NAMANGAN DAVLAT UNIVERSITETI
ILMIY AXBOROTNOMASI, [2026-8]
ISSN:2181-1458
ISSN:2181-0427

01.00.00-FIZIKA-MATEMATIKA
journal.namdu.uz
PHYSICS AND MATHEMATICS

UDK 539.3

UCH QATLAMLI KONUSSIMON QOBIQNING BO‘ YLAMA-RADIAL TEBRANISH TENGLAMALARI

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Annotatsiya. *Ushbu ishda transversal-izotrop uch qatlamli konussimon qobiqning bo‘ ylama-radial teranishlari masalasi qaraladi. Masala o‘ qqa simmetrik masala deb faraz qilinadi. Uch qatlamli konussimon qobiqqa silindrik koordinatalar sistemasi joylashtiriladi. Uch qatlamli konussimon qobiqning yon sirtidagi chegaraviy shartlardan foydalanilib bo‘ ylama-radial tebranish tenglamalari keltirib chiqariladi. Keltirib chiqarilgan bo‘ ylama-radial tebranish tenglamalarida uch qatlamli konussimon qobiqning markaziy o‘ qidan ixtiyoriy masofada joylashgan sirtdagi ko‘ chish vektori komponentalari izlanuvchi funksiyalar sifatida kiritiladi. So‘ ngra ichki va tashqi sirtdagi ko‘ chish va kuchlanishlar ham shu kiritilgan izlanuvchi funksiyalar orqali ifodalanadi.*

Kalit so‘ zlar: *qatlam, transversal-izotrop material, bo‘ ylama-radial tebranish, chegaraviy shart, tebranish tenglamasi, ko‘ chish, kuchlanish*

УРАВНЕНИЯ ПРОДОЛЬНО-РАДИАЛЬНЫХ КОЛЕБАНИЙ ТРЕХСЛОЙНОЙ КОНИЧЕСКОЙ ОБОЛОЧКИ

Аннотация. *В данной статье рассматривается задача о продольно-радиальных колебаниях поперечно изотропной трехслойной конической оболочки. Предполагается, что задача является осесимметричной. В трехслойную коническую оболочку вводится цилиндрическая система координат. Используя граничные условия на боковой поверхности трехслойной конической оболочки, выводятся уравнения продольно-радиальных колебаний. В выведенных уравнениях продольно-радиальных колебаний компоненты вектора смещения на поверхности, расположенной на произвольном расстоянии от центральной оси трехслойной конической оболочки, вводятся в качестве искомых (неизвестных) функций. Затем смещения и напряжения на внутренней и внешней поверхностях также выражаются через эти введенные искомые функции.*

Ключевые слова: *слой, поперечно изотропный материал, продольно-радиальные колебания, граничное условие, уравнение колебаний, смещение, напряжение*

LONGITUDINAL-RADIAL VIBRATION EQUATIONS OF A THREE-LAYERED CONICAL SHELL

Annotation. *This paper considers the problem of longitudinal-radial vibrations of a transversely isotropic three-layered conical shell. The problem is assumed to be axisymmetric. A cylindrical coordinate system is introduced into the three-layered conical shell. Using the boundary conditions on the lateral surface of the three-layered conical shell, the longitudinal-radial vibration equations are derived. In the derived longitudinal-radial vibration equations, the components of the displacement vector at a surface located at an arbitrary distance from the central axis of the three-layered conical shell are introduced as the sought (unknown) functions. Then, the displacements and stresses on the inner and outer surfaces are also expressed through these introduced sought functions.*

Keywords: *layer, transversely isotropic material, longitudinal-radial vibration, boundary condition, vibration equation, displacement, stress*

