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ESTIMATION OF THE ASYMPTOTIC NORMALIZATION COEFFICIENT (ANC) FOR THE VIRTUAL DECAYS $^{8}B \rightarrow {}^{7}Be + p$ and $^{8}Li \rightarrow {}^{7}Li + n$, WITHIN THREE-BODY MODEL

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Abstract: *The asymptotic normalization coefficient (ANC) of the virtual decay $^{8}B \rightarrow {}^{7}Be + p$ is one of the most important quantities for estimating the cross section and the rate of the radiative capture reaction $^{7}Be(p,\gamma)^{8}B$, a key process of the solar proton–proton chain and a major source of the high-energy ^{8}B solar neutrino flux. . This paper is devoted to the estimation of the ANC within a self-consistent three-body $\alpha+^{3}He+p$ potential cluster model based on the hyperspherical Lagrange-mesh method. The methodology of extracting the ANC from the asymptotic behavior of the overlap integral between the three-body ^{8}B (^{8}Li) and two-body $^{7}Be = \alpha + {}^{3}He$ ($^{7}Li = \alpha + {}^{3}H$) wave functions is described, and the resulting values are compared with earlier ab initio, cluster-model, and transfer-reaction estimates.*

Keywords: *asymptotic normalization coefficient, ^{8}B and ^{8}Li mirror nuclei, three-body cluster model, hyperspherical Lagrange-mesh method, capture reaction $^{7}Be(p, \gamma)^{8}B$.*

ОЦЕНКА АСИМПТОТИЧЕСКОГО НОРМИРОВОЧНОГО КОЭФФИЦИЕНТА (ANC) ДЛЯ ВИРТУАЛЬНЫХ РАСПАДОВ $^{8}B \rightarrow {}^{7}Be + p$ И $^{8}Li \rightarrow {}^{7}Li + n$ В РАМКАХ ТРЁХЧАСТИЧНОЙ МОДЕЛИ

Аннотация: *Асимптотический нормировочный коэффициент (ANC) виртуального распада $^{8}B \rightarrow {}^{7}Be + p$ является одной из ключевых величин для оценки сечения и скорости радиационной реакции захвата $^{7}Be(p, \gamma)^{8}B$. Данная реакция играет важную роль в протон-протонной цепочке термоядерных реакций Солнца и является основным источником высокоэнергетического потока солнечных нейтрино от распада ^{8}B. Настоящая работа посвящена теоретическоой оценке асимптотического нормировочного коэффициента, в рамках самосогласованной трёхчастичной потенциальной кластерной модели $\alpha+^{3}He+p$, основанной на методе гиперсферической Лагранжевой сетки. Описана методика извлечения ANC из асимптотического поведения интеграла перекрытия между трёхчастичной волновой функцией ядра ^{8}B (^{8}Li) и двухчастичной волновой функцией $^{7}Be = \alpha + {}^{3}He$ ($^{7}Li = \alpha + {}^{3}H$). Полученные значения асимптотических нормировочных коэффициентов сопоставлены с результатами, полученными ранее в рамках ab initio-подходов, кластерных моделей и методов, основанных на реакциях переноса.*

Ключевые слова: *асимптотический нормировочный коэффициент (ANC), зеркальные ядра ^{8}B и ^{8}Li, трёхчастичная потенциальная кластерная модель, гиперсферический метод Лагранжевой сетки, реакция захвата $^{7}Be(p, \gamma)^{8}B$.*

UCH-JISM MODEL DOIRASIDA $^{8}B \rightarrow {}^{7}Be + p$ VA $^{8}Li \rightarrow {}^{7}Li + n$ VIRTUAL PARCHALANISHLARI UCHUN ASIMPTOTIK NORMALLASHTIRISH KOEFFITSIYENTINI (ANC) BAHOLASH

