CHARGED BRANS-DICKE WORMHOLES

IN SCALAR-TENSOR THEORIES

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Abstract

This article analyzes charged wormholes in the Brans-Dicke theory for the case $\omega=0$ within the framework of scalar-tensor theory. A comparative analysis carried out through the Einstein and Jordan frames demonstrates that, using conformal continuation, static and traversable wormholes can be obtained in the Jordan frame.

Keywords

Einstein and Jordan frames, Wormholes, Singularity, Brans-Dicke theory, Perturbation equations.

ЗАРЯЖЕННЫЕ ЧЕРВЕВОПОДОБНЫЕ ПЕРЕХОДЫ БРАНС–ДИК В СКАЛЯРНО-ТЕНЗОРНЫХ ТЕОРИЯХ

Аннотация

В данной статье анализируются заряженные кротовые норы в теории Бранс-Дик для случая $\omega=0$ в рамках скалярно-тензорной теории. Сравнительный анализ, проведённый через рамки Эйнштейна и Джордана, показывает, что с помощью конформного продолжения в рамке Джордана можно получить статические и проходимые кротовые норы.

Ключевые слова: рамки Эйнштейна и Джордана, кротовые норы, сингулярность, теория Бранс-Дик, уравнение возмущений.

Introduction

Although General Relativity (GR) provides a certain accuracy in explaining gravitational phenomena, it cannot resolve problems in cosmology and highenergy physics. To address these problems, generalized models of General Relativity are required. One such model is the Brans-Dicke [1] theory, which belongs to the class of scalar-tensor theories. In this theory, gravitational interaction is described not only by the metric but also through an additional scalar field. Unlike General Relativity, this theory allows for the possibility that the strength of gravitational interaction may not be the same at different points of spacetime. In particular, the case of the Brans-Dicke parameter $\omega = 0$ (in the Brans-Dicke theory, ω is a dimensionless parameter that determines the coupling strength between the scalar field and gravity, specifying the variation of the scalar field and the gravitational constant across spacetime) is of special theoretical interest, since in this case transitions between conformal frames can lead to the emergence of nontrivial geometric structures.

In this work, the $\omega = 0$ case of the Brans-Dicke theory is analyzed in an electrovacuum setting based on the general formulation of scalar-tensor theory. The spacetime interpretations in the Einstein and Jordan frames are compared, and the physical significance of the wormhole-type spacetime configurations arising from conformal continuation is discussed.

Brans-Dicke Theory

Scalar-tensor [2,3,6] theories belonging to the Bergmann-Wagoner-Nordtvedt class describe gravitational dynamics through the combined behavior of the metric and the scalar field. The action integral of the Brans-Dicke theory is written as follows: [7,8]

$$S_{BD} = \frac{1}{16\pi} \int \sqrt{-g}\, d^4 x \left[\phi R + \frac{\omega}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - 2U(\phi) + L_m \right], \qquad (1)$$

Here R is the scalar curvature of spacetime, $g = det\,(g_{\mu\nu})$, $\omega \neq -3/2$is the Brans-Dicke coupling parameter, $U(\phi)$ is the interaction potential of the scalar field, and L_m describes the Lagrangian of matter other than gravity.

After the conformal transformation,

$$S_{STN} = \frac{1}{16\pi} \int \sqrt{-\bar{g}}\, d^4 x [\bar{R} + 2\varepsilon \bar{g}^{\mu\nu} \psi_{,\mu} \psi_{,\nu} - \frac{2U(\phi)}{\phi^2} + \frac{L_m}{\phi^2}] \qquad (2)$$

Here, all quantities marked with a bar belong to the Einstein frame, and $\varepsilon = sign(\omega + 3/2)$determines whether the scalar field is canonical $(\varepsilon = +1)$or phantom $(\varepsilon = -1)$in nature. [4,5]

It is well known that, in the Einstein frame, exact static, spherically symmetric solutions with an electric field corresponding to the action (2) have been known since the 1970s. These solutions branch into several families. A complete classification of such solutions has been analyzed in detail in our work [2].

