

NAMANGAN DAVLAT
UNIVERSITETI
ILMIY AXBOROTNOMASI, [2026-9]
ISSN:2181-1458
ISSN:2181-0427



01.00.00-FIZIKA-MATEMATIKA
journal.namdu.uz
PHYSICS AND MATHEMATICS

UDC 511.53 (517.978)

THE LIFE-LINE PROBLEM IN AN INERTIAL DIFFERENTIAL GAME WITH ENERGY-BASED CONTROL CONSTRAINTS

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Abstract: This paper considers the life-line problem in a differential game between inertial players (a pursuer and an evader) whose controls are subject to integral constraints. In this problem, the main objective of the evader is to avoid the pursuer and reach a prescribed life-line region, whereas the objective of the pursuer is to achieve a win by preventing the opponent from reaching this region. The relative motion of the players, their initial positions and velocities, as well as the integral energy constraints imposed on the controls are analyzed, based on which solvability conditions for the life-line problem are determined. The obtained results provide a mathematical description of the pursuit-evasion problem in inertial differential games with integral constraints and develop existing results related to pursuit-evasion and life-line problems.

Keywords: differential game, pursuer, evader, strategy, guaranteed capture time, "life-line".

ENERGIYA SARFI BO' YICHA CHEGARALANISHLI INERTSION DIFFERENSIAL O' YINDA QUTILISH CHIZIG' I MASALASI

Annotatsiya: Ushbu maqolada integral chegaralanishlarga ega inersion o' yinchilar — quvlovchi va qochuvchi o' rtasidagi differensial o' yinda "qutilish chizig' i" masalasi qaraladi. Mazkur masalada, qochuvchining asosiy maqsadi quvlovchidan qochib, belgilangan "qutilish chizig' i" sohasiga yetib borishdan iborat bo' lsa, quvlovchiniki esa raqibi ushbu sohaga yetib bormasdan yutuqga erishishdir. Bunda o' yinchilarning nisbiy harakati, boshlang' ich holatlari va tezliklari hamda boshqaruvlarning integral energiya chegaralanishlari tahlil qilinib, "qutilish chizig' i" masalasining yechiluvchanlik shartlari aniqlanadi. Olingan natijalar integral chegaralanishli inersion differensial o' yinlarda quvish-qochish masalasining matematik tavsifini beradi hamda quvish-qochish va "qutilish chizig' i" masalalariga oid mavjud natijalarni rivojlantiradi.

Kalit so' zlar: differensial o' yin, quvlovchi, qochuvchi, strategiya, kafolatlangan tutish vaqti, "qutilish chizig' i".

ЗАДАЧА ЛИНИИ ЖИЗНИ В ИНЕРЦИОННОЙ ДИФФЕРЕНЦИАЛЬНОЙ ИГРЕ С ОГРАНИЧЕНИЯМИ НА УПРАВЛЕНИЯ ПО ЗАТРАТАМ ЭНЕРГИИ

Аннотация: В данной статье рассматривается задача «линии жизни» в дифференциальной игре между инерционными игроками — преследователем и убегающим, управления которых удовлетворяют интегральным ограничениям. В рассматриваемой задаче основной целью убегающего является, избегая преследователя, достичь заданной области «линии жизни», тогда как целью преследователя является добиться выигрыша, не допустив достижения этой области соперником. При этом анализируются относительное движение игроков, их начальные положения и скорости, а также интегральные энергетические ограничения на управления, на основе чего определяются условия разрешимости задачи «линии жизни». Полученные результаты дают математическое описание задачи преследования-убегания в инерционных дифференциальных играх с интегральными ограничениями и развивают существующие результаты, относящиеся к задачам преследования-убегания и «линии жизни».

Ключевые слова: дифференциальная игра, преследователь, убегающий, стратегия, гарантированное время захвата, «линия жизни».

Kirish

Differensial o' yinlar nazariyasi qarama-qarshi manfaatlarga ega bo'lgan boshqariluvchi dinamik tizimlarda optimal boshqaruv masalalarini tadqiq etishning muhim matematik vositalaridan biridir. Mazkur nazariya texnika, mexanika, robototexnika, avtomatik boshqaruv va boshqa amaliy sohalarida uchraydigan ko'p bosqichli konfliktli jarayonlarning matematik modellarini qurish va tahlil qilish imkonini beradi. Xususan, quvlovchi va qochuvchi harakatlari bilan tavsiflanadigan differensial o' yinlarda tutish va qochish masalalari nazariyaning asosiy yo'nalishlaridan birini tashkil etadi.

