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WEAKLY PERIODIC p-ADIC GENERALIZED GIBBS MEASURES FOR THE ISING MODEL ON THE CAYLEY TREE OF ORDER TWO

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Abstract: *This paper investigates weakly periodic p-adic generalized Gibbs measures for the Ising model on a Cayley tree of order two. Since such measures depend on the choice of the normal subgroup H_A, we consider the case $|A|=2$. For this model, we prove the existence of six unbounded H_{A_1}-weakly periodic p-adic generalized Gibbs measures and at least one unbounded H_{A_2}-weakly periodic p-adic generalized Gibbs measure.*

Keywords: *p-adic number, Gibbs measure, Cayley tree, Ising model, weakly periodic p-adic generalized Gibbs measure, phase transition.*

IKKINCHI TARTIBLI KELI DARAXTIDA IZING MODELI UCHUN KUCHSIZ DAVRIY p-ADIK UMUMLASHGAN GIBBS O‘ LCHOVLARI

Annotatsiya: *Ushbu maqolada ikkinchi tartibli Keli daraxtidagi Izing modeli uchun kuchsiz davriy p-adik umumlashgan Gibbs o‘ lchovlari o‘ rganildan. Bunday o‘ lchovlar H_A normal qism gruppaning tanlanishiga bog‘ liq ekanligini hisobga olib, bu maoqlada $|A|=2$ bo‘ lgan hol qaralgan. Ushbu model uchun oltita chegaralanmagan H_{A_1}-kuchsiz davriy va kamida bitta chegaralanmagan H_{A_2}-kuchsiz davriy p-adik umumlashgan Gibbs o‘ lchovlari mavjud ekanligi isbotlandi.*

Kalit so‘ zlar: *p-adik son, Gibbs o‘ lchovlari, Keli daraxti, Izing modeli, kuchsiz davriy p-adik umumlashgan Gibbs o‘ lchovlari, faza almashishi.*

СЛАБОПЕРИОДИЧЕСКИЕ p-АДИЧЕСКИЕ ОБОБЩЁННЫЕ МЕРЫ ГИББСА ДЛЯ МОДЕЛИ ИЗИНГА НА ДЕРЕВЕ КЭЛИ ПОРЯДКА ДВА

Аннотация: *В данной работе исследуются слабопериодические p-адические обобщённые меры Гиббса для модели Изинга на двоичном дереве Кэли. Поскольку такие меры зависят от выбора нормальной подгруппы H_A, рассматривается случай $|A|=2$. Для данной модели доказано существование шести неограниченных H_{A_1}-слабопериодических p-адических обобщённых мер Гиббса и, по крайней мере, одной неограниченной H_{A_2}-слабопериодической p-адической обобщённой меры Гиббса.*

Ключевые слова: *p-адическое число, мера Гиббса, дерево Кэли, модель Изинга, слабопериодическая p-адическая обобщённая мера Гиббса, фазовый переход.*

Kirish

Klassik statistik mexanikada o‘ lchovlar nazariyasiga asoslangan yondashuv doirasida muayyan modelga mos Gibbs o‘ lchovlari to‘ plamini tadqiq etish asosiy masalalardan biri hisoblanadi [1]. Mazkur yondashuv Kolmogorovning ehtimollar nazariyasi aksiomalariga asoslanadi [2]. Boshqa tomondan, p-adik ehtimollik nazariy fizikada nisbatan yaqinda shakllangan va klassik ehtimollik nazariyasidan o‘ ziga xos xususiyatlari bilan farq qiluvchi matematik sistemani

ifodalaydi [4,5]. p -adik ehtimolliklar p -adik fizik modellarni o‘ rganishda tabiiy ravishda yuzaga keladi. Bunga Volovich tomonidan taklif etilgan p -adik tor modeli muhim misol bo‘ la oladi [22].

Ushbu maqolada ikkinchi tartibli Keli daraxtida p -adik Izing modeli uchun kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari tadqiq etiladi. Mazkur model amaliy va nazariy fanlarning turli sohalarida keng ko‘ lamli tatbiqlarga ega. Biroq, ushbu tadqiqotning o‘ ziga xos jihati shundaki, unda kuchsiz davriy, xususan, davriy bo‘ lmagan p -adik umumlashgan Gibbs o‘ lchovlarida fazaviy o‘ tish hodisasi o‘ rganiladi. Ierarxik daraxtlardagi modellar uchun fazaviy o‘ tishning mavjudligi odatda renormalizatsiya gruppa (RG) usuli yordamida o‘ rganiladi. Mazkur usul ierarxik panjaralarni hamda p -adik ratsional funksiyalar orqali aniqlanuvchi modellarni qurishga asoslanadi. p -adik Izing va Potts modellari uchun [8,9,11-13] ishlarida fazaviy o‘tish bilan RG almashtirishining xaotik xatti-harakati o‘rtasida muhim bog‘liqlik mavjudligi ko‘rsatilgan.