Annotatsiya: *$^{8}B \rightarrow {}^{7}Be + p$ virtual parchalanishining asimptotik normallashtirish koeffitsiyenti (ANC) $^{7}Be(p, \gamma)^{8}B$ radiatsion qamrash reaksiyasining kesimi va tezliklarini baholash uchun muhim kattaliklardan biri hisoblanadi. Mazkur reaksiya Quyoshdagi proton-proton termoyadro reaksiyalari zanjirining muhim bosqichlaridan biri bo‘ lib, yuqori energiyali ^{8}B Quyosh neytrinolari oqimining asosiy manbalaridan biridir. Ushbu ish ANC qiymatini gipersferik Lagranj to‘ r usuliga asoslangan o‘ z-o‘ ziga mos uch-jism $\alpha+^{3}He+p$ potensial klaster modeli doirasida baholashga bag‘ ishlangan. ANC ni uch-jismli ^{8}B (^{8}Li) yadrosining to‘ lqin funksiyasi bilan ikki-jismli $^{7}Be = \alpha+^{3}He$ ($^{7}Li = \alpha+^{3}H$) yadrosining to‘ lqin funksiyasi orasidagi qoplanish integralining*

asimptotikasidan ajratib olish metodikasi tavsiflanadi. Olingan asimptotik normallashtirish koeffitsiyentlari ab initio yondashuvlari, klaster modellari hamda uzatish reaksiyalari yordamida ilgari olingan natijalar bilan taqqoslanadi.

Kalit so‘ zlar: *asimptotik normallashtirish koeffitsiyenti, ^{8}B va ^{8}Li ko‘ zgu yadrolari, uch-jism potensial klaster modeli, gipersferik Lagranj to‘ r usuli, $^{7}Be(p,\gamma)^{8}B$ qamrash reaksiyasi.*

Introduction

The proton-radiative-capture reaction $^{7}Be(p,\gamma)^{8}B$ governs the production of high-energy ^{8}B solar neutrinos within the proton–proton chain of hydrogen burning in the Sun [1,2]. Because the relevant stellar energies lie far below the Coulomb barrier, the cross section of this reaction cannot be measured directly at astrophysically relevant energies. A central quantity in such an extrapolation is the asymptotic normalization coefficient (ANC), which fixes the amplitude of the tail of the bound-state wave function of the valence proton in ^{8}B beyond the range of the nuclear interaction. Since ^{8}B is a weakly bound one-proton halo nucleus with a separation energy of only about 0.1375 MeV, the capture process at low energies is dominated by this peripheral, long-range part of the wave function, so that the cross section becomes essentially proportional to the square of the ANC. The estimation of this coefficient accurately is therefore equivalent to determining the low-energy capture rate itself. This work reviews the problem of estimating the ANC for the $^{8}B \rightarrow {}^{7}Be + p$ virtual decay, with emphasis on a three-body potential cluster model in which ^{8}B is treated as an $\alpha + {}^{3}He + p$ system and ^{8}Li as $\alpha + {}^{3}H + n$, allowing a consistent study of both the ^{8}B and the ^{8}Li mirror nuclei. The aim is to summarise the physical basis of the approach, the main results obtained for the ANC.

Literature review

Empirical ANC values for $^{8}B \rightarrow {}^{7}Be + p$ have traditionally been extracted from peripheral nuclear transfer reactions analyzed within the distorted-wave Born approximation (DWBA). Proton-transfer measurements of reactions such as $^{10}B(^{7}Be,^{8}B)^{9}Be$ and $^{14}N(^{7}Be,^{8}B)^{13}C$ give a squared ANC of about 0.465 fm^{-1}, while an analysis of the $^{7}Be(d,n)^{8}B$ reaction yields a noticeably larger value about 0.613 fm^{-1}. Coulomb-breakup measurements of ^{8}B on heavy targets have produced intermediate results, typically in the range 0.45–0.55 fm^{-1}.

On the theoretical side, ab-initio calculations provide an independent route to the ANC. Variational Monte Carlo calculations with realistic Argonne v_{18} and Urbana IX potentials give a squared ANC about 0.537 fm^{-1}, close to the value obtained with the no-core shell model combined with the resonating-group method (NCSM/RGM), about 0.509 fm^{-1}. Earlier NCSM calculations using the CD-Bonn 2000 nucleon–nucleon potential produced a larger value, about 0.587 fm^{-1}. At the same time a more recent NCSM-with-continuum calculation based on chiral effective-field-theory potentials at fourth order, including three-nucleon forces, notably fails to reproduce the experimental S-wave $p + {}^{7}Be$ scattering length, a concern since the S-wave channel dominates the low-energy capture process and strongly constrains the reliability of the underlying wave function, and hence of the extracted ANC [1,2].