Klassik tutish masalalarida o' yinchilarning harakati odatda birinchi tartibli differensial tenglamalar orqali ifodalanadi va boshqaruvlarga nuqtaviy chegaralanishlar qo'yiladi. Biroq real mexanik jarayonlarda jismning harakati uning tezligi va tezlanishini hisobga olishni talab qiladi. Shu sababli inersion harakat modellariga asoslangan differensial o' yinlarni o'rganish muhim ahamiyat kasb etadi. Bunday hollarda o' yinchilarning dinamikasi ikkinchi tartibli differensial tenglamalar bilan tavsiflanadi va boshlang'ich holat bilan bir qatorda boshlang'ich tezliklar ham jarayonning keyingi rivojlanishiga ta'sir ko'rsatadi.

Boshqaruvlarga integral chegaralanishlar qo'yilgan hollarda masala yanada murakkablashadi. Bunda boshqaruvning har bir vaqttdagi qiymatini alohida chegaralash o'rniga, uning butun harakat davomida sarflanishini tavsiflovchi integral shartlardan foydalaniladi. Ushbu yondashuv real tizimlarda mavjud bo'lgan energiya, yoqilg'ich, resurs yoki boshqaruv xarajatlarini hisobga olish imkonini beradi.

Adabiyotlar tahlili

"Differensial o' yin" tushunchasi ilk marotaba Amerikalik matematik R. Ayzeks tomonidan kiritilgan bo'lib, dastlab uning monografiyasida bir qator o' yin masalalari keltirilgan. Keyinchalik ushbu yo'nalish bo'yicha ko'plab matematiklar tomonidan Ayzeks ilgari surgan g'oyalar rivojlantirildi va quvish-qochish masalalarini yechishning turli usullari ishlab chiqildi. Jumladan, L.S. Pontryagin [2] tomonidan taklif etilgan almashinuvchi integrallar usuli yordamida differensial o' yinlar nazariyasining klassik masalalaridan biri bo'lgan "Bola va Timsol" masalasi tadqiq etilgan. Keyinchalik N.N. Krasovskiy [3] va uning izdoshlari inersion harakatli o' yinchilardan tashkil topgan umumlashgan masalalarni ekstremal pozitsion strategiyalarni qurish usuli asosida o'rganidilar.

Quvish-qochish masalalarining rivojlanishi bilan bir qatorda, o' yinchilarning boshqaruvlariga turli xil chegaralanishlar qo'yilgan differensial o' yinlar ham tadqiq etila boshlandi. Xususan, boshqaruvlarga integral chegaralanishlar qo'yilgan quvish-qochish masalalari turli usullar yordamida o'rganiladi [4-8].

Mazkur yo'nalishdagi mavjud natijalarga tayangan holda, ushbu ishda integral chegaralanishlarga ega bo'lgan inersion differensial o' yinda **"qutilish chizig' i" masalasi** tadqiq etiladi. Bunda o' yinchilarning harakatlari ikkinchi tartibli differensial tenglamalar yordamida ifodalanadi, boshqaruv funksiyalari esa integral tipdagi chegaralanishlar sinfidan tanlanadi. Tadqiqotning asosiy maqsadi **"qutilish chizig' i" o' yinini har ikki o' yinchilarning nuqtai nazaridan o'rganib, yechiluvchanlik shartlarini topishdan iborat.**

Tadqiqot metodologiyasi

Mazkur tadqiqot ishida integral chegaralanishli inersion differensial o' yinda **"qutilish chizig' i" masalasi uchun yechiluvchanlik shartlarini** aniqlash hamda quvlovchi tomonidan qochuvchini chekli vaqtda tutib olish imkoniyatini tadqiq etish uchun differensial o' yinlar nazariyasi, boshqaruv nazariyasi va matematik analizning bir nechta klassik va umumlashtirilgan usullaridan foydalanildi. Bularga misol qilib keltirish mumkin:

Inersion harakat tenglamalari — o' yinchilarning dinamik harakatini tavsiflash va ularning boshlang'ich holatlari hamda tezliklarining keyingi harakatga ta'sirini aniqlash uchun qo'llanildi.

Geometrik usul — boshlang'ich holatlar va tezliklar orasidagi geometrik munosabatlarni tahlil qilish, shuningdek, quvlovchining qochuvchini aniq tutib olish imkoniyatini belgilovchi tutish chizig'ini aniqlash uchun ishlatildi.

Integral tenglamalar usuli — inersion harakat tenglamalarining yechimlarini integral ko‘rinishda ifodalash va boshqaruvlarning o‘ yinchilar trayektoriyalariga ta’sirini tadqiq qilishda foydalanildi.