Bizning asosiy e’tiborimiz ikkinchi tartibli Keli daraxtida Izing modeli uchun indeks ikki bo‘ lgan normal qism gruppaga ($|A|=2$) mos keluvchi kuchsiz davriy p -adik umumlashtirilgan Gibbs o‘ lchovlariga tadqiq qilishga qaratilgan. Natijada, model uchun faza almashishi mavjudligi isbotlanadi.

Adabiyotlar tahlili

So‘ nggi yillarda Keli daraxtida p -adik umumlashgan Gibbs o‘ lchovlarini tadqiq etish rivojlanib bormoqda. [3] maqolada O.Hakimov p -adik umumlashgan Gibbs o‘ lchovi tushunchasini kiritdi va p -adik Izing modeli uchun p -adik umumlashgan Gibbs o‘ lchovlarini o‘ rgandi. [17] maqolada esa bu xuddi shu model uchun H_{A_1} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari mavjud ekanligini isbotlangan. [18] maqolada esa ikkinchi tartibli Keli daraxtida Izing modeli uchun $|A|=1$ bo‘ lgan holga mos barcha H_{A_1} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari o‘ rganilgan. Bundan tashqari [8-13, 15, 16] maqolalarda Keli daraxtidagi turli modellar uchun p -adik (kvazi) Gibbs o‘ lchovlari o‘ rganilgan.

Tadqiqot metodologiyasi

Mazkur tadqiqotda ikkinchi tartibli Keli daraxtida Izing modeli uchun H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlarining mavjudligi va ularning xossalari o‘ rganildi.

Funksional tenglamalar — Gibbs o‘ lchovlarini tavsiflovchi tenglamalar sistemasi yechimlari orqali kuchsiz davriy va translyatsion-invariant o‘ lchovlar ajratib olindi.

Normal qism gruppalar — indeks $|A|=2$ bo‘ lgan hol alohida ko‘ rib chiqildi, chunki kuchsiz davriylik tushunchasi tanlangan normal qism gruppaga bog‘ liq.

Chegaralanganlik tahlili — topilgan yechimlarga mos Gibbs o‘ lchovlarining p -adik normasi hisoblanib, ularning chegaralanmaganligi isbotlandi.

Taqqoslash va solishtirish — yangi topilgan kuchsiz davriy o‘ lchovlar avvalgi translyatsion-invariant o‘ lchovlar bilan solishtirildi va ularning farqlari ko‘ rsatildi.

Metodologiya doirasida matematik fizika, p -adik analiz va ehtimollik nazariyasi vositalari uyg‘ unlashtirilib, qaralayotgan model uchun fazaviy o‘ tish yuz berishi ko‘ rsatib o‘ tildi.

Natijalar va ularning tahlili

1. Dastlabki tushunchalar. ***p -adik sonlar.*** p tub son bo‘ lsin. Ixtiyoriy noldan farqli $r \in \mathbb{Q}$ ratsional sonni

$$r = p^{v} \frac{a}{b}$$

ko‘ rinishda ifodalash mumkin, bu yerda $v \in \mathbb{Z}$, $a,b \in \mathbb{Z}$ va $EKUB(p,a)=1,\ EKUB(p,b)=1.$ Bunda $v = v_p(r)$ soni r ning p -adik tartibi (yoki p -adik bahosi) deb ataladi. Shuningdek, 0 soning p -adik tartibi sifatida 0 olinadi, ya’ni $v_p(0)=\infty.$ $\mathbb{Q}$ ratsional sonlar maydonida $|r|_p = p^{-v_p(r)}$ formula orqali aniqlangan $|\cdot|_p$ funksiya ultrametrik (noarximed) absolyut qiymat (yoki p -adik normasi) bo‘ lib, u quyidagi kuchli uchburchak tengsizligini qanoatlantiradi:

$$|x+y|_p \le \max\{|x|_p, |y|_p\}.$$

¤ ratsional sonlar maydonining $|\cdot|_p$ absolyut qiymatga (p-adik normaga) nisbatan to‘ ldirilishi natijasida p-adik sonlar maydoni deb ataluvchi ¤ $_p$ maydon hosil bo‘ ladi. Ma’lumki, har qanday $x \in$ ¤ $_p, x \neq 0,$ son yagona tarzda quyidagi ko‘ rinishda ifodalanadi:

$$x = \sum_{n=v_p(x)}^{\infty} x_n p^n, \qquad x_n \in \{0,1,\ldots,p-1\},$$

bunda $x_{v_p(x)} \neq 0.$ Bu yerda $v_p(x) \in \not\subset$ soni x ning p-adik bahosi deb ataladi.

Har qanday $a \in$ ¤ $_p$ va $r > 0$ uchun p-adik ochiq shar quyidagicha aniqlanadi:

$$B(a,r) = \{x \in \text{¤ }_p : |x-a|_p < r\}.$$

p-adik eksponenta funksiyasi esa quyidagi qator ko‘ rinishida aniqlanadi:

$$\exp_p(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

bu qator $p=2$ bo‘ lganda $x \in B(0, \frac{1}{2})$ sharda, $p \neq 2$ bo‘ lganda esa $x \in B(0,1)$ sharda yaqinlashadi.

