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MAKSIMAL UZUNLIKDAGI KVAZI-FILIFORM NILRADIKALGA EGA YECHILUVCHAN LEYBNITS ALGEBRALARIDAN QURILGAN LEYBNITS (3)-ALGEBRALARI

Mamadaliyev O' ktamjon Xasanboyevich

Namangan davlat universiteti dotsenti, University of Business and Science

ORCID: 0009-0001-2003-2320

E-mail: mamadaliyevuktamjon@mail.ru

Annotatsiya: Ushbu maqolada nilradikali Lie bo' limgan maksimal uzunlikdagi kvazifiliform Leybnits algebrasi bo' lgan echiluvchan Leybnits algebralardan Leybnits (3)-algebralari hosil qilingan va ularning differensiallashi topilgan.

Kalit so' zlar: Leybnits algebralari, yechiluvchan Leybnits algebralari, differensiallash, Leybnits (n)-algebralari.

3-АЛГЕБРЫ ЛЕЙБНИЦА, ПОСТРОЕННЫЕ ИЗ РАЗРЕШИМЫХ АЛГЕБР ЛЕЙБНИЦА С КВАЗИФИЛИФОРМНЫМ НИЛЬРАДИКАЛОМ МАКСИМАЛЬНОЙ ДЛИНЫ

Аннотация: В данной статье построены 3-алгебры Лейбница из разрешимых алгебр Лейбница, нильрадикалами которых являются не являющиеся алгебрами Ли квазифилиформные алгебры Лейбница максимальной длины, и найдены их дифференцирования.

Ключевые слова: алгебры Лейбница, разрешимые алгебры Лейбница, дифференцирования, n-алгебры Лейбница.

LEIBNIZ 3-ALGEBRAS CONSTRUCTED FROM SOLVABLE LEIBNIZ ALGEBRAS WITH A QUASI-FILIFORM NILRADICAL OF MAXIMAL LENGTH

Abstract: In this paper, Leibniz 3-algebras are constructed from solvable Leibniz algebras whose nilradicals are non-Lie quasi-filiform Leibniz algebras of maximal length, and their derivations are determined.

Keywords: Leibniz algebras, solvable Leibniz algebras, derivations, Leibniz n-algebras.

Hozirgi vaqtda zamonaviy algebraning muhim yo' naliqlaridan biri Leybnits algebralari nazariyasiga bag' ishlangan tadqiqotlar hisoblanadi. Eslatib o' tamizki, Lie algebralarni tabiiy ravishda umumlashtiruvchi va Leybnits ayniyati bilan aniqlanuvchi ushbu algebra sinfi ilk bor 1965-yilda Blox tomonidan D-algebral nomi ostida taklif etilgan [15]. Ancha keyinroq, o' tgan asrning 90-yillari boshlarida D-algebral J.-L. Lode va T. Pirashvili ishlarida [3] zamonaviy Leybnits algebralari atamasi ostida qayta paydo bo' ldi. J.-L. Lode va T. Pirashvilining [4], [5] ishlaridan so' ng Leybnits algebralari katta qiziqish uyg' otdi. Buni ushbu algebralarga bag' ishlangan ilmiy ishlar sonining tez sur'atlar bilan ortib borayotgani ham tasdiqlaydi [6–10].

Leybnits (n)-algebralari [2] da Nambu algebralari [11] yoki Lie (n)-algebralarning [12,13] antisimmetrik bo' limgan varianti sifatida kiritilgan. (n=2) bo' lganda, Leybnits (2)-algebralari oddiy Leybnits algebralari bilan ustma-ust tushadi. Shunday qilib, Leybnits (n)-algebralari bir vaqtning o' zida Lie algebralari, Leybnits algebralari va Lie (n)-algebralarning umumlashmasi hisoblanadi.

1-ta'rif. K maydon ustida aniqlangan L algebraning ixtiyoriy x, y, z elementlari uchun quyidagi Leybnits ayniyati bajarilsa,

$$[x, [y, z]] = [[x, y], z] - [[x, z], y],$$

L algebra Leybnits algebrasi deyiladi, bu yerda $[-, -]$ – L da aniqlangan ko' paytirish amali.