In this work, we focus only on the class of solutions that allow for conformal continuation. Such a solution exists in a single [+1] branch, corresponding to a canonical scalar field $(\varepsilon=+1)$. The harmonic radial coordinate is chosen so that the metric coefficients in the Einstein frame satisfy the condition $\alpha=2\beta+\gamma$. In this case, the metric takes the following form.

$$ds_E^2=\frac{h^2 dt^2}{q^2\sinh^2[h(u+u_1)]}-\frac{k^2 q^2\sinh^2[h(u+u_1)]}{h^2\sinh^2(ku)}\left[\frac{k^2 du^2}{\sinh^2(ku)}+d\Omega^2\right] \qquad (3)$$

$$\psi=Cu \qquad (4)$$

$$F_{\mu\nu}=h^2\frac{(\delta\text{¿¿}\mu 0\delta_{\nu 1}-\delta_{\nu 0}\delta_{\mu 1})}{q\sinh^2[h(u+u_1)]},\Rightarrow F_{\mu\nu}F^{\mu\nu}=\frac{-2q^2}{r^4}=\frac{-2h^4\sinh^4(ku)}{q^2k^4\sinh^4[h(u+u_1)]}\text{¿} \qquad (5)$$

Here q is the electric charge and C is the scalar charge [2].

The solutions (3)-(5) presented above are universal not only for General Relativity but also for any scalar-tensor theory in the Einstein frame. The corresponding solution in the Jordan frame of the Brans-Dicke theory is obtained through a conformal transformation.

As a result, the metric in the Jordan frame is written as follows:

$$ds_J^2=e^{-\sqrt{2}Cu/\varpi}\left[\frac{h^2 dt^2}{q^2\sinh^2[h(u+u_1)]}-\frac{4q^2\sinh^2[h(u+u_1)]}{\sinh^2(2hu)}\left(\frac{4h^2 du^2}{\sinh^2(\sinh^2(2hu))}+d\Omega^2\right)\right] \qquad (6)$$

We are particularly interested in the conformal continuation that arises when transitioning from the Einstein frame to the Jordan frame.

In the above metric (6) $e^{-\sqrt{2}Cu/\varpi}=e^{2hu}$ and the metric remains regular as $u=\infty$. Therefore, the Jordan spacetime M_J can be conformally continued beyond this limit. For this purpose, a new radial coordinate $y=e^{-2hu}$ which is finite at $u=\infty$ is introduced. As a result, the metric takes the following form:

$$ds_J^2=\frac{4h^2 dt^2}{[m+h-y(m-h)]^2}-\frac{4[m+h-y(m-h)]^2}{(1-y^2)^2}\left(\frac{4dy^2}{(1-y^2)^2}+d\Omega^2\right) \qquad (7)$$

Here, the new radial coordinate is defined in the range $y\in(-1,1)$. The quantity $m=\sqrt{h^2+q^2}$ corresponds to the Schwarzschild mass in the Einstein frame M_E. However, in the Jordan frame M_J due to the conformal factor, the mass takes a different value and is determined by the relation $m_J=\sqrt{h^2+q^2}-h=m-h$. The original spatial infinity corresponds to $u=0$ which maps to $y=1$ while $y=-1$ represents the second spatial infinity. Thus, the space time in the Jordan frame describes a static and traversable wormhole. Although it is asymptotically flat at both ends, the time flow is different, being determined by the relation

$$g_{00}^{J}\big|_{y=1}=1, \quad g_{00}^{J}\big|_{y=-1}=\frac{h^2}{m^2}=\frac{h^2}{h^2+q^2} \tag{8}$$

The throat is located at the point where the spherical radius $r_J=\sqrt{-g_{22}}$ attains its minimum.

$$y_{th}=\frac{\sqrt{m}-\sqrt{h}}{\sqrt{m}+\sqrt{h}}, \quad r_{th}=\left[\sqrt{m}+\sqrt{h}\right]^2 \tag{9}$$

Thus, in the case of a nonzero charge $q\neq 0$ the wormhole throat is asymmetric. If $q=0$ the metric (7) becomes symmetric, and it is convenient to use the spherical radius $r_J=4h/(1-y^2)$ as a result of which the metric takes the following form:

$$ds_J^2\big|_{q=0}=dt^2-\frac{dr_J^2}{1-\frac{4h}{r_J}}-r_J^2\,d\Omega^2 \tag{10}$$