1-Lemma [7,21]. Agar $x \in B\left(0, p^{-1/(p-1)}\right)$ bo‘ lsa, u holda quyidagi tengliklar o‘ rinli:

$$|\exp_p(x)|_p = 1, \qquad |\exp_p(x)-1|_p = |x|_p < 1.$$

F. Mukhamedov, O. Khakimovlar [14]-ishda p-adik normani hisoblashni yengillashtirish uchun yangi «o» belgisini kiritganlar. Bu belgi $\equiv (\mathrm{mod}\, p^k)$ yozuvini umumlashtiradi, lekin daraja k ni aniq ko‘ rsatmasdan ishlatiladi. Agar x p-adik son bo‘ lsa, unda $o[x]$ ifodasi p-adik normasi $p^{v_p(x)}$ dan qat’iy kichik p-adik sonni bildiradi, ya’ni

$$|o(x)|_p < |x|_p .$$

Misol. Agar $x = 1 - p + p^2$ bo‘ lsa, unda $o[1] = x - 1$ yoki $o[p] = x - 1 + p$ deb yozish mumkin. Demak, $o[\cdot]$ belgisi p-adik sonlar bilan hisoblashni soddalashtiruvchi qulay vosita hisoblanadi.

Quyidagi to‘ plamni aniqlaymiz:

$$E_p = \{x \in \text{¤ }_p : |x-1|_p < p^{-1/(p-1)}\}.$$

Shuningdek, p-adik butun sonlar va birlar quyidagicha aniqlanadi:

$$\not\subset_p = \{x \in \text{¤ }_p : |x|_p \le 1\}, \qquad \not\subset^*_p = \{x \in \text{¤ }_p : |x|_p = 1\}.$$

p-adik analiz va uning matematik fizika qo‘ llanishi haqida batafsil ma’lumot [20,21] manbalarda keltirilgan.

p-adik o‘ lchovlar. Faraz qilaylik (X, B) o‘ lchovli fazo bo‘ lsin, bu yerda B X ning qism to‘ plamlari algebrasi. $\mu : \mathrm{B} \to$ ¤ $_p$ funksiyasi *p-adik o‘ lchov* deyiladi, agar B dan olingan chekli sonli o‘ zaro kesishmaydigan to‘ plamlar $A_1, A_2, \ldots, A_n$ uchun (ya’ni $A_i \cap A_j = \varnothing$, $i \neq j$) quyidagi qo‘ shilish sharti bajarilsa:

$$\mu\left(\bigcup_{j=1}^{n} A_j\right) = \sum_{j=1}^{n} \mu(A_j).$$

p-adik o‘ lchov μ *ehtimollik o‘ lchovi* deyiladi, agar $\mu(X)=1.$

p-adik o‘ lchovlarning eng muhim xossalardan biri *chegaralangan*ligidir. μ p-adik o‘ lchov chegaralangan deyiladi, agar quyidagi shart o‘ rinli bo‘ lsa:

$$\sup\{|\mu(A)|_p : A\in \mathrm{B}\}<\infty.$$

Shuni ta’kidlash kerakki, p-adik ehtimollik o‘ lchovlari umumiy holda har doim ham chegaralangan bo‘ lavermaydi.

p-adik o‘ lchovlar haqida batafsil ma’lumot [6-20] manbalarda keltirilgan.

Keli daraxti. Γ^k Keli daraxti tartibi $k\geq 1$ bo‘ lgan, har bir uchdan aniq $(k+1)$ ta qirra chiquvchi cheksiz daraxtdir.

Γ^k Keli daraxti haqida batafsil ma’lumot [19] manbada keltirilgan.

Faraz qilaylik, Keli daraxtida ixtiyoriy uchni olib uni $x_0\in\Gamma^k$ deb fiksirlaylik. Endi quyidagi to‘ plamlarni aniqlaymiz ([19]):

$$W_n=\{x\in V: d(x,x_0)=n\},\qquad V_n=\bigcup_{m=0}^{n}W_m,\qquad L_n=\{\langle x,y\rangle\in L: x,y\in V_n\},$$

$$S(x)=\{y\in W_{n+1}: d(x,y)=1\},\qquad x\in V_n,$$

$$S_1(x)=\{y\in V: d(x,y)=1\},\qquad x_{\downarrow}=S_1(x),\quad S(x).$$

Izing modeli uchun p*-adik Gibbs o‘ lchovlari.* Biz ushbu maqolada Keli daraxtida p-adik Izing modelini ko‘ rib chiqamiz. Faraz qilaylik $\Phi=\{-1,1\}$ bo‘ lsin. Agar $A\subset V$ bo‘ lsa, u holda σ konfiguratsiya quyidagicha aniqlanadi:

$$x\in A \text{ a } \sigma(x)\in\Phi.$$

A to‘ plamdagi barcha konfiguratsiyalar to‘ plami $\Omega_A=\Phi^A$ bilan belgilanadi, xuddi shuningdek $\Omega_V:=\Omega$.