Ixtiyoriy L Leybnits algebrasi uchun quyidagi quyi markaziy va hosilaviy ketma-ketliklarni mos ravishda aniqlaymiz:

$$L1 = L, Lk+1 = [Lk, L1]; L[1] = L, L[k+1] = [L[k], L[k]], k1 \geq$$

2-ta'rif. L Leybnits algebrasi uchun shunday son mavjud bo' lib, (mos ravishda,) bo' lsa, u holda L yechiluvchan (mos ravishda, nilpotent) Leybnits algebrasi deyiladi. $S \in \mathbb{N} \quad L^{[S]} = 0 \quad L^S = 0$

3-ta'rif. Aytaylik L – n -o' lchamli Leybnits algebrasi bo' lsin. Agar uning nilindeksi $n-1$ ga teng bo' lsa, u holda L kvazi-filiform Leybnits algebrasi deyiladi.

Aytaylik L – nolga teng bo' limgan fazolarning chekli soni bilan Z -graduirlangan Leybnits algebrasi, ya'ni $L=V_i$ bo' lsin,

$$\text{bu yerda har qanday } i, j \in Z \text{ lar uchun } [V_i, V_j] \subseteq \bigoplus_{i \in Z} V_{i+j}.$$

L nilpotent Leybnits algebrasi, agar $L = \dots$ bo' lsa, bu yerda barcha i ($k_1 \leq i \leq k_t$) lar uchun $V_i = 0$, bog' langan graduirovkani

$$\text{qabul qiladi deymiz. } V_{k_1} \oplus V_{k_2} \oplus \dots \oplus V_{k_t} \neq 0$$

$\text{len}(\oplus L) = k_t - k_1 + 1$ soni graduirovkaning uzunligi deyiladi. Keyinchalik, $l(L) = \max\{\text{len}(\oplus L) = k_t - k_1 + 1 \mid L = V_{k_1} \oplus V_{k_2} \oplus \dots \oplus V_{k_t}$

bog' langan graduirovka} ni L Leybnits algebrasining uzunligi deb ataymiz.

4-ta'rif. L Leybnits algebrasi, agar $l(L) = \dim L$ bo' lsa, maksimal uzunlikdagi algebra deyiladi.

Ma'lumki [10], maksimal uzunlikdagi har qanday n -o' lchamli ($n \geq 6$) kvazi-filiform L bo' limgan Leybnits algebrasi quyidagi o' zaro izomorf bo' limgan algebra oilalaridan biriga izomorfdir:

$$M^{1,\delta} : \begin{cases} [e_1, e_1] = e_n, & [e_{n-1}, e_1] = e_2, & [e_i, e_1] = e_{i+1}, & 2 \leq i \leq n-3, \\ [e_{n-1}, e_{n-1}] = \delta e_4, & \delta \in \{0, 1\}, & [e_i, e_{n-1}] = \delta e_{3+i}, & 2 \leq i \leq n-5; \end{cases}$$

$$M^{2,\lambda} : \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-3, \\ [e_{n-1}, e_1] = e_n, \\ [e_1, e_{n-1}] = \lambda e_n, & \lambda \in C; \end{cases}$$

$$M^{3,\alpha} : \begin{cases} [e_1, e_1] = e_3, & [e_i, e_1] = e_{i+1}, & 3 \leq i \leq n-1, \\ [e_1, e_i] = -e_{i+1}, & 3 \leq i \leq n-1, \\ [e_3, e_3] = \alpha e_6, & \alpha = 0, \text{ agar } n > 6, \alpha \in \{0, 1\}, \text{ agar } n = 6, \end{cases}$$

bu yerda algebraning bazisi $\{e_1, e_2, \dots, e_n\}$

[1] ishda nilradikali maksimal uzunlikka ega kvazi-filiform Leybnits algebrasidan iborat bo' lgan va nilradikalni to' ldiruvchi fazoning o' lchami maksimal qiymatga teng bo' lgan yechiluvchan Leybnits algebralarining klassifikatsiyasi keltirilgan.