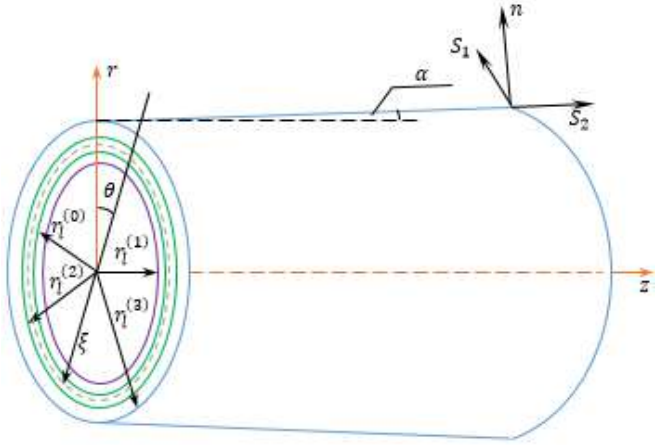
Kirish

Zamonaviy texnika va qurilish sohalarida yengil va yuqori mustahkamlikka ega konstruktiv elementlarga bo‘ lgan ehtiyoj tobora ortib bormoqda. Shu sababli uch qatlamli konussimon qobiq konstruksiyalari aviatsiya, gidrotexnika

inshootlari, transport vositalari hamda qurilish sanoatida keng qo‘ llanilmoqda. Bunday konstruksiyalarning asosiy afzalligi nisbatan kam og‘ irlikda yuqori qattiqlik va tebranishlarni so‘ ndirish xususiyatiga ega bo‘ lishidir. Konussimon shaklga ega qobiqlar, o‘ z navbatida, aylanma simmetriyaga ega bo‘ lgan turli texnik qurilmalarda alohida o‘ rin tutadi, chunki geometrik parametrlarning o‘ zgaruvchanligi tufayli ularning dinamik xarakteristikalari silindrik qobiqlarnikidan sezilarli farq qiladi. Bir qatlamli qobiqlarning tebranish nazariyasi klassik ishlarda yetarlicha o‘ rganilgan bo‘ lsa-da, ko‘ p qatlamli xususan, uch qatlamli qobiqlarning dinamik masalalari alohida yondashuvni talab qiladi, chunki qatlamlar orasidagi mexanik xossalarning farqi hisobga olinishi zarur. Uch qatlamli qobiqlarning statik va dinamik masalalari bir qator tadqiqotchilar tomonidan o‘ rganilgan. Xususan, Grigolyuk va Chulkovning [1] klassik ishlarida uch qatlamli qobiqlarning turg‘ unligi va tebranishlari, shu jumladan uch qatlamli silindrik va konussimon qobiqlarning kritik yuklamalari masalalari batafsil tadqiq etilgan. Ilyamov va hammualliflarining tadqiqotlarida elastik to‘ ldiruvchiga ega qobiqlarning mustahkamligi, turg‘ unligi va dinamikasi masalalari ko‘ rib chiqilgan [2], Latifovning ishida esa elastik va suyuq muhit bilan o‘ zaro ta’sirlashuvchi qobiqlarning tebranishlari o‘ rganilgan [3]. So‘ nggi yillarda O‘ zbekiston olimlari tomonidan ham bu yo‘ nalishda faol ishlar olib borilmoqda. Xususan, uch qatlamli qovushoq-elastik konussimon qobiqlarning tebranishlari masalasiga bag‘ ishlangan tadqiqotda harakat tenglamalari o‘ rta sirt siljishining asosiy komponentlariga nisbatan chiqarilgan hamda olingan yechimlar tahlil qilingan. [4] ishda sayoz o‘ rnatilgan uch qatlamli sferik qobiqlarning erkin tebranishlari tadqiq etilgan. Xudoynazarov va hammualliflari tomonidan qovushoq siqiluvchi suyuqlik bilan to‘ ldirilgan elastik silindrik qobiqning bo‘ ylama-radial tebranishlari masalasi o‘ rganilgan [5]. Bu ish, xususan, bo‘ ylama va radial tebranishlarni birgalikda tavsiflovchi tenglamalar tuzilishi jihatidan mazkur tadqiqotga metodik jihatdan yaqin turadi.

Shunga qaramay, uch qatlamli konussimon qobiqning bo‘ ylama-radial tebranishlarini tavsiflovchi to‘ liq shakllangan tenglamalar sistemasini qatlamlarning turli mexanik-geometrik parametrlarini hisobga olgan holda tahlil qilish masalasi to‘ liq yechilmagan bo‘ lib qolmoqda, bu esa mazkur tadqiqotning dolzarbligini belgilaydi.

