error. For a sample of N stars with star-by-star scatter σ, the statistical error of the mean falls as $\sigma/\sqrt{N}$, while a systematic offset δ caused by an analysis choice does not. The two are equal at

$$N_{crit} = (\sigma / \delta)2; \quad (6)$$

above N_{crit} the analysis choice, not the sample size, limits the accuracy.

Survival audit. For each analysis step that removes or fails to classify objects, the surviving fraction k/n is reported with a 95 % Wilson score interval [27], which behaves well for small n and for fractions near 0 or 1. All computations were carried out in Python with NumPy and Matplotlib; as a consistency check, applying Eqs. (2)–(3) to all seven bins reproduces the published full-sample mean and scatter (0.972 and 2.437 Gyr versus 0.97 and 2.43 Gyr; the bins contain 29,736 stars, 73 fewer than the full sample because of edge rounding).

Results and discussion

1. The selection-window multiverse. Figure 1 shows the reported mean offset for all 28 windows. The values span −0.66 to +2.71 Gyr. Narrow windows at the edges are extreme by construction, but the spread remains large even among windows that a careful analyst could plausibly choose: for the 13 windows that retain at least half of the sample, M_W ranges from +0.42 to +1.21 Gyr, i.e. the spread (0.79 Gyr) is almost as large as the offset itself. Moving only the cool boundary from 3500 to 4250 K – a change suggested in [24] – lowers the reported mean from 0.97 to 0.78 Gyr and the scatter from 2.44 to 2.15 Gyr while still keeping 87 % of the stars (Table 2).

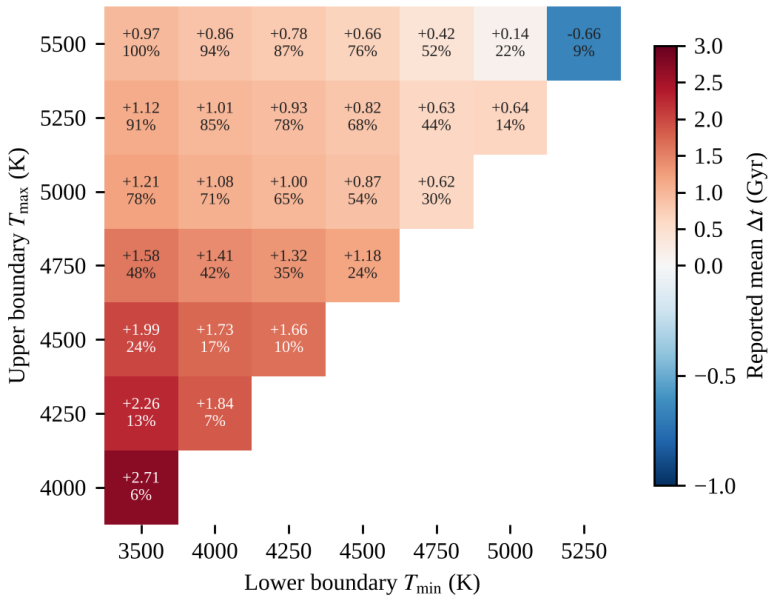


Figure 1. Reported mean age offset Δt (upper number) and retained fraction of stars (lower number) for every contiguous T_{eff} selection window

The cancellation index identifies a subtler problem. The two windows that come closest to «agreement» ($|M_W| < 0.5$ Gyr) are 4750–5500 K and 5000–5500 K. In both, the hottest bin has a negative offset (−0.66 Gyr) that partly cancels positive offsets in the neighbouring bins, giving $K_W = 0.22$ and 0.51 Gyr respectively. For the 5000–5500 K window, the mean of +0.14 Gyr hides a typical bin-level disagreement of 0.65 Gyr. By contrast, the 4750–5250 K window recommended in [25] has $K_W = 0$: its 0.63 Gyr offset is honest, with no cancellation. A small mean offset is therefore not evidence of agreement unless K_W is also small.

Table 2

Selected selection windows from the multiverse

Window (K)	N	Retained (%)	M_W (Gyr)	S_W (Gyr)	K_W (Gyr)
3500–5500 (adopted in [25])	29,736	100	+0.97	2.44	0.11
4000–5500	27,864	94	+0.86	2.27	0.12
4250–5500	25,814	87	+0.78	2.15	0.13
4500–5250	20,204	68	+0.82	1.98	0.00
3500–4750 (cool half)	14,216	48	+1.58	2.73	0.00
4750–5250	12,979	44	+0.63	1.81	0.00
4750–5500	15,520	52	+0.42	1.97	0.22
5000–5500	6,606	22	+0.14	2.11	0.51

2. Evolutionary-stage composition. The submitted study [25] found mean offsets of 1.23 Gyr for RGB and 0.19 Gyr for RC stars. Equation (5) turns this into a prediction for any sample (Figure 2). For the APOGEE–GALAH common sample (f = 24.6 %) the model returns 0.97 Gyr, identical to the observed value – a consistency check by construction, not an independent test. Among the 529 stars of the author's Kepler compilation that carry a seismic stage label (272 RGB/AGB, 257 RC), the RC fraction is 48.6 %; if the per-stage offsets apply, a comparison of the same two catalogues on such a sample would report 0.72 Gyr, i.e. 25 % less than the published value, without any change in either catalogue. A pure-RC sample would report 0.19 Gyr and might be read as «excellent agreement».