Izing modeli uchun V_n dagi p-adik Hamiltonian, ya’ni $H_n:\Omega_{V_n}\to ¤_p$ funksiya quyidagicha beriladi:

$$H_n(\sigma_n)=J\sum_{\langle x,y\rangle\in L_n}\sigma(x)\sigma(y)$$

bu yerda $J,\alpha\in B\left(0,p^{-1/(p-1)}\right),\quad \{0\}$.

Faraz qilaylik $h: x\in V \text{ a } \hat{h}_x\in ¤_p$ funksiya berilgan bo‘ lsin va Ω_{V_n} ustidagi p-adik ehtimollik o‘ lchovi $\mu_h^{(n)}$ quyidagicha aniqlanadi:

$$\mu_h^{(n)}(\sigma_n)=\frac{1}{Z_n^{(\hat{h})}}\exp_p\{H_n(\sigma_n)\}\prod_{x\in W_n}\hat{h}_x^{\sigma(x)},\qquad n=1,2,\ldots, \quad (2)$$

bu yerda $Z_n^{(\hat{h})}$ normallovchi bo‘ luvchi bo‘ lib,

$$Z_n^{(\hat{h})}=\sum_{\sigma_n(x)\in\Omega_{V_n}}\exp_p\{H_n(\sigma_n)\}\prod_{x\in W_n}\hat{h}_x^{\sigma(x)}. \quad (3)$$

$\mu_h^{(n)}$ p-adik ehtimollik o‘ lchovi *muvofiq* deyiladi, agar barcha $n\in ¥$ va $\sigma_{n-1}\in\Omega_{V_{n-1}}$ uchun quyidagi shart bajarilsa:

$$\sum_{\varphi^{(n)}\in\Omega_{W_n}} \mu_h^{(n)}\left(\sigma_{n-1}\vee\varphi^{(n)}\right)=\mu_h^{(n-1)}(\sigma_{n-1}). \quad (4)$$

Kolmogorov teoremasining p-adik analogiga ko‘ ra, Ω to‘ plamida yagona μ_h limit p-adik o‘ lchov mavjud bo‘ lib, u $n\in ¥$ barcha natural sonlar uchun quyidagi shartni qanoatlantiradi:

$$\mu_h\left(\{\sigma|_{V_n}\equiv\sigma_n\}\right)=\mu_h^{(n)}(\sigma_n).$$

(2) formula orqali aniqlangan bunday μ_h o‘ lchov p*-adik umumlashgan Gibbs o‘ lchovi* deb ataladi. Biz shuni alohida ta’kidlaymizki, agar barcha $x\in V$ lar uchun $\hat{h}_x\in\varepsilon_p$ bo‘ lsa, u holda bu o‘ lchov p-adik Gibbs o‘ lchovi deb ataladi. Ravshanki, har bir p-adik Gibbs o‘ lchovi p-adik umumlashgan Gibbs o‘ lchovi deyiladi, shuningdek, shunday p-adik umumlashgan Gibbs o‘ lchovlari mavjudki, bu o‘ lchovlar p-adik o‘ lchov bo‘ la olmaydi.

Qaralayotgan model uchun fazaviy o‘ tishlar yoki kvazi fazaviy o‘ tishlar μ_h o‘ lchovining chegaralangan yoki chegaralanmaganligiga qarab yuzaga keladi.

Agar qaralayotgan model uchun kamida ikkita turli μ_h va μ_s p-adik umumlashgan Gibbs o‘ lchovlari mavjud bo‘ lib, ulardan biri chegaralangan, ikkinchisi esa chegaralanmagan bo‘ lsa, u holda qaralayotgan model uchun fazaviy o‘ tish yuz beradi; aksincha, barcha topilgan p-adik umumlashgan Gibbs o‘ lchovlari yoki chegaralangan yoki chegaralanmagan bo‘ lsa, qaralayotgan model uchun kvazi fazaviy o‘ tishlar yuzaga keladi [10].

Quyidagi tasdiqda $\{\mu_h^{(n)}\}_{n\geq1}$ ehtimollik taqsimotlari ketma-ketligining muvofiq bo‘ lishini ta’minlovchi h funksiyaga qo‘ yiladigan shart keltiriladi.

1-Teorema [16]. (2) formula orqali aniqlangan $\{\mu_h^{(n)}\}_{n\geq1}$ p-adik ehtimollik taqsimotlari ketma-ketligi *muvofiq* bo‘ lishi uchun quyidagi tenglikning bajarilishi zarur va yetarli:

$$\hat{h}_x=\prod_{y\in S(x)}\frac{\theta\hat{h}_y+1}{\hat{h}_y+\theta}, \quad (5)$$

bu yerda $\theta=\exp_p(2J), \qquad \hat{h}_x=\exp_p(2h_x).$

Demak, ushbu teorema asosida p-adik Gibbs o‘ lchovlarini tavsiflash masalasi (5) funksional tenglama yechimlarini topishga keltiriladi.