Masalaning qo‘ yishi. Ichki hamda tashqi sirtlariga bo‘ ylama va radial kuchlar ta’sir qiluvchi uch qatlamli transversal-izotrop konussimon qobiqning bo‘ ylama-radial tebranishlarini tadqiq qilamiz. (r, θ, z) silindrik koordinatalar sistemasi boshini uch qatlamli konussimon qobiqning kichik ko‘ ndalang kesimiga joylashtiramiz (1-rasm).



1-rasm. Uch qatlamli konussimon qobiq

Bunda Oz – o‘ qini uch qatlamli konussimon qobiqning simmetriya o‘ qi bo‘ ylab, Or – o‘ qini uch qatlamli konussimon qobiqning ko‘ ndalang kesimi radiusi bo‘ ylab yo‘ naltiramiz. Bu yerda z-bo‘ ylama, r-radial va θ-burchak koordinatalaridir [6]. Uch qatlamli konussimon qobiqning qatlamlari materiallari transversal-izotrop deb hisoblaymiz.

Konussimon qobiqning yasovchisibilan markaziy o‘ qi orasidagi burchakni α bilan, uzunligini l bilan va $z = 0$ kichik ko‘ ndalang kesimi ichki radiusini r_0 bilan belgilaymiz. Ichki qatlamning qalinligi h_1, o‘ rta qatlamning qalinligi h_0 va tashqi qatlamning qalinligi h_2 bo‘ lsin deb faraz qilamiz. U holda $z = 0$ kesimdan l masofadagi kesim uchun ichki radius, ya’ni ichki qatlamning ichki radiusi $r_l^{(1)} = r_0 + l\tan\alpha$, ichki qatlamning tashqi radiusi ya’ni o‘ rta qatlam uchun ichki radius $r_l^{(0)} = r_0 + l\tan\alpha + h_1$, o‘ rta qatlamning tashqi radiusi ya’ni tashqi qatlam uchun ichki radius $r_l^{(2)} = r_0 + l\tan\alpha + h_1 + h_0$ va tashqi qatlam uchun tashqi radius $r_l^{(3)} = r_0 + l\tan\alpha + h_1 + h_0 + h_2$ bo‘ ladi. Ushbu uch o‘ lchovli uch qatlamli konussimon qobiq jismi elastiklik nazariyasi qonunlariga bo‘ ysinadi deb hisoblaymiz.

Masala o‘ qqa nisbatan simmetrik bo‘ lganligi uchun uch qatlamli konussimon qobiq nuqtalari ko‘ chish vektorlarining bo‘ ylama va radial komponentlari, kuchlanish tenzori komponentalarining bo‘ ylama, radial, buralma hamda bo‘ ylama-radial komponentalari, deformatsiya tenzorining esa $\varepsilon_{rr}^{(m)}, \varepsilon_{\theta\theta}^{(m)}, \varepsilon_{zz}^{(m)}$ va $\varepsilon_{rz}^{(m)}$ komponentalari noldan farqli bo‘ ladi. Qolgan komponentlari esa nolga teng bo‘ ladi.

Transversal-izotrop uch qatlamli konussimon qobiq nuqtalarida kuchlanishlar va deformatsiyalar orasida quyidagi bog‘ lanishlar mavjud bo‘ ladi [8]:

$$\sigma_{rr}^{(m)} = C_{11}^{(m)}\frac{\partial U_r^{(m)}}{\partial r} + C_{12}^{(m)}\frac{U_r^{(m)}}{r} + C_{13}^{(m)}\frac{\partial U_z^{(m)}}{\partial z};$$
$$\sigma_{\theta\theta}^{(m)} = C_{12}^{(m)}\frac{\partial U_r^{(m)}}{\partial r} + C_{11}^{(m)}\frac{U_r^{(m)}}{r} + C_{13}^{(m)}\frac{\partial U_z^{(m)}}{\partial z};$$
$$\sigma_{zz}^{(m)} = C_{13}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial r} + \frac{U_r^{(m)}}{r}\right) + C_{33}^{(m)}\frac{\partial U_z^{(m)}}{\partial z}; \quad (1)$$
$$\sigma_{rz}^{(m)} = C_{44}^{(m)}\left(\frac{\partial U_r^{(m)}}{\partial z} + \frac{\partial U_z^{(m)}}{\partial r}\right).$$

bu yerda $C_{ij}^{(m)}$-transversal-izotrop material uchun elastiklik o‘ zgarmaslari.