Boshqaruvni konstruktiv tanlash usuli — quvlovchi uchun qochuvchini chekli vaqtda aniq tutishni ta’minlovchi boshqaruvni qurish hamda uning integral chegaralanishni qanoatlantirishini asoslash uchun qo‘llanildi.

Tadqiqot ishining asosiy obyektlari sifatida ikki o‘ yinchi – quvlovchi va qochuvchi qaralib, o‘ yinchilarning boshlang‘ich tezliklari teng bo‘lgan hol uchun “qutilish chizig‘i” o‘ yinini yechiluvchanlik shartlarini aniqlashda quvish hamda o‘ yinchilarning uchrashish nuqtalarini qurish masalalarida olingan natijalar asosiy ahamiyat kasb etadi. Qochuvchi uchun quvlovchi har qanday strategiya qo‘llaganda ham “qutilish chizig‘i” deb nomlanuvchi sohaga tutilmasdan yetib bora olishini ta’minlovchi shartlarni topish ishning asosiy mazmunini tashkil qiladi. Olingan natijalar integral resurslar bilan chegaralangan inersion konfliktli tizimlarda tutish imkoniyatlarini aniqlash, shuningdek, quvlovchi uchun tutishni ta’minlovchi boshqaruv strategiyalarini qurish uchun nazariy asos bo‘lib xizmat qiladi.

Natijalar va ularning tahlili

Masalaning qo‘yilishi. Faraz qilaylik, boshqariluvchi o‘ yinchi boshqa bir boshqariluvchi o‘ yinchini Yevklid fazosida ta’qib qilayotgan bo‘lsin. Odatda differensial o‘ yinlarda birinchi o‘ yinchini quvlovchi, ikkinchi o‘ yinchini esa qochuvchi deb ataymiz. bilan quvlovchining fazodagi holat vektorini, bilan esa qochuvchining holat vektorini belgilasak, u holda o‘ yinchilarning harakat dinamikallari ushbu differensial tenglamalar orqali

ifodalanadi: $\dot{P} = E \cdot \dot{x} \cdot \dot{y}$

$$P : \ddot{x} = u, \quad x(0) = x_{10}, \quad \dot{x}(0) = x_{11}, \quad (1)$$

$$E : \ddot{y} = v, \quad y(0) = y_{10}, \quad \dot{y}(0) = y_{11}. \quad (2)$$

Bu yerda, va o‘ yinchilarning boshlang‘ich holatlarini ifodalovchi vektorlar bo‘lib, vaqtda munosabat bajariladi. va esa o‘ yinchilarning boshlang‘ich tezlik vektorlarini ifodalaydi hamda o‘ yin avvalida ular teng deb qaraladi. Tezlanish vektorlari hamda mos ravishda o‘ yinchilarning boshqaruv parametrlari deyiladi. Ushbu boshqaruv parametrlarining vaqt bo‘yicha o‘zgarishi va akslantirishlarni qanoatlantiruvchi o‘lchovli funksiyalar sifatida tanlanadi. $x_{10} y_{10} t = 0$

$$x_{10} \neq y_{10} \quad x_{11} y_{11} \quad u \vee v : [0, +\infty) \rightarrow \mathbb{R}^n \quad v(\cdot) : [0, +\infty) \rightarrow \mathbb{R}^n$$

1-Ta’rif. o‘lchovli funksiya quvlovchi uchun joiz boshqaruv deyiladi, agar $u(\cdot) = (u(t), t \geq 0)$

$$\int_0^t (t-s) |u(s)|^2 ds \leq \rho_0^2, \quad \rho_0 > 0, \quad t \geq 0, \quad (3)$$

integral chegaralanishni qanoatlantirsa.

2-Ta’rif. o‘lchovli funksiya qochuvchi uchun joiz boshqaruv deyiladi, agar $v(\cdot) = (v(t), t \geq 0)$

$$\int_0^t (t-s) |v(s)|^2 ds \leq \sigma_0^2, \quad \sigma_0 > 0, \quad t \geq 0, \quad (4)$$

integral chegaralanishni qanoatlantirsa.