H_A*-kuchsiz davriy p-adik umumlashgan Gibbs o‘ lchovi.* Faraz qilaylik G_k^* gruppa G_k ning indeks $r\geq1$ bo‘ lgan normal qism gruppasi bo‘ lsin va $G_k/G_k^*=\{H_0,H_1,\ldots,H_{r-1}\}$ faktor gruppa bo‘ lsin ([19]).

1-ta’rif. Barcha $x\in H_i$ lar uchun $h_x=h_i$ tenglik o‘ rinli bo‘ ladigan $h=\{h_x,x\in G_k\}$ miqdorlar to‘ plami G_k^*-kuchsiz davriy deb ataladi. G_k-davriy miqdorlar translyatsion-invariant deb ataladi.

2-ta’rif. Ixtiyoriy $x\in H_i$ va $x_\downarrow\in H_j$ lar uchun $h_x=h_{ij}$ o‘ rinli bo‘ ladigan $h=\{h_x,x\in G_k\}$ miqdorlar to‘ plami G_k^*-davriy deb ataladi.

3-ta’rif. G_k^*-(kuchsiz) davriy h ga mos μ umumlashgan p-adik Gibbs o‘ lchovi G_k^*-(kuchsiz) davriy umumlashgan p-adik Gibbs o‘ lchovi deb ataladi. G_k-davriy o‘ lchov translyatsion-invariant o‘ lchov deyiladi.

Shuni ta’kidlash kerakki, G_k gruppaning indeks ikki bo‘ lgan har qanday normal bo‘ luvchisi quyidagi ko‘ rinishga ega ([19]):

$$H_A=\{x\in G_k:\sum_{i\in A}\omega_x(a_i)-\text{juft son}\},$$

bu yerda A to‘plam N_k ning xos qism to‘plami, ya’ni $\varnothing \neq A \subseteq N_k = \{1,2,3,\ldots,k+1\}$ va $\omega_x(a_i)-$ $x \in G_k$ so‘zidagi a_i harflar sonini bildiradi. H_A gruppa G_k gruppaning normal qism gruppasi bo‘lib, kuchsiz davriylik tushunchasi tanlangan H_A normal qism gruppaga bog‘liq. Agar $|A|=k+1$ bo‘lsa, ya’ni $A=N_k$, u holda kuchsiz davriylik tushunchasi $G_k^{(2)}$-davriylik tushunchasi bilan ustma-ust tushib qoladi ($|A|$ - A to‘plamning quvvati). Shuning uchun biz $A \neq N_k$ bo‘lgan $A \subset N_k$ holni qaraymiz. U holda H_A-kuchsiz davriy h_x lar to‘plami quyidagi ko‘rinishda bo‘ladi:

$$h_x = \begin{cases} h_1, \text{ agar } x \in H_1,\ x_{\downarrow} \in H_1, \\ h_2, \text{ agar } x \in H_1,\ x_{\downarrow} \in H_2, \\ h_3, \text{ agar } x \in H_2,\ x_{\downarrow} \in H_1, \\ h_4, \text{ agar } x \in H_2,\ x_{\downarrow} \in H_2, \end{cases}$$

bu yerda $H_1 := H_A$ va $H_2 := G_k \setminus H_A$.

$|A|=1$ holda indeksi ikki bo‘lgan normal qism gruppa quyidagi ko‘rinishga ega bo‘ladi:

$$H_{A_1} = \{x \in G_k : \omega_x(a_i) \equiv 0 \pmod 2\}, \qquad i \in \{1,2,3\},$$

ya’ni $x \in G_k$ so‘zdagi a_i harflar soni juft.

Ushbu maqolada ikkinchi tartibli Keli daraxtida Izing modeli uchun A to‘plamning quvvati ikkiga teng bo‘lgan hol qaraladi, ya’ni $|A|=2$. Bu holda indeksi ikki bo‘lgan normal qism gruppa quyidagi ko‘rinishga ega bo‘ladi:

$$H_{A_2} = \{x \in G_k : (\omega_x(a_i) + \omega_x(a_j)) \equiv 0 \pmod 2\}, \qquad i,j \in \{1,2,3\}, i \neq j,$$

ya’ni $x \in G_k$ so‘zida a_i va a_j harflarining umumiy soni juft. Shunday qilib, (5) funksional tenglama quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} \hat{h}_1 = \left(\dfrac{\theta\hat{h}_3+1}{\theta+\hat{h}_3}\right)^2 \\ \hat{h}_2 = \dfrac{\theta\hat{h}_1+1}{\theta+\hat{h}_1}\cdot\dfrac{\theta\hat{h}_3+1}{\theta+\hat{h}_3} \\ \hat{h}_3 = \dfrac{\theta\hat{h}_2+1}{\theta+\hat{h}_2}\cdot\dfrac{\theta\hat{h}_4+1}{\theta+\hat{h}_4} \\ \hat{h}_4 = \left(\dfrac{\theta\hat{h}_2+1}{\theta+\hat{h}_2}\right)^2 \end{cases} \tag{6}$$