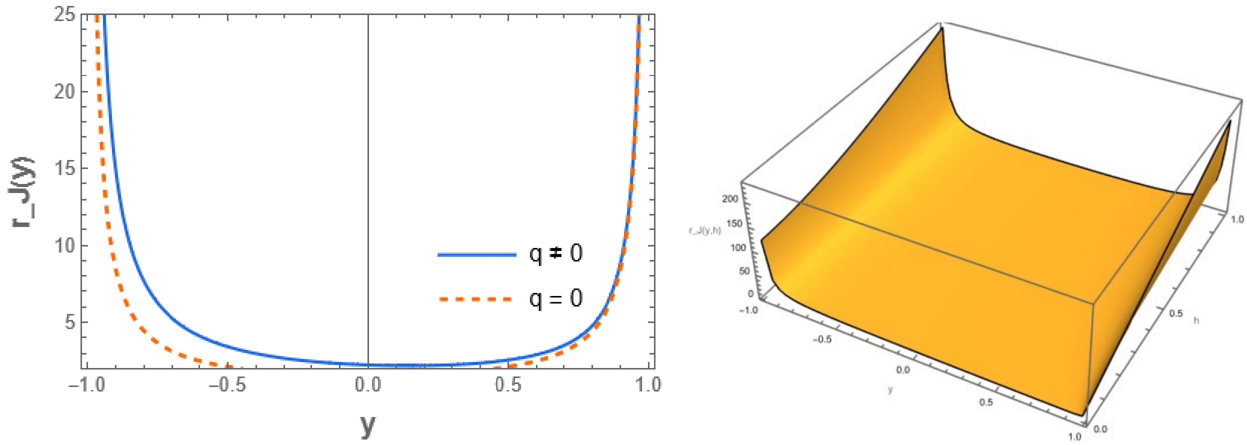

Figure 1. *The radius $r_J(y)$ for $y\in(-1,1)$ at different values of $h=\sqrt{m^2-q^2}$ with $m=1$. The upper curve corresponds to the case $h=1(q=0)$. The minimum point of each $r_J(y)$ function represents the throat of the wormhole.*

Figure 1 illustrates the behavior of the spherical radius $r_J(y)$ defined in the Jordan frame, over the interval $y\in(-1,1)$. As seen from the graph, the function $r_J(y)$ possesses a finite minimum, and this point identifies the throat of the wormhole. In the absence of electric charge $(q=0)$ the solution is symmetric and the throat is located at $(q=0)$. When a charge is introduced $(q\neq 0)$ the position of the throat shifts, and the spacetime geometry becomes asymmetric.

The problem of spherically symmetric perturbations

We now consider the problem of spherically symmetric perturbations for the charged wormhole solution. Following the approach adopted in previous studies, the perturbations in this work are analyzed on the basis of the background solution defined in the Einstein frame. This is because, precisely in the Einstein frame, the perturbation equations take a more compact and convenient form. At the same time, due to the geometric structure of the wormhole spacetime, it is necessary to treat the problem independently in two separate regions, the manifolds $M_{E+¿(y>0)¿}$ and $M_{E-¿(y<0)¿}$.

We consider the spacetime $M_{E-¿¿}$ which is the region in the Einstein frame that is conformally equivalent to the domain $y<0$ in the Jordan frame. For convenience, we continue to use y instead of $-y$ or $|y|$. As a result, the metric takes the following form:

$$ds_E^2=\frac{4h^2 y d t^2}{[m+h+y(m-h)]^2}\left(\frac{4 d y^2}{(1-y^2)^2}+d\Omega^2\right) \tag{11}$$

In this metric, there is a naked central singularity at $y=0$ and a spatial infinity at $y=1$ however, the Schwarzschild mass is now equal to h. The functions W and V_{eff} appearing in the equations are given in [5].