1-teorema. [1] Faraz qilaylik, R – yechiluvchan Leybnits algebra bo' lib, uning nilradikali ga izomorf va bo' lsin. U

holda R quyidagi algebra bilan izomorf bo' ladi: $M^{1,0}$ $\dim Q = 2$

$$R(M^{1,0}, 2) : \begin{cases} [e_1, e_1] = e_n, & [e_i, e_1] = e_{i+1}, & 2 \leq i \leq n-2, \\ [e_1, x_1] = e_1, & [e_i, x_1] = (i-2)e_i, & 3 \leq i \leq n-1, \\ [e_n, x_1] = 2e_n, & [x_1, e_1] = -e_1, \\ [e_i, x_2] = e_i, & 2 \leq i \leq n-1. \end{cases}$$

2-teorema. [1] oilasiga mansub ixtiyoriy algebra quyidagi algebralardan biriga izomorf bo' ladi: $R(M^{2,\lambda}, 2)$

$$R(M^{2,\lambda}, 2) : \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-3, \\ [e_{n-1}, e_1] = e_n, \\ [e_i, x_1] = ie_i, & 1 \leq i \leq n-2, \\ [e_n, x_1] = e_n, & [x_1, e_1] = -e_1, \\ [e_{n-1}, x_2] = e_{n-1}, & [e_n, x_2] = e_n. \end{cases}$$

$$R(M^{2,-1}, 2): \begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-3, \\ [e_{n-1}, e_1] = e_n, & [e_1, e_{n-1}] = -e_n, \\ [e_i, x_1] = ie_i, & 1 \leq i \leq n-2, \\ [x_1, e_1] = -e_1, & [e_n, x_1] = -[x_1, e_n] = e_n, \\ [e_{n-1}, x_2] = -[x_2, e_{n-1}] = e_{n-1}, \\ [e_n, x_2] = -[x_2, e_n] = e_n. \end{cases}$$

Leybnits (n)-algebrasi L — bu quyidagi

$$[[x_1, \dots, x_n], y_1, \dots, y_{n-1}] = \sum_{i=1}^n [x_1, \dots, x_{i-1}, [x_i, y_1, \dots, y_n]]. \quad (1)$$

asosiy ayniyatni qanoatlantiruvchi n-chiziqli qavs bilan jihozlangan K-vektorial fazo. n=2 bo'lgan holda (1) asosiy ayniyat Leybnits ayniyatiga aylanadi, shuning uchun Leybnits 2-algebrasi oddiy Leybnits algebrasidir [2].

$$[-, -, \dots, -]: L^{\otimes n} \rightarrow L$$

Har qanday Leybnits algebrasi L quyidagi n-arli ko'paytirish (n-qavs) yordamida Leybnits (n)-algebrasi sifatida ham qaralishi mumkin:

$$[x_1, x_2, \dots, x_n] := [x_1, [x_2, \dots, [x_{n-1}, x_n] \dots]].$$

Quyidagi tasdiqlarda 1-teorema va 2-teoremlarda keltirilgan, va yechiluvchan Leybnits algebralari asosida qurilgan

Leybnits 3-algebralarining ko'paytirish jadvali keltiriladi. $R(M^{1,0}, 2)$ $R(M^{2,0}, 2)$ $R(M^{2,-1}, 2)$

1-tasdiq. algebra algebra asosida qurilgan Leybnits (3)-algebrasi bo'lsin. U holda uning ko'paytirish jadvali

quyidagi ko'rinishga ega bo'ladi: $L_1 R(M^{1,0}, 2)$

$$L: \begin{cases} [e_1, e_1, x_1] = e_n, & [x_1, e_1, x_1] = -e_1, \\ [e_i, e_1, x_1] = e_{i+1}, & 2 \leq i \leq n-2, \\ [e_1, x_1, e_1] = -e_n, & [x_1, x_1, e_1] = e_1, \\ [e_i, x_1, e_1] = -e_{i+1}, & 2 \leq i \leq n-2. \end{cases}$$

Isbot. algebraning elementlaridan bo'lishi mumkin bo'lgan barcha uchliklarni tenglikka qo'yib, ternar

ko'paytmalarni aniqlab chiqamiz: $R(M^{1,0}, 2) [x_1, x_2, x_3] = [x_1, [x_2, x_3]]$

$$[e_1, e_1, x_1] = [e_1, [e_1, x_1]] = [e_1, e_1] = e_n, [x_1, e_1, x_1] = [x_1, [e_1, x_1]] = [x_1, e_1] = -e_n$$

$$[e_i, e_1, x_1] = [e_i, [e_1, x_1]] = [e_i, e_1] = e_{i+1}, \quad 2 \leq i \leq n-2,$$

$$[e_1, x_1, e_1] = [e_1, [x_1, e_1]] = [e_1, -e_1] = -e_n, [x_1, x_1, e_1] = [x_1, [x_1, e_1]] = [x_1, -e_1] = e_1$$

$$[e_i, x_1, e_1] = [e_i, [x_1, e_1]] = [e_i, -e_1] = -e_{i+1}, \quad 2 \leq i \leq n-2.$$

Qolgan barcha uchliklar nolga teng.