(6) tenglamalar sistemasini $I_1 = \{(h_1,h_2,h_3,h_4) \in ¤_p : h_1 = h_2 = h_3 = h_4\}$ invariant to‘plamdagi yechimi (5) tenglamaning translatsion-invariant yechimi bilan ustma-ust tushadi. Shu sababli bu holni qaramaymiz. Endi $I_2 = \{(h_1,h_2,h_3,h_4) \in ¤_p : h_1 = h_4,\ h_1 = h_4\}$ to‘plamdagi yechimlarini qaraylik. Bu holda (6) tenglamalar sistemasi quyidagiga ekvivalent:

$$\begin{cases} \hat{h}_1 = \left(\dfrac{\theta \hat{h}_2 + 1}{\theta + \hat{h}_2} \right)^2 \\ \hat{h}_2 = \dfrac{\theta \hat{h}_1 + 1}{\theta + \hat{h}_1} \cdot \dfrac{\theta \hat{h}_2 + 1}{\theta + \hat{h}_2} \end{cases} \quad (7)$$

$h_1 = x,\ h_2 = y$ belgilashdan so‘ ng (7) tenglamalar sistemasini quyidagicha yozishamiz:

$$\begin{cases} x = \left(\dfrac{\theta y + 1}{\theta + y} \right)^2 \\ y = \dfrac{\theta x + 1}{\theta + x} \cdot \dfrac{\theta y + 1}{\theta + y} \end{cases} \quad (8)$$

(8) tenglamalar sistemasining 1-tenglamasini 2-siga qo‘ yib, uni ko‘ paytuvchiga ajratsak quyidagini hosil qilamiz:

$$\theta(y-1)(y+1)(y^2 - (\theta^2 - 2\theta - 1)y + 1) = 0 \quad (9)$$

Bu (9) tenglamani ildizlari quyidagicha ekanligini oson topish mumkin:

$$y_1 = 1,\ y_2 = -1,\ y_{3,4} = \frac{\theta^2 - 2\theta - 1 \pm (\theta - 1)\sqrt{\theta^2 - 2\theta - 3}}{2}$$

Topilgan yechimlardan foydalanib, quyidagiarni topamiz:

$$x_1 = 1,\ x_2 = 1,\ x_{3,4} = \frac{\theta^2 - 2\theta - 1 \pm (\theta - 1)\sqrt{\theta^2 - 2\theta - 3}}{2}$$

Demak, (6) tenglamalar sistemasining yechimlari quyidagicha:

$$\begin{cases} 1, \text{ agar } x \in H_1,\ x_\downarrow \in H_1, \\ 1, \text{ agar } x \in H_1,\ x_\downarrow \in H_2, \\ 1, \text{ agar } x \in H_2,\ x_\downarrow \in H_1, \\ 1, \text{ agar } x \in H_2,\ x_\downarrow \in H_2, \end{cases} \text{ va } \begin{cases} -1, \text{ agar } x \in H_1,\ x_\downarrow \in H_1, \\ 1, \text{ agar } x \in H_1,\ x_\downarrow \in H_2, \\ 1, \text{ agar } x \in H_2,\ x_\downarrow \in H_1, \\ -1, \text{ agar } x \in H_2,\ x_\downarrow \in H_2, \end{cases} \quad (10) \begin{cases} y_3, \text{ agar } x \in H_1,\ x_\downarrow \in H_1, \\ y_3, \text{ agar } x \in H_1,\ x_\downarrow \in H_2, \\ y_3, \text{ agar } x \in H_2,\ x_\downarrow \in H_1, \\ y_3, \text{ agar } x \in H_2,\ x_\downarrow \in H_2, \end{cases} \text{ va}$$

$$\begin{cases} y_4, \text{ agar } x \in H_1,\ x_\downarrow \in H_1, \\ y_4, \text{ agar } x \in H_1,\ x_\downarrow \in H_2, \\ y_4, \text{ agar } x \in H_2,\ x_\downarrow \in H_1, \\ y_4, \text{ agar } x \in H_2,\ x_\downarrow \in H_2, \end{cases}$$

Biz topgan kuchsiz davriy yechimlarimizning uchtasi translatsion-invariant yechimlar bilan ustma-ust tushar ekan. x_2 va y_2 lar orqali hosil qilingan yechimlar esa translyatsion-invariant bo‘ lmagan kuchsiz davriy yechimlardir.

2-teorema. Faraz qilaylik, $|A| = 2$ bo‘ lsin. U holda har qanday p tun son uchun ikkinchi tartibli Keli daraxtidagi Izing modeli uchun kamida bitta translyatsion-invariant bo‘ lmagan -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovi mavjud.