Harakat tenglamalariga (1) ifodalarni qo‘ yish orqali, uch qatlamli konussimon qobiq qatlamlari nuqtalarining $U_z^{(m)}$- bo‘ ylama va $U_r^{(m)}$- radial ko‘ chishlarini ifodalovchi quyidagi harakat tenglamalari sistemasiga ega bo‘ lamiz:

$$\tilde{C}_{11}^{(m)}\left(\frac{\partial^2 \tilde{U}_r^{(m)}}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{U}_r^{(m)}}{\partial r} - \frac{1}{r^2}\tilde{U}_r^{(m)}\right) -$$
$$-\tilde{C}_{44}^{(m)}k^2\tilde{U}_r^{(m)} - k\left(\tilde{C}_{13}^{(m)} + \tilde{C}_{44}^{(m)}\right)\frac{\partial \tilde{U}_z^{(m)}}{\partial r} = \rho_m p^2 \tilde{U}_r^{(m)};$$
$$\tilde{C}_{44}^{(m)}\left(\frac{\partial^2 \tilde{U}_z^{(m)}}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{U}_z^{(m)}}{\partial r}\right) - k^2\tilde{C}_{33}^{(m)}\tilde{U}_z^{(m)} + \quad (2)$$
$$+k\left(\tilde{C}_{44}^{(m)} + \tilde{C}_{13}^{(m)}\right)\left(\frac{\partial \tilde{U}_r^{(m)}}{\partial r} + \frac{\tilde{U}_r^{(m)}}{r}\right) = \rho_m p^2 \tilde{U}_z^{(m)};$$

Uch qatlamli konussimon qobiqning bo‘ ylama-radial tebranishlari ichki va tashqi $f_n^{(i)}(z,t)$, $f_{S_2}^{(i)}(z,t)$ dinamik kuchlar ta’siri natijasida vujudga keladi, deb faraz qilinadi. Shu sababli, transversal-izotrop xususiyatga ega uch qatlamli konussimon qobiqning $r_l^{(3)}$-tashqi va $r_l^{(1)}$- ichki sirtlarida quyidagi chegaraviy shart o‘ rinli bo‘ ladi:

$$\sigma_{rr}^{(i)} - 2F_i'(z)\sigma_{rz}^{(i)} + \left(F_i'(z)\right)^2\sigma_{zz}^{(i)} = \left(1 + \left(F_i'(z)\right)^2\right)f_n^{(i)};$$
$$F_i'(z)\left(\sigma_{rr}^{(i)} - \sigma_{zz}^{(i)}\right) + \left(1 - \left(F_i'(z)\right)^2\right)\sigma_{rz}^{(i)} = \left(1 + \left(F_i'(z)\right)^2\right)f_{S_2}^{(i)}. \quad (3)$$

(3) ifoda $i = 1$ da ichki sirt uchun chegaraviy shartni va $i = 2$ da tashqi sirt uchun chegaraviy shartni ifodalaydi.

Bundan tashqari, o‘ rta qatlam bilan ichki va tashqi qatlamlar orasida quyidagi dinamik va kinematik kontakt shartlar o‘ rinli bo‘ ladi:

$r_l^{(0)}$ sirtda

$$U_r^{(1)} = U_r^{(0)};\ U_z^{(1)} = U_z^{(0)};\ \sigma_{rr}^{(1)} = \sigma_{rr}^{(0)};\ \sigma_{zz}^{(1)} = \sigma_{zz}^{(0)}. \quad (4)$$

$r_l^{(2)}$ sirtda

$$U_r^{(0)} = U_r^{(2)};\ U_z^{(0)} = U_z^{(2)};\ \sigma_{rr}^{(0)} = \sigma_{rr}^{(2)};\ \sigma_{zz}^{(0)} = \sigma_{zz}^{(2)}. \quad (5)$$

bu yerda $U_r^{(m)}(r,z,t)$ va $U_z^{(m)}(r,z,t)$ uch qatlamli konussimon qobiq qatlamlari nuqtasi radial va bo‘ ylama ko‘ chishlari.