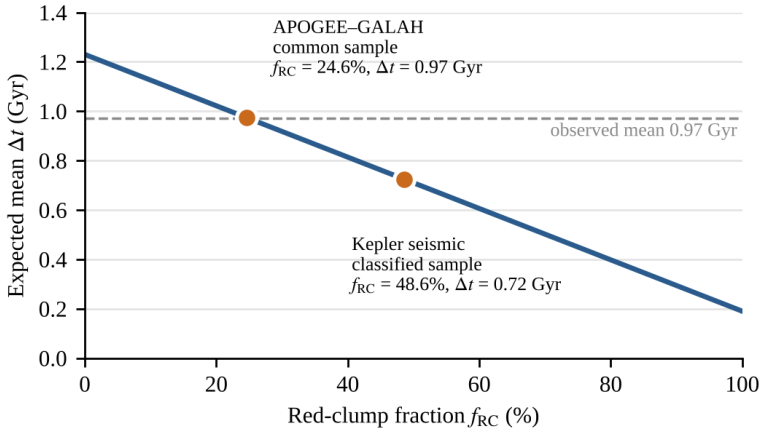


Figure 2. Expected mean age offset as a function of the red-clump fraction, Eq. (5)

The same mechanism explains an apparent inconsistency between two of our earlier reports. The Chirchiq contribution [23], which considered ascending giants only, quoted a median offset of +1.3 Gyr, whereas the full mixed sample [25] gave +0.95 Gyr. Both are correct; they describe different stage mixtures. Similarly, [23] described a dependence of the Δt distribution on metallicity, while [25] found a Pearson coefficient of only r = 0.01 with [Fe/H]. These statements also do not contradict each other: a linear correlation coefficient measures a trend in the mean, while [23] described a change in the width and symmetry of the distribution. Reporting which statistic was used is part of reporting the result.

3. Statistical versus systematic limits. With σ = 2.43 Gyr, Eq. (6) gives N_{crit} ≈ 6, 24 and 94 stars for systematic floors of 1.0, 0.5 and 0.25 Gyr (Figure 3). Since defensible choices move the mean by several tenths of a Gyr, the analysis choice dominates the error budget above about a hundred stars. For the full common sample the formal statistical error of the mean is only 2.43/√29,809 ≈ 0.014 Gyr – over fifty times smaller than the 0.79 Gyr multiverse range. Quoting the statistical error alone therefore overstates the precision of a catalogue-level offset by more than an order of magnitude.

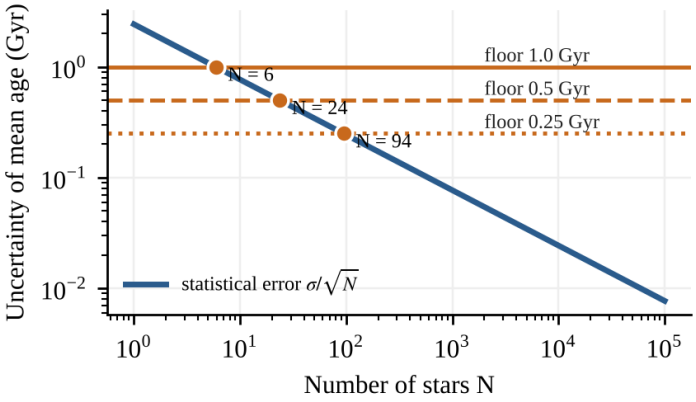


Figure 3. Statistical error of a mean age compared with fixed systematic floors

4. Cross-study survival audit. Figure 4 and Table 3 extend the comparison to the other studies. In each case a large part of the data does not survive a single analysis step. Only 20 % (95 % interval 12–32 %) of globular clusters have stable hard two-family labels [21], whereas 62 % (49–74 %) have a reproducible soft radial statistic: the choice between hard and soft assignment roughly triples the usable sample. Of the 1,850 Kepler giants, only 28.6 % (26.6–30.7 %) carry a seismic stage label, so any stage-resolved conclusion rests on a minority of the catalogue. In the APOGEE–GALAH comparison, 63 % of the cross-matched stars pass the quality cuts and 44 % also pass the temperature window [24, 25]. In the chemical-transport study [22], the outcome depended on the element: of the labels examined, only [Ni/Fe] met the equivalence rule in both directions, while the alpha-sequence morphology test remained indeterminate.