2-tasdiq. algebra asosida qurilgan Leybnits (3)-algebrasi bo'lsin. U holda uning ko'paytirish jadvali quyidagi

ko'rinishga ega bo'ladi: $L_2 R(M^{2,0}, 2)$

$$L_2: \begin{cases} [e_i, e_1, x_1] = e_{i+1}, & 1 \leq i \leq n-3, \\ [e_i, x_1, e_1] = -e_{i+1}, & 1 \leq i \leq n-3, \\ [e_{n-1}, e_1, x_1] = e_n, & [e_{n-1}, x_1, e_1] = -e_n, \\ [x_1, e_1, x_1] = -e_1, & [x_1, x_1, e_1] = e_1. \end{cases}$$

Isbot. algebraning elementlaridan bo'lishi mumkin bo'lgan barcha uchliklarni tenglikka qo'yib, ternar

ko'paytmalarni aniqlab chiqamiz: $R(M^{2,0}, 2) [x_1, x_2, x_3] = [x_1, [x_2, x_3]]$

$$\begin{aligned} [e_i, e_1, x_1] &= [e_i, [e_1, x_1]] = [e_i, e_1] = e_{i+1}, \quad 1 \leq i \leq n-3, \\ [e_i, x_1, e_1] &= [e_i, [x_1, e_1]] = [e_i, -e_1] = -e_{i+1}, \quad 1 \leq i \leq n-3, \\ [e_{n-1}, e_1, x_1] &= [e_{n-1}, [e_1, x_1]] = [e_{n-1}, e_1] = e_n, [e_{n-1}, x_1, e_1] = [e_{n-1}, [x_1, e_1]] = [e_{n-1}, -e_1] = -e_n \\ [x_1, e_1, x_1] &= [x_1, [e_1, x_1]] = [x_1, e_1] = -e_1, [x_1, x_1, e_1] = [x_1, [x_1, e_1]] = [x_1, -e_1] = e_1. \end{aligned}$$

Qolgan barcha uchliklar nolga teng.

3-tasdiq. algebra asosida qurilgan Leybnits (3)-algebrasi bo'lsin. U holda uning ko'paytirish jadvali quyidagi

ko'rinishga ega bo'ladi: $L_3 R(M^{2,-1}, 2)$

$$L_3 : \begin{cases} [e_i, e_1, x_1] = e_{i+1}, & 1 \leq i \leq n-3, & [e_{n-1}, e_1, x_1] = e_n, & [e_{n-1}, x_1, e_1] = -e_n, \\ [e_i, x_1, e_1] = -e_{i+1}, & 1 \leq i \leq n-3, & [e_1, x_2, e_{n-1}] = e_n, & [e_1, e_{n-1}, x_2] = -e_n, \\ [x_1, e_1, e_{n-1}] = e_n, & [x_1, e_{n-1}, e_1] = -e_n, & [x_1, x_1, e_1] = e_1, & [x_1, e_1, x_1] = -e_1, \\ [x_1, e_n, x_1] = -e_n, & [x_1, x_1, e_n] = e_n, & [x_2, e_{n-1}, x_2] = -e_{n-1}, & [x_2, x_2, e_{n-1}] = e_{n-1}, \\ [x_2, e_n, x_1] = -e_n, & [x_2, x_1, e_n] = e_n, & [x_1, e_n, x_2] = -e_n, & [x_1, x_2, e_n] = e_n, \\ [x_2, e_n, x_2] = -e_n, & [x_2, x_2, e_n] = e_n, & [x_2, e_1, e_{n-1}] = e_n, & [x_2, e_{n-1}, e_1] = -e_n. \end{cases}$$

Isbot. Isboti yuqoridagi teoremlarning isboti kabi isbotlanadi.