Quvlovchi uchun barcha joiz boshqaruvlari sinfini, xuddi shu kabi, qochuvchining barcha joiz boshqaruvlari sinfini orqali belgilaylik. $u(\cdot) \in U_I \quad v(\cdot) \in V_I$

(3) va (4) formulalarda va mos ravishda o‘ yinchilarning eng katta resurslarini ifodalaydi. $\rho_0^2 \sigma_0^2$

3-Ta’rif. Ushbu

$$\rho(t) = \rho_0^2 - \int_0^t (t-s) |u(s)|^2 ds, \quad \rho(0) = \rho_0^2,$$

$$\sigma(t) = \sigma_0^2 - \int_0^t (t-s) |v(s)|^2 ds, \quad \sigma(0) = \sigma_0^2$$

qiymatlar quvlovchi va qochuvchining joriy, vaqtdagi qoldiq resurslari deyiladi. $t \geq 0$

(1) va (2) tenglamalar orqali, o‘ yinchilarning va boshqaruvlari uchun, har bir va uchliklar mos ravishda quvlovchi va qochuvchining $u(\cdot) \in U_I \quad v(\cdot) \in V_I \quad (x_{10}, x_{11}, u(\cdot)) \quad (y_{10}, y_{11}, v(\cdot))$

$$x(t) = x_{10} + x_{11}t + \int_0^t (t-s)u(s)ds, \quad y(t) = y_{10} + y_{11}t + \int_0^t (t-s)v(s)ds$$

trayektoriyalarini ifodalaydi.

(1)-(4) differensial o' yin uchun quvish hamda o' yinchilarning uchrashish nuqtalarini qurish masalasi [9] ishda o' rganilgan bo' lib, biz ushbu ishda bu o' yin uchun oldingi olingan natijalarga asoslangan holda "qutilish chizig' i" differensial o' yinini yechiluvchanlik shartlarini aniqlaymiz.

L – fazoning yopiq qism to' plami bo' lsin va o' yin avvalida qochuvchi ushbu to' plamdan tashqarida joylashgan deb faraz qilamiz, ya'ni,. Ushbu ishda, hol uchun (1)-(4) differensial o' yinda "qutilish chizig' i masalasini quvlovchi va qochuvchining nuqtai nazaridan o' rganiladi. Markur o' yinda quvlovchining asosiy maqsadi qochuvchini tutish, shunday vaqt mavjud bo' lishiga erishishdan iboratki, ushbu vaqtda quvlovchi va qochuvchining trayektoriyalari ustma-ust tushsin, ya'ni, tenglik qochuvchi sohada bo' lgan vaqtda bajarilsin. Qochuvchining asosiy maqsadi esa, quvlovchi tomonidan tutilishidan oldin sohaga yetib borishdan yoki tengsizlikning barcha lar uchun bajarilishini ta'minlashdan iborat. Boshqacha aytganda, qochuvchi sohaning hatto, chegarasiga yetib borgan taqdirda ham, shu vaqtdan boshlab quvlovchi qanday strategiyani qo' lllamasin, o' yinda g' alaba qozona olmaydi. Agar qochuvchi uchun bunday imkoniyat

mavjud bo' lmasa, u holda uning asosiy maqsadi tutilish vaqtini imkon qadar kechiktirishdan iborat bo' ladi. $\square^n$

$$y_{10} \notin L, x_{11} = y_{11}, L, t_* > 0, x(t_*) = y(t_*) \square^n \setminus L, L, x(t) \neq y(t), t \in [0, +\infty) \setminus L$$

4-Ta'rif. Ushbu

$$u_0(v) = v - \lambda_0(v)\xi_{10} \quad (5)$$

boshqaruv funksiyasi quvish o' yinida quvlovchining -strategiyasi deyiladi, bu yerda Π

$$\lambda_0(v) = \max\{0, \delta + 2\langle v, \xi_{10} \rangle\}, \quad \xi_{10} = z_{10}/|z_{10}|, \quad \delta = (\rho_0^2 - \sigma_0^2)/|z_{10}|, \quad z_{10} = x_{10} - y_{10}.$$

1-Teorema [9]. Agar bo' lsa, u holda hol uchun, quvlovchi (5) -strategiyani qo' llash orqali vaqt intervalida yutuqli

bo' ladi, bu yerda – kafolatlangan tutish vaqti va. $\rho_0 > \sigma_0, x_{11} = y_{11}, \Pi[0, T_0], T_0 = \sqrt{2}|z_{10}|/(\rho_0 - \sigma_0)$

5-Ta'rif. Ushbu to' plam

$$(6)W^*(x_{10}, y_{10}, \rho_0, \sigma_0) = \bigcup_{t=0}^{T_0} \{W(x_{10}, y_{10}, \rho_0, \sigma_0) + tx_{11}\}$$