Halo effective field theory at next-to-leading order gives a squared ANC of 0.564 fm^{-1}, in reasonable agreement with the VMC and CD-Bonn NCSM results. A two-body effective-potential model, tuned to reproduce the measured S-wave $p + {}^{7}Be$ scattering length, yields a squared ANC of about 0.538 fm^{-1}, consistent with several of the ab initio estimates, though it does not explicitly include ^{7}Be core-excitation effects.

Three-body cluster-model calculations of the ^{8}B ANC were first carried out within the hyperspherical harmonics method, but were restricted to a maximal hypermomentum of $K_{max} = 12$, which is insufficient to guarantee convergence for such a weakly bound halo system. This limitation motivates renewed three-body calculations carried to substantially larger K_{max}.

For the mirror decay $^{8}Li \rightarrow {}^{7}Li + n$, the main empirical constraint is a squared ANC of 0.432 ± 0.044 fm^{-1}, obtained from a DWBA analysis of the $^{13}C(^{7}Li,^{8}Li)^{12}C$ neutron-transfer reaction [3]. This value is used below as the principal point of comparison for the present three-body ^{8}Li result, and is noticeably smaller than the three-body cluster-model estimate, a discrepancy discussed further in the results section.

Overall, the literature shows that squared ANC estimates for $^{8}B \rightarrow {}^{7}Be + p$ span a fairly wide range from about 0.45 fm^{-1} obtained from transfer-reaction analyses up to about 0.59 fm^{-1} from CD-Bonn-based NCSM and related ab initio calculations underlining the continuing need for independent, well-converged theoretical estimates of this coefficient.

Research Methodology

The structure of the ^{8}B nucleus is described within a three-body α+^{3}He+p potential cluster model. The two-body interactions between the constituent clusters are represented by phenomenological potentials, which are chosen to reproduce the corresponding two-body scattering and bound-state properties.

The α–^{3}He, $\alpha - N$ and 3He-N interaction potentials are particularly important for the present ANC analysis because they determine the structure of the ^{7}Be(^{7}Li) core and ^{8}B(^{8}Li) halo nucleus. They are described by a Gaussian nuclear potential supplemented by the Coulomb interaction,

$$V(r) = V_0 exp(-\alpha_0 r^2) + V_C(r). \quad (1)$$

The parameters of the α–N, $^3He(^3H)$-N and α–$^3He(^3H)$ potential are given in Refs. [4], [5], [6], respectively. The potential depth V_0 and width α_0 parameters fitted to the scattering and bound state data for each partial wave. The resulting three-body wave function is subsequently used to calculate the ^7Be+p overlap function, whose asymptotic behavior provides the ANC.

The three-body geometry underlying is defined through the relative coordinates **r** (between the α-particle and ^3He) and **R** (between the ^7Be core and the proton), illustrated schematically in figure.

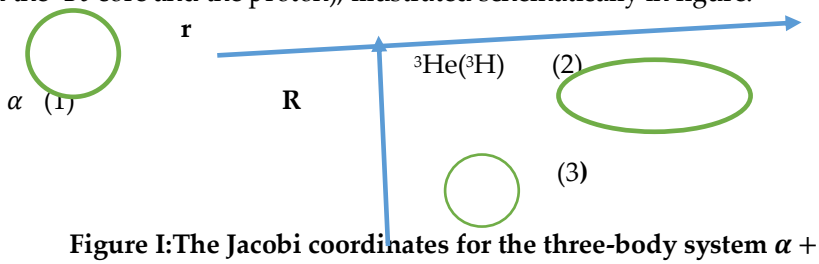


Figure I:The Jacobi coordinates for the three-body system $\alpha + ^3He(^3H)+p(n)$