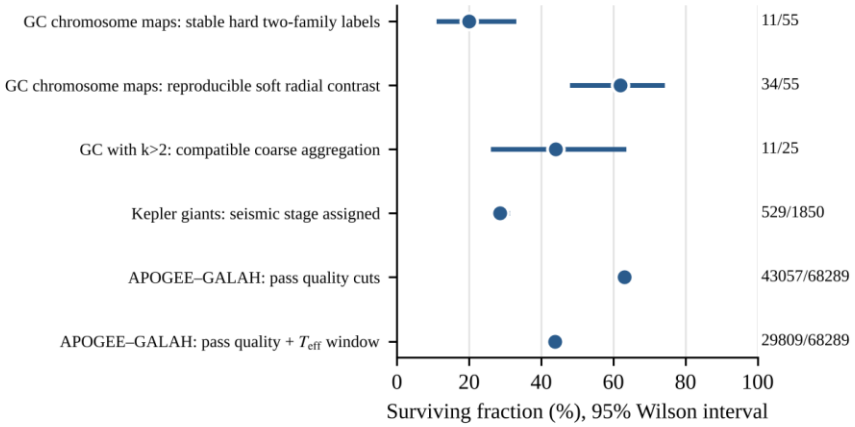


Figure 4. Fraction of objects surviving individual analysis steps in the four studies, with 95 % Wilson intervals

Table 3

Effect of single analysis choices on headline results across the four studies

Analysis choice	Alternatives compared	Effect on the reported result
T_{eff} window (ages)	28 contiguous windows	M_W from −0.66 to +2.71 Gyr; +0.42 to +1.21 Gyr at ≥50 % retention
RGB/RC composition (ages)	f_{RC} = 0–100 %	Mean offset from 1.23 to 0.19 Gyr
Summary statistic (ages)	mean trend vs distribution shape	[Fe/H] dependence absent (r = 0.01) or present
Population labels (GCs)	hard GMM labels vs posterior weights	11/55 vs 34/55 clusters usable
Radial statistic (GCs)	broad-family contrast vs extreme tails	0 of 4 tests robust vs ϱ = 0.500, p = 0.0026
Element / footprint (chemistry)	five labels; full vs balanced footprint	equivalence only for [Ni/Fe]
Stage label required (seismic)	labelled vs all stars	529 of 1,850 stars usable

Taken together, the four studies show a common pattern: the dependence on analysis choices is not a peculiarity of one data set but a structural property of red-giant population work. Two practical recommendations follow. First, every catalogue-level offset, gradient or correlation should be accompanied by the selection window, the stage composition and the summary statistic that produced it; Fig. 1 and Eq. (5) allow readers to translate the result to their own sample. Second, where several choices are equally defensible, the range over the multiverse – not the formal statistical error – should be quoted as the systematic uncertainty.

Limitations. The analysis is restricted to summary statistics published in the input studies; windows that do not coincide with the bin edges cannot be evaluated, and the true star-by-star multiverse may be somewhat wider. The stage-mixing model assumes that the RGB and RC offsets measured in the APOGEE–GALAH sample are transferable to the Kepler field, which has a different metallicity and mass distribution. Finally, the age comparison mixes survey and pipeline effects [25], so the present results describe the sensitivity of the catalogue-level comparison as encountered by users, not a decomposition of its causes.

Conclusions

1. For the APOGEE–GALAH common red-giant sample, the reported mean age offset varies from −0.66 to +2.71 Gyr over the 28 possible temperature windows and from +0.42 to +1.21 Gyr over the 13 windows that retain at least half of the stars; the analysis-choice spread is comparable to the offset itself.

2. The proposed cancellation index K_W reveals that the windows with the smallest mean offset owe it partly to cancellation of opposite-sign systematic errors (K_W up to 0.51 Gyr); a small mean is not evidence of agreement.

3. Because RGB and RC stars carry different offsets (1.23 and 0.19 Gyr), the same two catalogues would report 0.72 Gyr instead of 0.97 Gyr for a sample with the RC fraction of the Kepler seismic sample (48.6 %).

4. With a star-by-star scatter of 2.43 Gyr, analysis-choice systematics exceed the statistical error for samples larger than ≈24–94 stars; for the full sample the statistical error (0.014 Gyr) understates the real uncertainty by more than a factor of fifty.

5. Across globular-cluster, chemical and asteroseismic studies, single analysis steps retain between 20 % and 63 % of the data, and the choice between hard and soft classification changes the usable globular-cluster sample by a factor of three. Reporting results as multiverse ranges is recommended as standard practice for red-giant population studies.

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