Demak uch qatlamli konussimon qobiqning ichki va tashqi dinamik yuklanishlar ta’siri ostida yuzaga keladigan bo‘ ylama-radial tebranishlari haqidagi masalani yechish (3) tenglamalarni (4) va (5) kinematik va dinamik kontakt shartlarhamda nolga teng boshlang‘ ich shartlar bilan yechishdan iborat bo‘ lar ekan.

Natijalar va ularning tahlili

Masalaning yechimi. Uch qatlamli transversal-izotrop konussimon qobiqning bo‘ ylama-radial tebranish tenglamalarini chiqarish uchun (2) harakat differensial tenglamalar sistemasini (3) chegaraviy, (4) va (5) kontakt shartlar bilan birgalikda yechamiz. Bir qancha matematik amallar va soddalashtirishlarni bajarganimizdan so‘ ng quyidagi differensial tenglamalar sistemasini hosil qilamiz:

$$\left[K_{i1}\frac{\partial}{\partial z} + K_{i2}\right]U_{r,0}^{(0)} + \left[L_{i1}\frac{\partial^3}{\partial t^2\partial z} + L_{i2}\frac{\partial^3}{\partial z^3} + L_{i3}\frac{\partial^2}{\partial t^2} + \right.$$
$$\left. + L_{i4}\frac{\partial^2}{\partial z^2} + L_{i5}\frac{\partial}{\partial z} + L_{i6}\right]\xi U_{r,1}^{(0)} + \left[M_{i1}\frac{\partial}{\partial z} + M_{i2}\right]U_{z,0}^{(0)} +$$

$$+\left[N_{i1}\frac{\partial^2}{\partial z^2}+N_{i2}\frac{\partial}{\partial z}+N_{i3}\right]\xi U_{z,1}^{(0)}=\left(1+\left(F_i'(z)\right)^2\right)f_n^{(i)};$$
$$\left[Q_{i1}\frac{\partial}{\partial z}+Q_{i2}\right]U_{r,0}^{(0)}+\left[J_{i1}\frac{\partial^3}{\partial t^2\partial z}+J_{i2}\frac{\partial^3}{\partial z^3}+J_{i3}\frac{\partial^2}{\partial t^2}+\right.$$
$$\left.+J_{i4}\frac{\partial^2}{\partial z^2}+J_{i5}\frac{\partial}{\partial z}+J_{i6}\right]\xi U_{r,1}^{(0)}+\left[I_{i1}\frac{\partial}{\partial z}+I_{i2}\right]U_{z,0}^{(0)}+$$
$$+\left[R_{i1}\frac{\partial^2}{\partial z^2}+R_{i2}\frac{\partial}{\partial z}+R_{i3}\right]\xi U_{z,1}^{(0)}=\left(1+\left(F_i'(z)\right)^2\right)f_{S_2}^{(i)};\quad (6)$$

bu yerda K_{ij}, L_{ij}, M_{ij}, N_{ij}, Q_{ij}, J_{ij}, I_{ij} va R_{ij} uch qatlamli konussimon qobiqning geometrik va fizik parametrlariga bog‘ liq bo‘ lgan o‘ zgarmas koeffitsiyenlar. Masalan:

$$K_{11}=F_1'(z)r_l^{(1)}C_{44}^{(1)};\ \dots\dots$$
$$L_{14}=-\frac{C_{44}^{(0)}}{2C_{11}^{(0)}}\left(C_{11}^{(1)}+C_{12}^{(1)}+2\left(F_1'(z)\right)^2C_{13}^{(1)}\right)\left(\ln\frac{r_l^{(0)}}{\xi}-\frac{1}{2}\right);\ \dots\dots$$