Isbot. (10) orqali berilgan yechim (6) sistemaning H_{A_2} -kuchsiz davriy (translyatsion-invariant bo‘ lmagan) yechimi hisoblanadi. 3-Ta’rifga ko‘ ra, bu yechim H_{A_2} -kuchsiz davriy (translyatsion-invariant bo‘ lmagan) p -adik umumlashgan Gibbs o‘ lchovining mavjudligini bildiradi. Isbot yakunlandi.

1-Izoh. [18] maqolada biz $|A| = 1$ holni qarab chiqganmiz. Bu holda I_2 to‘ plamda quyidagi yechim topilgan.

$$\begin{cases} 1, \text{ agar } x\in H_1,\ x_\downarrow \in H_1, \\ -1, \text{ agar } x\in H_1,\ x_\downarrow \in H_2, \\ -1, \text{ agar } x\in H_2,\ x_\downarrow \in H_1, \\ 1, \text{ agar } x\in H_2,\ x_\downarrow \in H_2, \end{cases} \quad (11)$$

Ushbu ishda esa biz $|A|=2$ bo‘ lgan holni qaradik. Biz shuni alohida ta’kidlashimiz kerakki, (11) yechim va (10) yechimlar farqlidir. Shuning uchun, 3-Ta’rifga ko‘ ra bu yechimlar aniqlaydigan H_{A_1} - va H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari ham turlidir.

2-Izoh.

Gibbs o‘ lchovlari G_k ning H_A normal qism gruppasini tanlashga bog‘ liq ekanligini alohida ta’kidlaymiz. Bu esa |A| ning turli qiymatlarini tanlash H_A ning turli qism gruppalarini tanlashga mos kelishini anglatadi. O‘ z navbatida, bu turli tenglamalar sistemalarining hosil bo‘ lishiga va, natijada, H_A -kuchsiz davriy Gibbs o‘ lchovlarining turli sinflari yuzaga kelishiga olib keladi. Kuchsiz davriy Gibbs o‘ lchovlari to‘ plami $G_k^{(2)}$ -davriy Gibbs o‘ lchovlari to‘ plamini ham, xususan, translyatsion-invariant Gibbs o‘ lchovlarini ham o‘ z ichiga oladi.

Ikkinchi tartibli Keli daraxtida Izing modeli uchun translyatsion-invariant p -adik umumlashgan Gibbs o‘ lchovlari [3] ishida o‘ rganilgan. Xususan, $|A|=1$ bo‘ lgan holga mos keluvchi H_{A_1} -kuchsiz davriy Gibbs o‘ lchovlari [18] maqolada tadqiq etilgan. Bu ishda biz ikkinchi tartibli Keli daraxtida Izing modeli uchun oltita chegaralanmagan H_{A_1} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovining mavjudligini isbotlaganmiz.

Agar $|A|=k+1$ bo‘ lsa, u holda $H_{A_{k+1}}$ -kuchsiz davriy Gibbs o‘ lchovi $G_k^{(2)}$ -davriy Gibbs o‘ lchovi bilan ustma-ust tushadi. [3] ishida O. Hakimov ikkinchi tartibli Keli daraxtida Izing modeli uchun $G_k^{(2)}$ -davriy Gibbs o‘ lchovlarini o‘ rgangan.

Endi yuqorida topilgan H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlarining chegaralanganligi haqidagi quyidagi teoremani keltiramiz:

3-Teorema. Faraz qilaylik $p>3$ bo‘ lsin. Ikkinchi tartibli Keli daraxtidagi Izing modeli uchun oltita chegaralanmagan H_{A_1} -kuchsiz davriy va kamida bitta chegaralanmagan H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari mavjud.

Isbot. Faraz qilaylik $p>3$ tub son bo‘ lsin. [18] maqolada biz oltita chegaralanmagan H_{A_1} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari mavjud ekanligni isbotlagan edik. Endi biz 2-Teoremada topilgan H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovining chegaralanmaganligini isbotlaylik. (2) formuladan foydalanib, biz quyidagiga ega bo‘ lamiz:

$$\left|\mu_h^{(n)}(\sigma_n)\right|_p = \left|\frac{1}{Z_n^{(\hat{h})}}\right|_p \left|\exp_p\{H_n(\sigma_n)\}\right|_p \prod_{x\in W_n}\left|\hat{h}_x^{\sigma(x)}\right|_p$$

1-Lemmaga ko‘ ra,

$$|\exp_p\{H_n(\sigma_n)\}|_p = 1.$$

Shuningdek, $|1|_p = |-1|_p = 1$ bo‘ lgani uchun quyidagini hosil qilamiz:

$$\left|\exp_p\{H_n(\sigma_n)\}\right|_p \prod_{x\in W_n}\left|\hat{h}_x^{\sigma(x)}\right|_p = 1$$