$$W_{-¿}(y)=\frac{6h^2(1-y)^4-6m^2(1+y)^4-48my(1-y^2)}{y\left[h(1-y)^3+m(1+y)^3\right]^2},¿ \tag{12}$$

$$V_{eff-¿}(y)=\frac{-h^2(1-y^2)^3}{16y^2[h+m+(m-h)y]^6\left[h(1-y)^3+m(1+y)^3\right]^2}[h^4(1-y)^7(1-25y-25y^2+y^3)-m^4(1+y)^7(-1-25y+25y^2+y^3)+6h^2m^2(1-y^2)^3(1+70y^2+y^4)+4h^3m(1-y)^4(1-12y+ \tag{13}$$

In expression (13), the function $V_{eff-¿(y)¿}$ represents the effective potential obtained for spherically symmetric perturbations in the spacetime $M_{E-¿¿}$. The shape of $V_{eff-¿(y)¿}$ plays a crucial role in determining the behavior of perturbations and the existence of stable or unstable regimes. Therefore, its dependence on the parameters m and h is analyzed separately in graphical form (Figure 2).

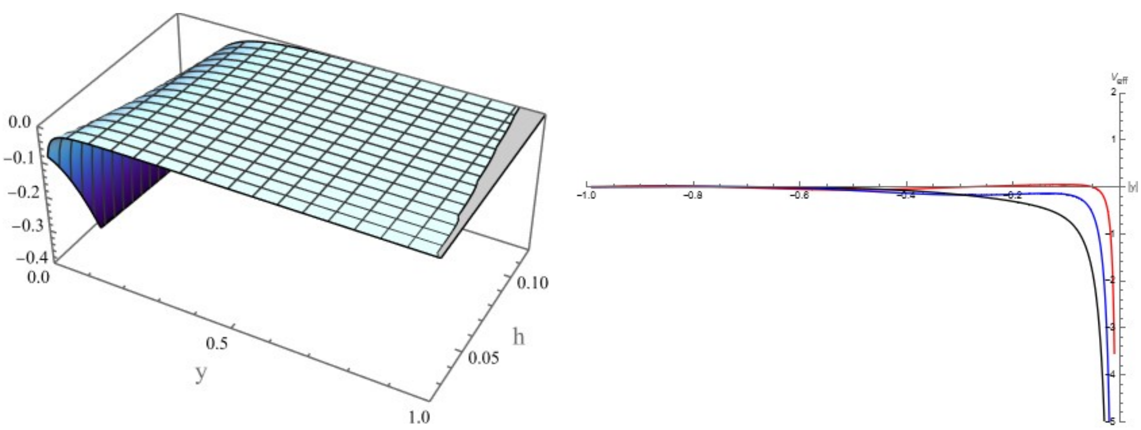

Figure 2. *The dependence of the effective potential* V_{eff} *for perturbations on* $|y|$ *in the spacetime* $M_{E-¿¿}$ *is shown, corresponding to the region* $y<0$*. The parameters are taken as* $m=1$ *and* $h=0.1,.035,1$*. Here, the case* $h=1$ *corresponds to* $q=0$

Figure 2 shows the dependence of the effective potential $V_{eff-¿¿}$on $¿y\vee¿$ for spherically symmetric perturbations in the spacetime $M_{E-¿¿}$ $(y<0)$. As seen from the graph, in the region $M_{E-¿¿}$ the behavior of the potential strongly depends on the spacetime geometry: it varies sharply near the central singularity $(y\to 0)$ and decays toward spatial infinity $(y\to 1)$.

Conclusion

In this work, electrovacuum spacetime configurations in the Brans-Dicke theory for the case $\omega=0$ were analyzed within the framework of scalar-tensor theory. A comparative analysis carried out in the Einstein and Jordan frames has shown that, through conformal continuation, static and traversable wormholes arise in the Jordan frame. These wormholes are asymptotically flat at both ends.

In charged cases, the wormhole throat is asymmetric and is characterized by the maximum and minimum values of the spherical radius. Furthermore, the perturbation analysis carried out in the Einstein frame shows that the perturbation equations and the effective potential V_{eff}vary at different points of spacetime, which is of crucial importance for assessing the stability of wormholes.

Overall, this work scientifically demonstrates that, in the Brans-Dicke theory with the parameter $\omega=0$ conformal continuation gives rise to new geometric structures in both charged and uncharged wormholes, describing their spherically symmetric spacetime configurations and stability properties.

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