The ANC values are extracted from the matching of the overlap integral presented below with its asymptotics described by the Whittaker function: $I_{l_R I}(R) = \langle \Psi_3(\vec{r},\vec{R},\xi_2,\xi_3) \,|\psi_I(\vec{r},\vec{R},\xi_2,\xi_3)\rangle = C_I W_{\eta;l_{R+1/2}}(2kR)/R$ (2)

where W is known Whittaker function, k is the wave number, η is the Sommerfeld parameter, and

$$\psi_I(\vec{r},\hat{R},\xi_2,\xi_3) = \frac{u_2(r)}{r} \times \{\{[Y_l(\hat{r}) \otimes \chi_{1/2}(\xi_3)] \otimes \chi_{1/2}(\xi_2)\} \otimes Y_{l_R}(\hat{R})\}_{2M} \quad (3)$$

is an effective wave function with $u_2(r)$/r the $^7Be = \alpha + ^3He$ ($^7Li = \alpha + ^3H$) two-body radial wave function, and ξ_2, ξ_3 the spin variables of the valence nucleon and of the ^3He (^3H) cluster, respectively. Since the orbital momentum $l_R = 1$ is fixed throughout, the resulting coefficient is denoted simply as C_I for each spin channel. The ANC is then obtained self-consistently from Eq. (2) as $C_I = I_I(R) \cdot R / W_{\eta,l_R+1/2}(2\kappa R)$, evaluated at asymptotic distances R between the core and the valence nucleon; the physically meaningful value of C_I is identified from the stable plateau region of this ratio as a function of the matching point R_0, as illustrated in Fig. II below.

Within the present three-body model, the overlap integral $I_I(R)$ reproduces the expected Whittaker-function asymptotic behaviour over the interval $R \approx 3$–10 fm. For the $^8B(2^+)$ ground state, the spin-1 channel asymptotics is matched at $R_0 = 4$ fm, while the dominant spin-2 channel is matched at $R_0 = 6$ fm; for the mirror $^8Li(2^+)$ ground state, the corresponding matching points are $R_0 = 4.5$ fm and $R_0 = 8.5$ fm, respectively.

Results and discussion

Table I compares the calculated bound-state energies with the corresponding experimental values. The obtained results are in reasonable agreement with the experimental data, providing a consistent description of the ^8B and ^8Li bound states used in the subsequent ANC analysis.

	^8B	Exp	^8Li	Exp
$E_b(2^+)$	-1.718	-1.724 [7]	-4.340	-4.499 [7]
$E_b(1^+)$	-1.075	-0.954 [7]	-3.559	-3.518 [7]
$\langle r^2_{mat}\rangle^{1/2}$	2.76	2.58(6) [8]	2.56	2.50 [9]

Table-I: Calculated bound state energies (MeV) and matter radii (fm)

The proton separation energy of the ^8B ground state is an important quantity for the determination of the ANC of the $^8B \rightarrow ^7Be$+p virtual transition. In the three-body model, the proton separation energy is calculated from the difference between the binding energies of ^8B and ^7Be:

$$E_p = E_b(8B) - E_b(7Be) \quad (3)$$

The calculated values depend strongly on the maximal hypermomentum K_{max}. The ^7Be binding energy obtained with the same α–^3He interaction is $E_b(^7Be)$=1.5924 MeV. The dependence of the calculated ^8B binding energy and proton separation energy on K_{max} is presented in Table II.

K_{max}	8	12	16	20	22	Exp
$E_b(^8B)$ (MeV)	1.3901	1.6221	1.6895	1.7121	1.7175	1.7236
E_p (MeV)	0.2023	0.0297	0.0971	0.1197	0.1251	0.1375

$E_b(^7Be)$ (MeV)	4.1005	4.2846	4.3274	4.3380	4.3399	4.4992
E_n (MeV)	1.6349	1.8190	1.8618	1.8724	1.8743	2.0323

Table-II: Dependance of the of the calculated binding and separation energies of ^8B and ^8Li nuclei on the maximal hypermomentum K_{max}

As shown in Table II, the calculated proton separation energy increases systematically with increasing *Kmax*, from −0.2023 MeV at *Kmax*=8 to 0.1251 MeV at *Kmax*=22 approaching the experimental value of 0.1375 MeV. The results indicate that sufficiently large *Kmax* is required to achieve a stable description of the weakly bound 8B ground state. In particular, the values obtained at *Kmax*=20 and 22 demonstrate improved convergence and are therefore used as a basis for the subsequent ANC analysis. By contrast, the neutron separation energy of ^8Li converges markedly faster, already at $Kmax \approx 12$, reflecting the absence of the Coulomb barrier between the ^7Li core and the neutron.