$x_{11} = y_{11}$ hol uchun quvish o' yinida o' yinchilarning uchrashish nuqtalar to' plami deyiladi, bu yerda – markazi

koordinatalar boshida radiusi birga teng dagi birlik shar. $W(x_{10}, y_{10}, \rho_0, \sigma_0) = x_{10} + \frac{\rho_0 \sigma_0}{\delta} B - \frac{\rho_0^2}{\delta} \xi_{10}, B \square^n$

6-Ta'rif. vaqt intervalida (5) -strategiya quvlovchi uchun "qutilish chizig' i" o' yinida yutuqli bo' lishni kafolatlaydi

deyiladi, agar shunday vaqt mavjud bo' lsaki, ushbu shartlar o' rinli bo' lsa, $[0, T_0] \cap \hat{t} \in [0, T_0]$

$$a); x(\hat{t}) = y(\hat{t})$$

$$b) \text{ barcha } y(t) \in L, t \in [0, \hat{t}]$$

2-Teorema. va bo' lsin. U holda (5) -strategiya quvlovchi uchun "qutilish chizig' i" o' yinida yutuqli bo' lishni

$$\text{kafolatlaydi. } \rho_0 > \sigma_0, W^*(x_{10}, y_{10}, \rho_0, \sigma_0) \cap L = \emptyset \square^n$$

Isbot. Ushbu Teorema isboti bevosita 1-Teorema isbotidan kelib chiqadi.

Navbatda biz "hayot chizig' i" o' yinini qochuvchining nuqtai nazaridan tahlil qilamiz.

7-Ta'rif. boshqaruvni qo' llash orqali qochuvchi "qutilish chizig' i" o' yinida yutuqli bo' ladi deyiladi, agar ixtiyoriy

$$\text{uchun } v_L(\cdot) \in V_I, u(\cdot) \in U_I$$

a) shunday, vaqt mavjud bo' lsaki, da va bo' lsa; $T_L, T_L > 0, t \in [0, T_L), y(T_L) \in L, x(t) \neq y(t)$

b) barcha larda bo' lsa. $t \geq 0, x(t) \neq y(t)$

Ushbu to' plamni qaraylik:

$$W_E(x_{11}, y_{10}, w) = \bigcup_{w \in W(x_0, y_0, \rho_0, \sigma_0)} \left\{ w_E : w_E = x_{11} \sqrt{2|w - y_{10}|/\Delta} + w \right\}. \quad (7)$$

Bu yerda, va. $\Delta = \sigma_0 \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} \right) / |z_{10}| \xi^* = (w - y_{10}) / |w - y_{10}| \delta_0 = \rho_0^2 - \sigma_0^2$

“Qutilish chizig’i” o’ yinida (7) to’ plam qochuvchining yetishish sohasi deyiladi.

[4] ishda keltirilganidek, ushbu ko’ rinishdagi

$$W(x(t), y(t), \rho(t), \sigma(t)) = \left\{ w : |w - x(t)| \geq \sqrt{\frac{\rho(t)}{\sigma(t)}} |w - y(t)| \right\},$$

ko’ p qiymatli akslantirish orqali vaqtda $t = 0$

$$W(x_{10}, y_{10}, \rho_0, \sigma_0) = \left\{ w : |w - x_{10}| \geq \frac{\rho_0}{\sigma_0} |w - y_{10}| \right\}$$

hosil qilamiz.

Bundan

$$|w - x_{10}| \geq \frac{\rho_0}{\sigma_0} |w - y_{10}|. \quad (8)$$

tengsizlikni inobatga olib, kabi belgilash kiritilsa va (8) ni har ikki tarafini kvadratga ko’ tarsak, quyidagi ko’ rinishni

$$\text{hosil qilamiz } \bar{w} = w - y_{10} \tilde{n}_0 = \frac{\rho_0}{\sigma_0}$$

$$|\bar{w}|^2 \left(1 - \frac{1}{\tilde{n}_0^2} \right) + \frac{2}{\tilde{n}_0^2} \langle \bar{w}, z_{10} \rangle - \frac{1}{\tilde{n}_0^2} |z_{10}|^2 \leq 0.$$