Differensiallashlar. Barcha tenzor ko'paytmalar K maydoni ustida olinadi. Faraz qilaylik, ω - vektor fazo bo'lib,

unda n -chiziqli amal berilgan bo'lsin. dan ga akslantiruvchi akslantirish, agar $\mathbf{A} : \omega \rightarrow \mathbf{A}$

$$d : \mathbf{A} \rightarrow \mathbf{A}$$

$$d(\omega(a_1, \dots, a_n)) = \sum_{i=1}^n \omega(a_1, \dots, f(a_i), \dots, a_n)$$

tenglik bajarilsa, ga nisbatan differensiallash deyiladi. Bu holda ω - differensiallash deb ham ataymiz. Barcha -

differensiallashlar to'plamini kabi belgilaymiz. Quyidagi tasdiq yaxshi ma'lumdir. $\omega \in \text{Der}_\omega(\mathbf{A})$

4-tasdiq. [2] 1) to'plam endomorfizmlar Li algebrasining Li qism algebrasi bo'ladi. $\text{Der}_\omega(\mathbf{A}) \subseteq \text{End}(\mathbf{A})$

2) Agar σ va ω bo'lsa, u holda bo'ladi. Bu yerda ham ω - chiziqli amaldir. $d \in \text{Der}_\omega(\mathbf{A}) \implies d \in \text{Der}_\sigma(\mathbf{A}) \implies d \in \text{Der}_{\omega+\sigma}(\mathbf{A})$

$$\sigma : \mathbf{A}^{\otimes n} \rightarrow \mathbf{A}^{\otimes n}$$

5-tasdiq. [2] Aytaylik, ikki chiziqli amal bo'lsin va amali $[-, -] : \mathbf{A}^{\otimes 2} \rightarrow \mathbf{A}$ $\omega : \mathbf{A}^{\otimes n} \rightarrow \mathbf{A}$

$$\omega(x_1, \dots, x_n) := [x_1, [x_2, \dots, [x_{n-1}, x_n] \dots]]$$

ko'rinishida berilgan bo'lsin. Agar akslantirish amalga nisbatan differensiallash bo'lsa, u holda bo'ladi. $d[-, -]$

$$d \in \text{Der}_\omega(\mathbf{A})$$

d Leybnits 3-algebrasining differensiallashi bo'lsin. Ta'rifga ko'ra, bizda quyidagi bor, bu yerda

$$d(\omega(x_1, x_2, x_3)) = \omega(d(x_1), x_2, x_3) + \omega(x_1, d(x_2), x_3) + \omega(x_1, x_2, d(x_3)) \quad \omega(x_1, x_2, x_3) := [x_1, [x_2, x_3]].$$

L_1 Leybnits (3)-algebrasining differensiallashini keltiramiz.

3-teorema. algebrasining ixtiyoriy differensiallashi quyidagi ko'rinishga ega. $L_1 d$

$$d(e_1) = a_{1,1}e_1 + a_{1,n-2}e_{n-2} + a_{1,n-1}e_{n-1} + a_{1,n}e_n, \quad d(e_2) = \sum_{i=2}^n a_{2,i}e_i + a_{2,n+2}x_2,$$

$$d(e_i) = ((i-2)a_{1,1} + a_{2,2})e_i + \sum_{j=i+1}^{n-1} a_{2,j-i+2}e_j, \quad 3 \leq i \leq n-2,$$

$$d(e_{n-1}) = ((n-3)a_{1,1} + a_{2,2})e_{n-1}, \quad d(e_n) = \sum_{i=4}^{n-1} a_{1,i-1}e_i + 2a_{1,1}e_n,$$

$$d(x_1) = -a_{1,n}e_1 - a_{1,n-2}e_{n-3} - a_{1,n-1}e_{n-2} + b_{1,n-1}e_{n-1} + b_{1,n}e_n + b_{1,n+2}x_2,$$

$$d(x_2) = b_{2,n-1}e_{n-1} + b_{2,n}e_n + b_{2,n+2}x_2.$$

Isbot. d - algebraning differensiallashi bo' lsin. d differensiallashni algebraning hosil qiluvchi elementlari uchun aniqlanish yetarli bo' lganligi sababli, biz quyidagini o' rnatamiz. $L_1 \quad L_1$

$$d(e_1) = \sum_{i=1}^n a_{1,i}e_i + a_{1,n+1}x_1 + a_{1,n+2}x_2, \quad d(e_2) = \sum_{i=1}^n a_{2,i}e_i + a_{2,n+1}x_1 + a_{2,n+2}x_2,$$

$$d(x_1) = \sum_{i=1}^n b_{1,i}e_i + b_{1,n+1}x_1 + b_{1,n+2}x_2, \quad d(x_2) = \sum_{i=1}^n b_{2,i}e_i + b_{2,n+1}x_1 + b_{2,n+2}x_2.$$