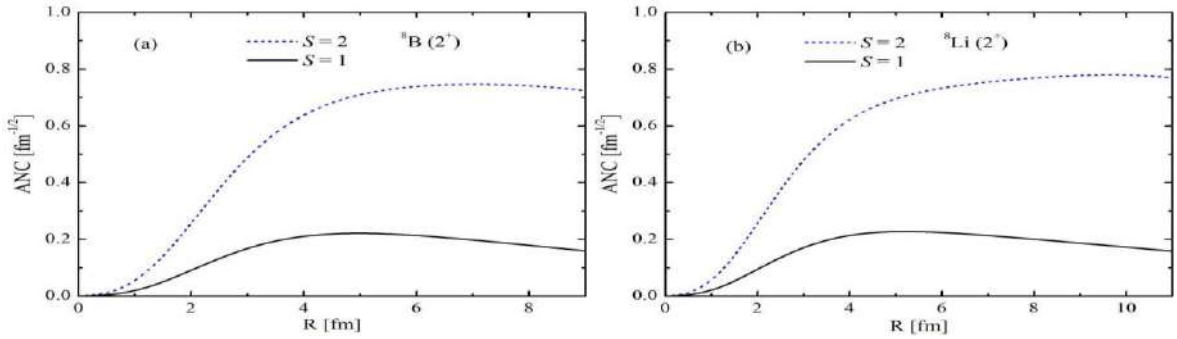


Figure II: Calculated ANC values for the ground (2^+) states of the ^8B and ^8Li nuclei in dependence on the matching point of the overlap integral with it's asymptotics

For the $^8B \rightarrow ^7Be + p$ decay, the extracted ANC values are $C_1 = 0.211$ fm$^{-1/2}$ in the spin-1 channel and $C_2 = 0.739$ fm$^{-1/2}$ in the spin-2 channel, giving a total squared ANC $C^2(^8B) = 0.591$ fm^{-1}. This lies above the ab initio NCSM/RGM value (~0.509 fm^{-1}) [10] but is consistent, within uncertainties, with the CD-Bonn-based NCSM result and with halo effective field theory. For the mirror decay $^8Li \rightarrow ^7Li + n$, the corresponding values are $C_1 = 0.220$ fm$^{-1/2}$ and $C_2 = 0.774$ fm$^{-1/2}$, giving a total squared ANC $C^2(^8Li) = 0.648$ fm^{-1} which is noticeably above the empirical transfer-reaction estimate of about 0.432 fm^{-1} obtained from the $^{13}C(^7Li,^8Li)^{12}C$ reaction [11].

The α+$^3He(^3H)$+$p(n)$ three-body cluster model, based on the hyperspherical Lagrange-mesh method and realistic two-body interactions, yields a consistent description of the structure of ^8B and ^8Li, with good convergence of binding energies (with K_{max} =22 for ground states, and K_{max}=28 for excited states) and matter radii. The ANCs were extracted self-consistently by matching the overlap integral of the three-body and two-body wave functions to its asymptotics expressed by the Whittaker functions.

Conclusion

The asymptotic normalization coefficients of the virtual decays $^8B \rightarrow ^7Be + p$ and $^8Li \rightarrow ^7Li + n$ can be determined self-consistently within a three-body $\alpha + ^3He(^3H) + p(n)$ potential cluster model, by matching the overlap integral of the three-body and two-body wave functions to its Whittaker-function asymptotics at large core–nucleon separation. Carried out with a sufficiently large hyperspherical basis (*Kmax* up to 22–28), this approach yields converged binding energies, nucleon separation energies, and matter radii for both ^8B and ^8Li mirror nuclei. The developed method yields ANC square values — about 0.591 fm^{-1} for ^8B and 0.648 fm^{-1} for ^8Li —broadly consistent with several ab initio and effective-field-theory estimates, while differing from certain transfer-reaction extractions, particularly for ^8Li.

The comparison of the two mirror ANCs points to a measurable breaking of the mirror symmetry of the nuclear force, attributable to the interplay of the Coulomb interaction and three-body dynamical effects. Together with the converged proton and neutron separation energies and matter radii, these ANC values provide a consistent and self-contained basis for further studies of the low-energy $^7Be(p,\gamma)^8B$ and $^7Li(n,\gamma)^8Li$ capture reactions.

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D. S. Toshova: Software, Investigation.

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