Oxirgi tengsizlikning har ikki tarafiga ko’ paytirib, qo’ shsak, $\tilde{n}_0^2 / (\tilde{n}_0^2 - 1) |z_{10}|^2 / (\tilde{n}_0^2 - 1)^2$

$$\left| \bar{w} + \frac{1}{\tilde{n}_0^2 - 1} z_{10} \right|^2 \leq \left(\frac{\tilde{n}}{\tilde{n}_0^2 - 1} |z_{10}| \right)^2,$$

yoki

$$\left| w - y_{10} + \frac{\sigma_0^2}{\delta} \xi_{10} \right| \leq \frac{\rho_0 \sigma_0}{\delta}, \quad (9)$$

hosil bo’ ladi. (9) ni ikki tarafini kvadratga ko’ tarilsa

$$|w - y_{10}|^2 + 2 \frac{\sigma_0^2}{\delta} \langle w - y_{10}, \xi_{10} \rangle + \frac{\sigma_0^4}{\delta^2} \leq \frac{\rho_0^2 \sigma_0^2}{\delta^2},$$

yoki

$$|w - y_{10}|^2 + 2 \frac{\sigma_0^2 |z_{10}|}{\delta_0} \langle \xi^*, \xi_{10} \rangle |w - y_{10}| - \frac{\sigma_0^2 |z_{10}|^2}{\delta_0} \leq 0.$$

yoza olamiz. Bundan

$$\begin{aligned} |w - y_{10}| &\leq \sqrt{\frac{\sigma_0^4 |z_{10}|^2}{\delta_0^2} \langle \xi^*, \xi_{10} \rangle^2 + \frac{\sigma_0^2 |z_{10}|^2}{\delta_0} - \frac{\sigma_0^2 |z_{10}|^2}{\delta_0} \langle \xi^*, \xi_{10} \rangle} = \\ &= \frac{\sigma_0 |z_{10}|}{\delta_0} \left[\sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} - \sigma_0 \langle \xi^*, \xi_{10} \rangle \right] = \frac{\sigma_0 |z_{10}|}{\sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} + \sigma_0 \langle \xi^*, \xi_{10} \rangle}, \end{aligned}$$

va nihoyat,

$$|w - y_{10}| \leq \frac{\sigma^2}{\Delta}. \quad (10)$$

hosil bo' ladi.

3-Teorema. bo' lsin va quyidagi ikki shartlardan biri bajarilsin: $W_E(x_{11}, y_{10}, w) \cap L \neq \emptyset$

$$a); \rho_0^2 \geq 2\sigma_0^2(1 - \langle \xi^*, \xi_{10} \rangle)$$

$$b) \text{ va. } \rho_0^2 < 2\sigma_0^2(1 - \langle \xi^*, \xi_{10} \rangle) \quad \rho(t) \geq \sigma(t)$$

U holda shunday boshqaruv mavjudki, bu boshqaruvni qo' llash orqali qochuvchi "qutilish chizig' i" o' yinida yutuqli

bo' ladi. $\mathbf{v}_L(\cdot) \in \mathbf{V}_I$

Isbot. Teoremaning shartiga ko' ra, kamida bitta nuqta mavjudki, (7) dagi tenglik o' rinli bo' ladigan. Faraz qilaylik, qochuvchi

$$\text{nuqtaga } W_E(x_{11}, y_{10}, w) \cap L \neq \emptyset \quad w_E \in W_E(x_{11}, y_{10}, w) \cap L \quad w_E$$

$$T_L = \sqrt{\frac{2|w - y_{10}|}{\Delta}} \quad (11)$$

vaqtda

$$\mathbf{v}_L = \Delta \xi^*. \quad (12)$$

boshqaruvni qo' llash orqali kelsin.

Dastlab, qochuvchining chekli (11) vaqtda (12) boshqaruv orqali nuqtaga yetib bora olishini ko' rsataylik. ekanligidan

$$w_E x_{11} = y_{11}$$

$$\begin{aligned} y(T_L) &= y_{10} + x_{11}T_L + \mathbf{v}_L T_L^2 / 2 = y_{10} + x_{11} \sqrt{\frac{2|w - y_{10}|}{\Delta}} + \xi^* |w - y_{10}| = \\ &= y_{10} + x_{11} \sqrt{\frac{2|w - y_{10}|}{\Delta}} + w - y_{10} = w + x_{11} \sqrt{\frac{2|w - y_{10}|}{\Delta}} = w_E. \end{aligned}$$

kelib chiqadi.

Endi 7-ta'rifning a) shartini bajarilishini ko' rsataylik ya'ni qochuvchining tutilmasligini. Teskarisidan faraz qilaylik, quvlovchi shunday boshqaruv qo' llasinki, tenglik dan kichik, vaqtda bajarilsin. U holda $u^*(\cdot) \in \mathbf{U}_I$ $x(t) = y(t)T_L$

$$\tilde{t} \tilde{t} < T_L$$

$$z(\tilde{t}) = z_{10} + \int_0^{\tilde{t}} (\tilde{t} - s) (u^*(s) - \mathbf{v}_L) ds, \quad (13)$$

bu yerda. Bundan, ekanligini ko' rsatish oson. $z(t) = x(t) - y(t) \quad z(\tilde{t}) = 0$

$$a) \text{ bo' lsin. U holda (3) va } \rho_0^2 \geq 2\sigma_0^2(1 - \langle \xi^*, \xi_{10} \rangle)$$

$$\int_0^t (t-s) |u(s)| ds \leq \left(\int_0^t (t-s) ds \right)^{1/2} \left(\int_0^t (t-s) |u(s)|^2 ds \right)^{1/2} \leq \frac{\rho_0}{\sqrt{2}} t$$