(6) tenglamalar sistemasidagi $U_{r,0}^{(0)}$, $U_{z,0}^{(0)}$, $U_{r,1}^{(0)}$ va $U_{z,1}^{(0)}$ izlanuvchi funksiyalar. Ular uch qatlamli konussimon qobiqning markaziy o‘ qidan ξ masofada joylashgan o‘ rta qatlamdan tanlangan sirtdagi ko‘ chish vektorining komponentalari hisoblanadi. Uch qatlamli konussimon qobiqning qatlamlaridagi ko‘ chish va kuchlanishlarni topish uchun uch qatlamli konussimon qobiqning qatlamlaridagi ko‘ chish va kuchlanishlarni ham $U_{r,0}^{(0)}$, $U_{z,0}^{(0)}$, $U_{r,1}^{(0)}$ va $U_{z,1}^{(0)}$ izlanuvchi funksiyalar orqali ifodalaymiz:

Uch qatlamli konussimon qobiqning ichki qatlamidagi ko‘ chish va kuchlanishlar quyidagicha:

$$\sigma_{rr}^{(1)}=\left(-\rho_1-\frac{C_{12}^{(1)}\rho_1}{2C_{11}^{(1)}}\right)\frac{\partial^2U_{r,0}^{(0)}}{\partial t^2}+\left(C_{44}^{(1)}+\frac{C_{12}^{(1)}C_{44}^{(1)}}{2C_{11}^{(1)}}\right)\frac{\partial^2U_{r,0}^{(0)}}{\partial z^2}+$$
$$+\rho_0\rho_1\left(1-2\ln\left(\frac{r_l^{(0)}}{\xi}\right)\right)\frac{\left(2C_{11}^{(1)}+C_{12}^{(1)}\right)}{4C_{11}^{(0)}C_{11}^{(1)}}\xi\frac{\partial^4U_{r,1}^{(0)}}{\partial t^4}+$$
$$+\frac{\left(2C_{11}^{(1)}+C_{12}^{(1)}\right)}{4C_{11}^{(0)}C_{11}^{(1)}}\left(2C_{44}^{(0)}\rho_1\ln\left(\frac{r_l^{(0)}}{\xi}\right)-C_{44}^{(0)}\rho_1+2C_{44}^{(1)}\rho_0\ln\left(\frac{r_l^{(0)}}{\xi}\right)-C_{44}^{(1)}\rho_0\times\right.$$
$$\left.\times\xi\frac{\partial^4U_{r,1}^{(0)}}{\partial t^2\,\partial z^2}+\left(\frac{\rho_1\xi\left(-2C_{11}^{(1)}-C_{12}^{(1)}\right)}{C_{11}^{(1)}\left(r_l^{(0)}\right)^2}\right)\frac{\partial^2U_{r,1}^{(0)}}{\partial t^2}+\frac{C_{44}^{(0)}C_{44}^{(1)}}{4C_{11}^{(0)}C_{11}^{(1)}}\times\right.$$
$$\left.\times\left(1-2\ln\left(\frac{r_l^{(0)}}{\xi}\right)\right)\left(2C_{11}^{(1)}+C_{12}^{(1)}\right)\xi\frac{\partial^4U_{r,1}^{(0)}}{\partial z^4}+\right.\quad (7)$$
$$+\frac{C_{44}^{(1)}}{C_{11}^{(1)}\left(r_l^{(0)}\right)^2}\left(2C_{11}^{(1)}+C_{12}^{(1)}\right)\xi\frac{\partial^2U_{r,1}^{(0)}}{\partial z^2}-\frac{C_{13}^{(1)}\rho_1}{C_{11}^{(1)}}\frac{\partial^3U_{z,0}^{(0)}}{\partial t^2\,\partial z}+\frac{C_{13}^{(1)}C_{44}^{(1)}}{C_{11}^{(1)}}\frac{\partial^3U_{z,0}^{(0)}}{\partial z^3}+$$
$$+\frac{\rho_1B_1^{(0)}}{4C_{11}^{(1)}}\left(2C_{11}^{(1)}\left(2\ln\left(\frac{r_l^{(0)}}{\xi}\right)-1\right)+2C_{12}^{(1)}\ln\left(\frac{r_l^{(0)}}{\xi}\right)-C_{12}^{(1)}-\right.$$
$$\left.-4\frac{C_{13}^{(1)}}{B_1^{(0)}}\ln\left(\frac{r_l^{(0)}}{\xi}\right)\xi\frac{\partial^3U_{z,1}^{(0)}}{\partial t^2\,\partial z}++\frac{C_{44}^{(1)}B_1^{(0)}}{4C_{11}^{(1)}}\left(2C_{11}^{(1)}\left(1-2\ln\left(\frac{r_l^{(0)}}{\xi}\right)\right)-\right.\right.$$
$$\left.\left.-2C_{12}^{(1)}\ln\left(\frac{r_l^{(0)}}{\xi}\right)+C_{12}^{(1)}+4\frac{C_{13}^{(1)}}{B_1^{(0)}}\ln\left(\frac{r_l^{(0)}}{\xi}\right)\xi\right)\frac{\partial^3U_{z,1}^{(0)}}{\partial z^3};\right.$$