Birinchi quyidagi tenglikni qaraymiz:

$$d(\omega(x_1, x_1, e_1)) = \omega(d(x_1), x_1, e_1) + \omega(x_1, d(x_1), e_1) + \omega(x_1, x_1, d(e_1))$$

Bu tenglikdan quyidagi tengliklarni olamiz:

$$a_{1,2} = a_{1,n+1} = a_{1,n+2} = 0, \quad b_{1,1} = -a_{1,n}, \quad b_{1,i} = -a_{1,i+1}, \quad 2 \leq i \leq n-2.$$

Bundan tashqari, bu tengliklardan quyidagi natijalarni olish mumkin:

$$d(e_1) = a_{1,1}e_1 + \sum_{i=3}^n a_{1,i}e_i$$

$$d(x_1) = -a_{1,n}e_1 - \sum_{i=2}^{n-2} a_{1,i+1}e_i + b_{1,n-1}e_{n-1} + b_{1,n}e_n + b_{1,n+1}x_1 + b_{1,n+2}x_2.$$

Endi quyidagi tenglikni qaraymiz:

$$d(\omega(x_1, e_1, x_1)) = \omega(d(x_1), e_1, x_1) + \omega(x_1, d(e_1), x_1) + \omega(x_1, e_1, d(x_1))$$

Bu ko' paytmani hisoblaymiz:

$$-d(e_1) = [d(x_1), e_1] + [x_1, d(e_1), x_1] + [x_1, e_1, d(x_1)],$$

$$-d(e_1) = -a_{1,n}e_1 - \sum_{i=3}^{n-1} a_{1,i}e_i - b_{1,n+1}e_1 - a_{1,1}e_1 - b_{1,n+1}e_1$$

$d(e_1)$ ni o' rniga qo' yamiz va tenglikni ikki tomonini tenglaymiz.

$$-a_{1,1}e_1 - \sum_{i=3}^{n-1} a_{1,i}e_i - a_{1,n}e_n = -a_{1,n}e_1 - \sum_{i=3}^{n-1} a_{1,i}e_i - 2b_{1,n+1}e_1 - a_{1,1}e_1$$

Ayniyatni hisoblash orqali biz quyidagi tengliklarni olamiz:

$$b_{1,n+1} = 0$$

Navbatdagi ko' paytma uchun differensiallash xossasini qaraymiz:

$$d(\omega(e_1, e_1, x_1)) = \omega(d(e_1), e_1, x_1) + \omega(e_1, d(e_1), x_1) + \omega(e_1, e_1, d(x_1))$$

$$d(e_n) = [d(e_1), e_1] + [e_1, d(e_1), x_1] + [e_1, e_1, d(x_1)]$$

Biz bu ko' paytmaning differensialini hisoblash orqali ni olamiz: $d(e_n)$

$$d(e_n) = \sum_{i=4}^{n-1} a_{1,i-1}e_i + 2a_{1,1}e_n.$$

Quyidagi ko' paytma uchun differensiallash xossasini qaraymiz

$$d(\omega(e_n, e_1, x_1)) = \omega(d(e_n), e_1, x_1) + \omega(e_n, d(e_1), x_1) + \omega(e_n, e_1, d(x_1)).$$

$$\begin{aligned}
d(\omega(e_n, e_1, x_1)) &= [d(e_n), e_1] + [e_n, [d(e_1), x_1]] + [e_n, [e_1, d(x_1)]], \\
0 &= \left[\sum_{i=4}^{n-1} a_{1,i-1} e_i + 2a_{1,1} e_n, e_1 \right] + [e_n, [a_{1,1} e_1 + \sum_{i=3}^n a_{1,i} e_i, x_1]] + \\
&\quad + [e_n, [e_1, -a_{1,n} e_1 - \sum_{i=2}^{n-2} a_{1,i+1} e_i + b_{1,n-1} e_{n-1} + b_{1,n} e_n + b_{1,n+2} x_2]] \\
0 &= \sum_{i=5}^{n-1} a_{1,i-2} e_i
\end{aligned}$$