Koshi-Bunyakovskiy tengsizligiga ko' ra, (13) ning o' ng tomonidan quyidagi kabi baholashni olamiz:

$$\left| z_{10} - \frac{\tilde{t}^2}{2} \mathbf{v}_L \right| \leq \int_0^{\tilde{t}} (\tilde{t} - s) |u^*(s)| ds \leq \frac{\rho_0 \tilde{t}}{\sqrt{2}},$$

yoki qisqacha,

$$\left| z_{10} - \frac{\tilde{t}^2}{2} \mathbf{v}_L \right| \leq \frac{\rho_0 \tilde{t}}{\sqrt{2}}.$$

Oxirgi tengsizlikning har ikki tarafini kvadratga ko' tarsak,

$$(14) \left(\frac{\tilde{t}^2}{2} \right)^2 |\mathbf{v}_L|^2 - \frac{\tilde{t}^2}{2} (\rho_0^2 + 2 \langle \mathbf{v}_L, \mathbf{z}_{10} \rangle) + |\mathbf{z}_{10}|^2 \leq 0$$

hosil bo' ladi. Agar va tengliklar inobatga olinsa, bundan

$$\begin{aligned} \langle \mathbf{v}_L, \mathbf{z}_{10} \rangle &= \sigma_0 \langle \xi^*, \xi_{10} \rangle \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} \right) \\ |\mathbf{v}_L| |\mathbf{z}_{10}| &= \sigma_0 \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} \right) \\ \rho_0^2 + 2 \langle \mathbf{v}_L, \mathbf{z}_{10} \rangle &= \sigma_0^2 + \frac{|\mathbf{v}_L|^2 |\mathbf{z}_{10}|^2}{\sigma_0^2}. \quad (15) \end{aligned}$$

hosil qilinadi.

Boshqa tomondan, teoremaning a) shartini inobatga olsak va qochuvchining (12) ko' rinishdagi boshqaruvi orqali keyingi

$$\begin{aligned} \rho_0^2 &\geq 2\sigma_0^2(1 - \langle \xi^*, \xi_{10} \rangle) \\ |\mathbf{v}_L| &= \Delta = \frac{\sigma_0}{|\mathbf{z}_{10}|} \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \delta_0} \right) \geq \\ &\geq \frac{\sigma_0}{|\mathbf{z}_{10}|} \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sqrt{\sigma_0^2 \langle \xi^*, \xi_{10} \rangle^2 + \sigma_0^2 - 2\sigma_0^2 \langle \xi^*, \xi_{10} \rangle} \right) = \\ &= \frac{\sigma_0}{|\mathbf{z}_{10}|} \left(\sigma_0 \langle \xi^*, \xi_{10} \rangle + \sigma_0(1 - \langle \xi^*, \xi_{10} \rangle) \right) = \frac{\sigma_0^2}{|\mathbf{z}_{10}|}, \end{aligned}$$

yoki,

$$\sigma_0^2 \leq |\mathbf{v}_L| |\mathbf{z}_{10}|. \quad (16)$$

tengsizliklar hosil bo' ladi. (15) va (16) dan (14) uchun,

$$\frac{\sigma_0}{|\mathbf{v}_L|} \sqrt{2} \leq \tilde{t} \leq \frac{|\mathbf{z}_{10}|}{\sigma_0} \sqrt{2}$$

tengsizlik hosil qilinadi.

(10) ga tayanib, keyingi natijamiz

$$T_L = \sqrt{\frac{2|\mathbf{w} - \mathbf{y}_{10}|}{\Delta}} \leq \frac{\sigma_0}{\Delta} \sqrt{2} = \frac{\sigma_0}{|\mathbf{v}_L|} \sqrt{2} \quad (17)$$

ga erishamiz.