$\sigma_{\theta\theta}^{(1)}$, $\sigma_{zz}^{(1)}$ va $\sigma_{rz}^{(1)}$ kuchlanishlar ham xuddi shunday ifodalanadi. Xuddi shunday uch qatlamli konussimon qobiqning o‘ rta qatlamidagi ko‘ chish va kuchlanishlar quyidagicha:

$$\sigma_{rr}^{(0)}=\frac{\rho_0\left(2C_{11}^{(0)}+C_{12}^{(0)}\right)}{C_{11}^{(0)}}\frac{\partial^2U_{r,0}^{(0)}}{\partial t^2}-\frac{C_{44}^{(0)}\left(2C_{11}^{(0)}+C_{12}^{(0)}\right)}{C_{11}^{(0)}}\frac{\partial^2U_{r,0}^{(0)}}{\partial z^2}-$$
$$-\frac{\left(2C_{11}^{(0)}+C_{12}^{(0)}\right)}{2\left(C_{11}^{(0)}\right)^2}\frac{\partial^4U_{r,1}^{(0)}}{\partial t^4}+\frac{C_{44}^{(0)}\rho_0\left(2C_{11}^{(0)}+C_{12}^{(0)}\right)}{\left(C_{11}^{(0)}\right)^2}\xi\times$$
$$\times\frac{\partial^4U_{r,1}^{(0)}}{\partial t^2\,\partial z^2}-\frac{2C_{13}^{(0)}\rho_0\xi}{B_1^{(0)}C_{11}^{(0)}}\frac{\partial^2U_{r,1}^{(0)}}{\partial t^2}+\frac{\left(C_{44}^{(0)}\right)^2\left(-2C_{11}^{(0)}-C_{12}^{(0)}\right)}{2\left(C_{11}^{(0)}\right)^2}\xi\frac{\partial^4U_{r,1}^{(0)}}{\partial z^4}+$$

$$+\frac{2C_{13}^{(0)}C_{44}^{(0)}\xi}{B_1^{(0)}C_{11}^{(0)}}\frac{\partial^2 U_{r,1}^{(0)}}{\partial z^2}+\frac{2C_{13}^{(0)}\rho_0}{C_{11}^{(0)}}\frac{\partial^3 U_{z,0}^{(0)}}{\partial t^2\,\partial z}-\frac{2C_{13}^{(0)}C_{44}^{(0)}}{C_{11}^{(0)}}\frac{\partial^3 U_{z,0}^{(0)}}{\partial z^3}+ \quad (8)$$

$$+\frac{\rho_0\xi B_1^{(0)}}{2C_{11}^{(0)}}\left(-2C_{11}^{(0)}+C_{12}^{(0)}+2\frac{C_{13}^{(0)}}{B_1^{(0)}}\right)\frac{\partial^3 U_{z,1}^{(0)}}{\partial t^2\,\partial z}+$$

$$+\frac{B_1^{(0)}C_{44}^{(0)}\xi}{2C_{11}^{(0)}}\left(2C_{11}^{(0)}-C_{12}^{(0)}-2\frac{C_{13}^{(0)}}{B_1^{(0)}}\right)\frac{\partial^3 U_{z,1}^{(0)}}{\partial z^3};$$

Uch qatlamli konussimon qobiqning bo‘ ylama-radial tebranishlarida hosil bo‘ ladigan o‘ rta qatlamning qolgan $\sigma_{\theta\theta}^{(0)}$, $\sigma_{zz}^{(0)}$ va $\sigma_{rz}^{(0)}$ larni ham izlanuvchi $U_{r,0}^{(0)}$, $U_{z,0}^{(0)}$, $U_{r,1}^{(0)}$ va $U_{z,1}^{(0)}$ funksiyalar orqali ifodalaymiz.