Bu tenglikdan quyidagi natijani olamiz:, shuning uchun $a_{1,i} = 0, \quad 3 \leq i \leq n-3 \Rightarrow b_{1,i} = 0, \quad 2 \leq i \leq n-4$

$$\begin{aligned}
d(e_1) &= a_{1,1} e_1 + a_{1,n-2} e_{n-2} + a_{1,n-1} e_{n-1} + a_{1,n} e_n, \\
d(x_1) &= -a_{1,n} e_1 - a_{1,n-2} e_{n-3} - a_{1,n-1} e_{n-2} + b_{1,n-1} e_{n-1} + b_{1,n} e_n + b_{1,n+2} x_2.
\end{aligned}$$

Endi ushbu tenglikni ko'rib chiqaylik:

$$\begin{aligned}
d(\omega(x_2, e_1, x_1)) &= \omega(d(x_2), e_1, x_1) + \omega(x_2, d(e_1), x_1) + \omega(x_2, e_1, d(x_1)), \\
d(\omega(x_2, e_1, x_1)) &= [d(x_2), e_1] + [x_2, [d(e_1), x_1]] + [x_2, [e_1, d(x_1)]], \text{ yani} \\
0 &= \left[\sum_{i=1}^n b_{2,i} e_i + b_{2,n+1} x_1 + b_{2,n+2} x_2, e_1 \right] + [x_2, [a_{1,1} e_1 + a_{1,n-2} e_{n-2} + a_{1,n-1} e_{n-1} + a_{1,n} e_n, x_1]] + \\
&\quad + [x_2, [e_1, -a_{1,n} e_1 - a_{1,n-2} e_{n-3} - a_{1,n-1} e_{n-2} + b_{1,n-1} e_{n-1} + b_{1,n} e_n + b_{1,n+2} x_2]], \\
0 &= \sum_{i=3}^{n-1} b_{2,i-1} e_i - b_{2,n+1} e_1 + b_{1,1} e_n
\end{aligned}$$

Shundan kelib chiqib, biz ega bo'lamiz. Demak, $b_{2,i} = 0, \quad 1 \leq i \leq n-2, \quad b_{2,n+1} = b_{2,n+2} = 0$

$$d(x_2) = b_{2,n-1} e_{n-1} + b_{2,n} e_n + b_{2,n+2} x_2.$$

Endi,

$$d(\omega(e_1, e_2, x_1)) = \omega(d(e_1), e_2, x_1) + \omega(e_1, d(e_2), x_1) + \omega(e_1, e_2, d(x_1)),$$

Tenglikni qarab ni olamiz. Shunga o'xshab, $a_{2,1} = 0$

$$d(\omega(e_1, e_2, e_1)) = \omega(d(e_1), e_2, e_1) + \omega(e_1, d(e_2), e_1) + \omega(e_1, e_2, d(e_1))$$

tenglikdan ekanligini olamiz. Natijada, $a_{2,n+1} = 0$

$$d(e_2) = \sum_{i=2}^n a_{2,i} e_i + a_{2,n+2} x_2.$$

hosil bo' ladi. Endi

$$d(\omega(e_2, e_1, x_1)) = \omega(d(e_2), e_1, x_1) + \omega(e_2, d(e_1), x_1) + \omega(e_2, e_1, d(x_1))$$

tenglikdan olamiz: $d(e_3)$

$$d(e_3) = \sum_{i=4}^{n-1} a_{2,i-1} e_i + (a_{1,1} + a_{2,2}) e_3.$$

Huddi shu kabi quyidagi tengliklar ketma-ketligidan

$$d(\omega(e_i, e_1, x_1)) = \omega(d(e_i), e_1, x_1) + \omega(e_i, d(e_1), x_1) + \omega(e_i, e_1, d(x_1))$$

uchun, matematik induksiya yordamida quyidagicha isbotlash mumkin: $2 \leq i \leq n-2$

$$d(e_i) = ((i-2)a_{1,1} + a_{2,2}) e_i + \sum_{j=i+1}^{n-1} a_{2,j-i+2} e_j, \quad 3 \leq i \leq n-2.$$