[18] maqoladagi 2.21-Lemmadan va (2.42) - (2.44) tengliklardan foydalanib, quyidagiga ega bo‘ lamiz:

$$Z_n^{(h)} = A_{n-1,h} Z_{n-1,h} = \prod_{k=1}^{n-1} A_{k,h} Z_{1,h} =$$

$$= \prod_{k=1}^{n-1} \left(\frac{(\theta h_1 + 1)(\theta + h_1)}{\theta h_1} \right)^{\frac{\alpha_k + \delta_k}{2}} \left(\frac{(\theta h_2 + 1)(\theta + h_2)}{\theta h_2} \right)^{\frac{\beta_k + \gamma_k}{2}} Z_{1,h},$$

bu yerda $\alpha_n, \beta_n, \gamma_n, \delta_n$ lar mos ravishda W_n sferadagi h_1, h_2, h_3, h_4 miqdorlar soni. $h_1 = 1,\ h_2 = -1$ va $\theta \in \varepsilon_p$ ekanligidan

$$\left| \frac{(\theta h_1 + 1)(\theta + h_1)}{\theta h_1} \right|_p = |\theta + 1|_p^2 = 1,$$

$$\left| \frac{(\theta h_2 + 1)(\theta + h_2)}{\theta h_2} \right|_p = |\theta - 1|_p^2 \le \frac{1}{p}.$$

Ushbu natijalar va [18] maqoladagi 2.15-Lemmadan foydalanib, o‘ lchovning p -adik normasi uchun quyidagiga ega bo‘ lamiz:

$$\left|\mu_h^{(n)}(\sigma_n)\right|_p = \left|\frac{1}{Z_n^{(\hat{h})}}\right|_p \left|\exp_p\{H_n(\sigma_n)\}\right|_p \prod_{x \in W_n} \left|\hat{h}_x^{\sigma(x)}\right|_p =$$

$$= \left|\frac{1}{Z_n^{(\hat{h})}}\right|_p = \prod_{k=1}^{n-1} \left|\frac{\theta h_2}{(\theta h_2 + 1)(\theta + h_2)}\right|_p^{\frac{\beta_k + \gamma_k}{2}} \frac{1}{|Z_{1,h}|_p} \ge p^{2^{n-2} - 0.5} \cdot \frac{1}{|Z_{1,h}|_p}$$

$0 < |Z_1, _h|_p \le 1$ ekanligini tekshirish oson. Endi o‘ lchov normasining limitni hisoblasak,

$$\lim_{n\to\infty} \left|\mu_h^{(n)}(\sigma_n)\right|_p = \lim_{n\to\infty} \left|\frac{1}{Z_n^{(\hat{h})}}\right|_p \left|\exp_p\{H_n(\sigma_n)\}\right|_p \prod_{x \in W_n} \left|\hat{h}_x^{\sigma(x)}\right|_p \ge \lim_{n\to\infty} p^{2^{n-2} - 0.5} \cdot \frac{1}{|Z_{1,h}|_p} = \infty.$$

Oxirgi olingan tenglik (10) yechimga mos keluvchi o‘ lchovning chegaralangan ekanligini ko‘ rsatib turibdi. Bu esa teorema isboti nihoyasiga yetganini anglatadi.

3-Izoh. [18] maqolada $p \equiv 1 (\mathrm{mod}\, 4)$ bo‘ lganda oltita chegaralanmagan Gibbs o‘ lchovi mavjud ekanligi isbotlangan. [3] maqolada esa aynan shu shart ostida bitta chegaralangan va ikkita chegaralanmagan translyatsion-invariant p -adik umumlashgan Gibbs o‘ lchovi mavjud ekanligi ko‘ rsatilgan. 3-Teoremada topilgan o‘ lchov esa har qanday tub sonda mavjud. Demak, har qanday tub son uchun qaralayotgan modelda faza almashishi yuz beradi. Shuni ta’kidlash joizki, Izing modeli uchun faza almashishi yuzaga kelish sharti [18] dagi shart bilan bir xil bo‘ ladi.

Xulosa

Ushbu maqolada ikkinchi tartibli Keli daraxtidagi Izing modeli uchun kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovlari tadqiq qilingan, bunda A to‘ plamning quvvati ikki bo‘ lgan holdagi normal qism gruppaga mos keluvchi H_{A_2} -kuchsiz davriylik o‘ rganilgan. Dastavval funksional tenglamaning kuchsiz davriy h_x yechimlarni aniqlandi. Bu yechimlarga mos p -adik umumlashgan Gibbs o‘ lchovlarining uchtasi [3] ishdagi translyatsion-invariant p -adik umumlashgan Gibbs o‘ lchovlari bilan ustma-ust tushdi. Qolgan bitta yechimga mos keluvchi p -adik umumlashgan Gibbs o‘ lchovi tranlyatsion-invariant bo‘ lmagan H_{A_2} -kuchsiz davriy p -adik umumlashgan Gibbs o‘ lchovi bo‘ lishi aniqlandi. Bundan tashqari topilgan yangi o‘ lchovning p -adik normasini hisoblash orqali uning chegaralanmaganligi isbotlandi.

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