Uch qatlamli konussimon qobiqning tashqi qatlamida hosil bo‘ ladigan kuchlanishlarni ham $U_{r,0}^{(0)}$, $U_{z,0}^{(0)}$, $U_{r,1}^{(0)}$ va $U_{z,1}^{(0)}$ izlanuvchi funksiyalar orqali ifodalaymiz.

Masalan:

$$\sigma_{rz}^{(2)}=\frac{C_{44}^{(2)}\left(r_l^{(2)}\right)^2}{2r}\frac{\partial U_{r,0}^{(0)}}{\partial z}+\frac{C_{44}^{(2)}\left(r_l^{(2)}\right)^2\rho_0\xi}{4C_{11}^{(0)}r}\frac{\partial^3 U_{r,1}^{(0)}}{\partial t^2\,\partial z}-\frac{C_{44}^{(0)}C_{44}^{(2)}\left(r_l^{(2)}\right)^2\xi}{4C_{11}^{(0)}r}\times$$

$$\times\frac{\partial^3 U_{r,1}^{(0)}}{\partial z^3}+\frac{C_{44}^{(2)}\xi}{r}\left(2\ln\left(\frac{r_l^{(2)}}{\xi}\right)-1\right)\frac{\partial U_{r,1}^{(0)}}{\partial z}-\frac{C_{44}^{(2)}}{r}U_{z,0}^{(0)} \quad (9)$$

$$+\frac{B_1^{(0)}C_{44}^{(2)}\left(r_l^{(2)}\right)^2\xi}{4r}\left(2\ln\left(\frac{r_l^{(2)}}{\xi}\right)-1\right)\frac{\partial^3 U_{z,1}^{(0)}}{\partial z^3}+\frac{C_{44}^{(2)}\xi}{r}\ln\left(\frac{r_l^{(2)}}{\xi}\right)\frac{\partial U_{z,1}^{(0)}}{\partial z}.$$

XULOSA

Ushbu maqolada transversal-izotrop uch qatlamli konussimon qobiqning bo‘ ylama-radial tebranishlari masalasi o‘ qqa nisbatan simmetrik masala sifatida qaraldi va uning to‘ liq matematik modeli ishlab chiqildi. Uch qatlamli konussimon qobiqqa silindrik koordinatalar sistemasi joylashtirilib, ichki, o‘ rta va tashqi qatlamlarning geometrik parametrlari (qalinliklari, radiuslari, konusning yasovchi burchagi) orasidagi bog‘ lanishlar aniq analitik ko‘ rinishda ifodalandi. Bu esa qobiqning istalgan ko‘ ndalang kesimidagi geometriyasini yagona formulalar orqali tavsiflash imkonini berdi. Matematik amallar va soddalashtirishlar natijasida qobiqning bo‘ ylama-radial tebranishlarini to‘ liq tavsiflovchi yakuniy tenglamalar sistemasi olindi. Olingan natijalar uch qatlamli konussimon qobiqlarning dinamik xususiyatlarini nazariy jihatdan chuqurroq tushunishga xizmat qilib, aviatsiya, gidrotexnika inshootlari, transport vositalari va qurilish sanoatida qo‘ llaniladigan yengil va yuqori mustahkamlikka ega ko‘ p qatlamli konstruksiyalarni loyihalashda muhim amaliy ahamiyatga ega. Xususan, ishlab chiqilgan model konstruksiyaning rezonans chastotalarini, tebranish amplitudalarini va kuchlanish konsentratsiyasi hududlarini oldindan baholash imkonini beradi. Keyingi olingan (6) tenglamalar sistemasi aniq chegaraviy shartlar bilan birgalikda sonli usullar yordamida yechiladi. Uch qatlamli konussimon qobiqning qatlamlaridagi ko‘ chish va kuchlanishlar grafiklarda tasvirlanadi. Uch qatlamli konussimon qobiq qatlamlari materiallarining tebranish jarayonidagi kuchlanganlik-deformatsiyalanganlik holatlari baholanadi.

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