Biz bu orqali yozib olamiz. $d(e_{n-2})$

$$d(e_{n-2}) = ((n-4)a_{1,1} + a_{2,2})e_{n-2} + a_{2,3}e_{n-1}$$

Quyidagi ko' paytmari qaraylik:

$$d(\omega(e_{n-2}, e_1, x_1)) = \omega(d(e_{n-2}), e_1, x_1) + \omega(e_{n-2}, d(e_1), x_1) + \omega(e_{n-2}, e_1, d(x_1))$$

Bundan esa diferensialini hisoblaymiz. $d(e_{n-1})$

$$d(e_{n-1}) = ((n-4)a_{1,1} + a_{2,2})e_{n-1} + a_{1,1}e_{n-1}$$

Natijada quyidagini olamiz:

$$d(e_{n-1}) = ((n-3)a_{1,1} + a_{2,2})e_{n-1}.$$

Qolgan ko' paytmalar uchun differensiallash xossasini tekshirish ayniyatni beradi. $\square$

4-teorema. algebrasining ixtiyoriy differensiallashi quyidagi ko' rinishga ega: L_2 d

$$d(e_1) = \sum_{i=1}^{n-2} a_{1,i}e_i + a_{1,n}e_n, \quad d(e_i) = ia_{1,1}e_i + \sum_{j=i+1}^{n-2} a_{1,j-i+1}e_j, \quad 2 \leq i \leq n-3,$$

$$d(e_{n-2}) = (n-2)a_{1,1}e_{n-2}, \quad d(e_{n-1}) = \sum_{i=n-3}^n a_{n-2,i}e_i + a_{n-1,n+2}x_2$$

$$d(e_n) = a_{n-1,n-3}e_{n-2} + (a_{1,1} + a_{n-1,n-1})e_n,$$

$$d(x_1) = -\sum_{i=1}^{n-3} a_{1,i+1}e_i + b_{1,n-2}e_{n-2} - a_{1,n}e_{n-1} + b_{1,n}e_n + b_{1,n+2}x_2,$$

$$d(x_2) = b_{2,n-2}e_{n-2} + b_{2,n}e_n + b_{2,n+2}x_2.$$

Isbot. Isbot algebrasining differensiallash xossasini bevosita tekshirish orqali amalga oshiriladi. L_2

5-teorema. algebrasining ixtiyoriy differensiallashi quyidagi ko' rinishga ega. L_3 d

$$d(e_1) = \sum_{i=1}^{n-2} a_{1,i}e_i + a_{1,n}e_n, \quad d(e_i) = ia_{1,1}e_i + \sum_{j=i+1}^{n-2} a_{1,j-i+1}e_j, \quad 2 \leq i \leq n-2,$$

$$d(e_{n-2}) = (n-2)a_{1,1}e_{n-2}, \quad d(e_{n-1}) = a_{n-1,n-1}e_{n-1} + a_{1,2}e_n,$$

$$d(e_n) = (a_{1,1} + a_{n-1,n-1})e_n, \quad d(x_1) = -\sum_{i=1}^{n-3} a_{1,i+1}e_i + b_{1,n-2}e_{n-2} + b_{1,n}e_n,$$

$$d(x_2) = b_{2,n-2}e_{n-2} + a_{1,n}e_{n-1} + b_{1,n}e_n.$$

Isbot. Isbot algebrasining differensiallash xossasini bevosita tekshirish orqali amalga oshiriladi. L_3

XULOSA

Ushbu maqolada [1] ishdagi maksimal uzunlikdagi kvazi-filiform nilradikalga ega uchta turdagi yechiluvchan Leybnits algebralari asosida Leybnits (3)-algebralari qurildi. Qurilgan ternar algebralarning ko' paytirish jadvallari keltirilib, ularning differensiallashlari to' liq tavsiflandi. Natijalar Leybnits n-algebralari nazariyasini rivojlantirishga xizmat qiladi va kelgusida ko' homologiylar, deformatsiyalar hamda yuqori o' lchamli umumlashtirishlarni o' rganish uchun asos bo' la oladi.

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