Oxirgi (17) natija ekanligini ko' rsatadi, bu esa dastlabki qilingan farazga ziddir. $T_L \leq \tilde{t}$

b) shart bajarilsin deylik. Agar $\rho_0^2 < 2\sigma_0^2(1 - \langle \xi^*, \xi_{10} \rangle)$

$$\left| \mathbf{z}_{10} - \frac{\tilde{t}^2}{2} \mathbf{v}_L \right| \leq \int_0^{\tilde{t}} (\tilde{t} - s) |u^*(s)| ds,$$

tengsizlikni har ikki tarafini kvadratga ko' tarilsa, u holda

$$\left(\frac{\tilde{t}^2}{2} \right)^2 |\mathbf{v}_L|^2 - \tilde{t}^2 \langle \mathbf{v}_L, \mathbf{z}_{10} \rangle + |\mathbf{z}_{10}|^2 = \left(\int_0^{\tilde{t}} (\tilde{t} - s) u^*(s) ds \right)^2$$

tenglikni yoza olamiz.

Yuqoridagi kabi Koshi-Bunyakovskiy tengsizligini qo' llasak va quvlovchining qoldiq resursini ko' rinishini inobatga olsak

$$\left(\int_0^{\tilde{t}} (\tilde{t} - s) u^*(s) ds \right)^2 \leq \frac{\tilde{t}^2}{2} \int_0^{\tilde{t}} (\tilde{t} - s) |u^*(s)|^2 ds = \frac{\tilde{t}^2}{2} (\rho_0^2 - \rho(\tilde{t})).$$

tengsizlikga kelimiz. Xuddi shu kabi qochuvchining, qoldiq resursini ko' rinishiga ko' ra

$$\frac{\tilde{t}^2}{2} |\mathbf{v}_L|^2 = \sigma_0^2 - \sigma(\tilde{t}).$$

ga kelimiz, u holda

$$\begin{aligned} \tilde{t}^2 (\sigma_0^2 - \sigma(\tilde{t})) - 2\tilde{t}^2 \langle \mathbf{v}_L, \mathbf{z}_{10} \rangle + 2|\mathbf{z}_{10}|^2 &\leq \tilde{t}^2 (\rho_0^2 - \rho(\tilde{t})) \Rightarrow \\ \Rightarrow \tilde{t}^2 (\delta_0 - \delta(\tilde{t}) + 2\langle \mathbf{v}_L, \mathbf{z}_{10} \rangle) &\geq 2|\mathbf{z}_{10}|^2 \end{aligned}$$

bu yerda. $\delta(\tilde{t}) = \rho(\tilde{t}) - \sigma(\tilde{t})$

$\rho(t) \geq \sigma(t)$ shartdan ko' rinadiki, bo' ladi. (15) dan esa $\delta(\tilde{t}) \geq 0$

$$\tilde{t} \geq |\mathbf{z}_{10}| \sqrt{\frac{2}{\delta_0 + 2\langle \mathbf{v}_L, \mathbf{z}_{10} \rangle}} = |\mathbf{z}_{10}| \sqrt{\frac{2\sigma_0^2}{|\mathbf{v}_L|^2 |\mathbf{z}_{10}|^2}} = \frac{\sigma_0}{|\mathbf{v}_L|} \sqrt{2}$$

kelib chiqadi. (17) ga ko' ra, ekanligi ko' rinadi. Bu esa dastlabki farazga ziddir. Shu bilan teorema isbotlandi. $\blacksquare T_L \leq \tilde{t}$

$$T_L > \tilde{t}$$

1-Natija. o' rinli bo' ladi. $W_E(x_{11}, y_{10}, w) \subset W^*(x_{10}, y_{10}, \rho_0, \sigma_0)$

Xulosa

Ushbu maqolada integral chegaralanishlarga ega inersion o' yinchilar o' rtasidagi differensial o' yinda "qutilish chizig' i" masalasi tadqiq etildi. Masalani yechishda asosiy usul sifatida parallel quvishning Π -strategiyasi qo' llanildi.

O' yinchilarning boshlang' ich tezliklari teng bo' lgan holatda Π -strategiya yordamida ularning uchrashish nuqtalari to' plami qurildi va shu holat uchun quvlovchi foydasiga "qutilish chizig' i" masalasi tadqiq etildi. Shuningdek, masala qochuvchining nuqtai nazaridan ham o' rganilib, quvlovchining ixtiyoriy boshqaruvi uchun "qutilish chizig' i" masalasining yechiluvchanlik shartlari topildi. Olingan natijalar integral chegaralanishli inersion differensial o' yinlarda quvish va "qutilish chizig' i" masalalarini yechish imkoniyatlarini kengaytiradi hamda kelgusida turli boshqaruv chegaralanishlariga ega chiziqli va nochiziqli konflikt-boshqariluvchi tizimlarga umumlashtirish uchun asos bo' lib xizmat qiladi.

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